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LECTURES

ON

NATURAL AND EXPERIMENTAL

Philoſophy,

CONSIDERED IN ITS PRESENT STATE OF
IMPROVEMENT.

DESCRIBING, IN A FAMILIAR AND EASY MANNER,

THE PRINCIPAL PHENOMENA OF NATURE;

AND SHOWING

THAT THEY ALL CO-OPERATE IN DISPLAYING THE GOODNESS,
WISDOM, AND POWER OF GOD.

BY THE LATE

GEORGE ADAMS,

MATHEMATICAL INSTRUMENT MAKER TO HIS MAJESTY, &c.

IN FOUR VOLUMES.

Illuſtrated with forty three large copperplates, elegantly engraved.

THIS AMERICAN EDITION,

PRINTED FROM THE LAST LONDON EDITION, EDITED BY
WILLIAM JONES, MATHEMATICAL INSTRUMENT MAKER,
IS CAREFULLY REVISED AND CORRECTED, BY

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LECTURES
ON
NATURAL PHILOSOPHY.

OF THE
NATURE AND PROPERTIES
OF
MATTER.

LECTURE XXIV.

LET us for a moment compare this universe to a palace erected by a Divine Architect; and the unphilosophical spectator to a foreigner, who only sees the external part of the building. From so superficial a view, it is evident he can obtain but a very unsatisfactory idea of the skill and contrivance of the Great Designer. To form a more accurate conception, to perceive its beauties, to discover its conveniences, it is necessary to enter the building, to view each apartment separately, to compare the convenience of every room singly, with the symmetry and elegance of the whole.

In the same manner the beauties of nature strike our view: we find curiosity allured by a variety of objects,—animals, vegetables, minerals, air, water, and fire: all put on different appearances to please, assist, or astonish us; but, in order to come at a knowledge of their nature, we must approach them closely; we must consider each as divested of all its accidental qualities of figure and colour; and turn from nature's external ornaments, to view her external simplicity.*

The question you would probably first ask in this view of things, is, What is matter, that substance which serves as the foundation of all bodies, and which, while it is itself unseen, gives existence to all other qualities that strike the senses?

As we cannot penetrate the mist which hides so much from human eyes, nor follow composites to their different and original principles; we must content ourselves with the knowledge originating in our external sensations. To avoid every thing equivocal, we shall consider *matter as that substance which, by its different modifications, become the object of our five senses, and hence also of physics.* I mean here not only what our senses really perceive, but also what by their nature they are capable of perceiving. Thus, for instance, the animalcules in infusions are objects of sight, although we are obliged to use a microscope to discover them; it is the same with the objects of the touch, although their impressions may be too feeble for us to perceive; it is the same with respect to hearing, taste, and smell: in a word, these object are withdrawn from our sensations by their minuteness, not by their nature; and we are persuaded that they posses the qualities of matter, though our senses are not acute enough to be affected by them. The invisible fluids, air and fire, are palpable, and thus the immediate objects of our senses; but there are other fluids which are not perceived by our senses, such as the magnetic fluid, the fluid causing gravity, and such others as reason may conceive to be productive of physical phenomena; and which may therefore be reckoned objects of the senses. Every thing which is by its nature an object of the five senses, although on account of its minuteness it may not affect them, is to be considered as a part of the physical world, and consequently as matter.†

---

* *Goldsmith's* Survey of Philosophy, vol. i. p. 17.

† De *Luc's* Lettres Physiques & Morales sur l'Histoire de la Terre, vol. i. p. 181 to 204, &c.

An investigation into the properties of matter is not an arbitrary affair, neither is the word, property, to be considered in a vague and indeterminate way. This word, in its philosophical sense, necessarily includes that of a primitive or primary cause, independent of every thing else but of a clear idea of the substance to which it is attributed. Thus a research into properties, is a research into primary causes; I do not say *first*, because this term belongs only to the Supreme Being, the First Cause of all things.

It is necessary to give you the sense in which I here use the word cause, that it may not be confounded with other senses of the same word. Whatever produces an effect is generally called a cause, but such causes may at the same time be effects of other causes, and thus not primary. But in physics, a primary cause is that from which originate, physically, all the physical effects occasioned by, or resulting from it. Whenever, therefore, what is called a cause, does not include clearly this idea of a primary cause, it ought not to be denominated the property of a substance in the abovementioned sense; for it is, or may be, a modification of the substance, that is to say, the effect of a cause, or of a chain of causes: in a word, it is nothing more than a phenomenon, and shows that we have not attained to the first link of the chain of the effects whose origin we seek.

The investigation of properties in the sense here used, consists in examining the phenomena belonging to the substance in question; and ascending from one effect to another, till you arrive at something which is a cause, agreeably to the foregoing definition; that is, which evidently and clearly arises from the idea of the substance considered, and which cannot be separated from it without annihilating the substance, or which is inseparable therefrom by any human conception.

It has been usual to reckon among the properties of matter, *impenetrability*, *extension*, *figure*, *divisibility*, *hardness*, *inertia*, *motion*, *attraction*, and *repulsion*. I know of no other which have a right to be examined, as effecting the senses, which, as I have often observed, can alone constitute the science of physics. Let us then compare the ideas of these properties with the preceding definition, and we will soon see, that the three last mentioned must be excluded from the class of properties so defined, and cannot, with strict propriety, be termed the characters of materiality.

Impenetrability, considered as expressing, that two particles of what constitutes matter, whatever it may be, cannot exist at the same time in the same place, *i. e.* so long as one retains its place it must necessarily exclude the other,—is undoubtedly a physical axiom: the contrary position would be a contradiction.

The physical world would vanish from the eyes of the understanding, and we should no longer have any notion thereof, if impenetrability, as here considered, were not an essential property of matter. There could be no existence of this substance, unless the idea of impenetrability immediately arose from it. As it is neither the existence of the substance, nor the cause of its existence, that we are considering, but its properties, there can be no doubt of this first principle.

For as soon as matter exists, it is impenetrable; this is the first thing which constitutes its existence as matter, that is, as a part of the physical world, an object of the five senses.

Every thing therefore, which is deduced from the impenetrability of matter, will be the result of what I have called a primary cause, and is subordinate to nothing else but the cause of the existence of matter.

Extension is another property of the same kind; it is indeed included in the idea of impenetrability, when it is considered with respect to space, namely, that two particles of matter cannot exist at the same time in the same place; they are therefore extended, that is, they occupy a certain portion of space. Extension is therefore an essentially constituent property of matter.

Figure is another property that necessarily flows from the preceding definition, or rather from the existence of matter itself. Whatever is material must have figure or shape; every finite extension is terminated or comprehended under some figure.

So far there cannot be two ways of thinking concerning the properties of matter, unless some other species of being is meant by the word matter ; in which case it would not belong to our physical world, that which affects our senses. But the remaining properties require a closer examination.

Divisibility. If this expression be confined to the possibility of conceiving that every atom of matter may be indefinitely divided by a sufficient power ; that is to say, that when considered as extended, a right and a left may be always separated by the mind, and by a sufficient power could be continually separated, it necessarily flows from the idea of matter as here defined.

Hardness. This property must be entered into more fully, as at first sight it seems contrary to the divisibility of matter ; but here you are to consider, that the divisibility of matter does not include the idea of an actual division. The atoms or first element of matter, that is, the smallest actual divisions, had an undivided (an indivisible) extension at some period of the physical world.

I do not here speak of the origin of matter ; but the existence thereof being once admitted, it must necessarily possess indivisible particles. An actual infinite division is an expression without any sense or meaning, it is contrary to the very existence of matter ; if then, in order to avoid absurdity, it be necessary that the idea of matter should contain that of indivisible particles, we must admit that there is a minimum, below which, though bodies may be capable of being reduced, there is no power in the physical world by which they can be divided or reduced. That is, no shock or impact that takes place in the physical universe can break or subdivide these elements ; and this is what I here mean by hardness, a quality easily understood, and which implies no other cause than the existence of matter. I therefore admit it without repugnance as a hypothetical property, which is rendered still more probable as it connects itself with the phenomena.

Thus then I have pointed out to you three properties which are essential to matter as above defined, and without which you can form no idea thereof. 1. Impenetrability, whereby it excludes other matter from the space it occupies. 2. Whatever is material must have size and extension, and yet the largest ultimate particle of matter is in all probability incomparably smaller than any point philosophers can form a distinct idea of, or than they can have any real occasion to suppose subservient to human researches in nature. 3. Whatever is material must have figure or shape; if this were not a property of every even the smallest unit of matter, no collection of them could acquire it by combination or cohesion. 4. Divisibility, when considered, not as an assertion of the actual possibility of dividing any parcel of matter without end, but as the possibility of conceiving extension as continually divisible into lesser and lesser extensions: a unit of nature is not composed of mathematical points cohering together by attraction, or any power a name can be invented for; a mathematical point is a nothing, a non-entity, and an infinite number of such can never form one atom of matter. 5. Hardness, or the indivisibility of atoms or first elements of matter: one atom may be divided from another, but no atom can be divided from itself. These properties are so essential to matter, that they may be termed the characters of materiality, as they are each of them inseparable from any possible conception human creatures can form of the particles of the material or physical universe.

Before I proceed to examine the other properties that I heretofore mentioned to you, it will be proper to guard you against the mistakes that some have fallen into in treating on the properties already explained. Thus Mr. *Locke*, to show that solidity was a property of water, using the word solidity as synonymous with impenetrability, mentions the Florentine experiment; but he has certainly applied this experiment to an improper purpose, and so far from clearing the idea he meant to explain, it renders it more confused and perplexed.

The experiment is as follows: a hollow globe of gold was filled with water and exactly closed; the globe thus filled being pressed with the extreme force of screws, the water made its way through the pores of that close and compact metal, and appeared upon the surface in little drops like those of dew, before the sides of the globe could be made to yield to the violent compression of the screws that squeezed it. This experiment may be used to show the porosity of gold, or the great force necessary to compress water, but by no means to demonstrate the impenetrability of water. Must we have granted, that water was not impenetrable or solid, if the globe had yielded while all the water was in it? And yet if the globe had been filled with air, it might have been pressed inward, though none of the air had escaped; and that not because the particles of air are not impenetrable, but because it is a more compressible fluid, so that the particles may be squeezed closer together, and the same quantity thus be made to occupy a smaller space.

An optical illusion has been introduced by some, to prove that there may be extension without solidity. By an image formed at a certain distance, between a concave mirror and any one looking into it, extension and form become an object of sense, where there is no sensible resistance; but this does not prove that an image is formed in empty space, or where there is no matter; for as the senses can be affected only by matter, they infallibly determine where it is, so that we are as certain matter exists where we see any thing, though we cannot feel it, as we should be certain there was matter where we felt it, though we could not see it.

It is here necessary to say something concerning space, of which some ideas have been entertained very contrary to sound physics. The only positive idea that can be applied to space is extension; but no idea should be applied to any subject, which the subject itself does not impress.

Matter forces upon our senses the idea or image of its dimensions or extension: but it is a kind of philosophical felony to steal an image which nature gives us, and invest a subject with it that never excited any idea in us, and consequently has no existence to us; we thus create nothing into a being, by applying ideas to it which we derive from something.

Space is only one of those ideas that are excited in the mind by matter, and the power in the mind of abstracting from its subject, just as we can image a colour to ourselves, without connecting in our apprehension a subject with it wherein in exists; a little more of the same metaphysics which can prove that nothing is extended, will prove that space is purple. It is a very ingenious contrivance in philosophers to render *nothing* a subject of inquiry and conception, by dressing it in a suit of clothes borrowed from *something*, and then gravely telling us, that this nothing is the form of God.*

In the same manner, much false reasoning has been introduced in treating of the infinite divisibility of matter; and the truth of this hypothesis is said to rest on mathematical demonstration. The arguments drawn from the mathematics in favour of this question, demonstrate the possible infinite divisibility of that body, which is the object of such investigation; but there is an essential distinction between body mathematical and body physical.

From not attending to this distinction, many philosophers of the first eminence have deceived themselves, and asserted, that all matter was as really for ever divisible, as were those ideal lines which the human imagination could create. Body mathematical is clothed only with two forms, extension and figure. These alone are the attributes contemplated by the mathematician.

Body physical, besides these, possesses other properties, with which it is always habited, and by which it becomes the object of philosophical investigation. Body mathematical has few properties, and those of the most precise nature, whilst the properties of body physical are complicated and manifold. In no part of the physical universe can the existence be discovered of that body the mathematician chooses to contemplate; gifted only with extension and figure, it is purely ideal, the figment of mental abstraction.

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* See Dr. *Wilson's* Observations on Principles and Moving Powers assumed by the present System.

Since then all mathematical reasoning concerning body regards only body mathematical, with what propriety can it be thought, that physical body is ever divisible; because that body to which the mind can at pleasure give indefinite degrees of magnitude or divisibility, is found to be so divisible? How far soever matter is divisible in its own nature, we can conceive no idea of its composition, but as it consists of units. To assert, that matter is ever compounded, and ever divisible, is glaringly absurd. All substance is measured either by number or weight; number is made up of units, and weight is no practical measure, till some elements are fixed upon, as the units of which all other weights are composed: we are therefore under the necessity of supposing all matter as made up of atoms,* or units. If it were the nature of body to be as multiple in all its parts as in the sensible whole, there would be nothing but number and plurality without limitation, and without ever finding the thing which is meant to be numbered and divided. The men, who assert the infinite divisibility of matter, should also have remembered, that infinity is an inexhaustible fund, and can never be completed; and that how capable soever matter may be of such division, it can never be effected. So that you must at last necessarily conclude, agreeable to what I have said before, that there are particles in nature which never were, nor will be, less than they are.

Though it is not easy to adduce a precise refutation of all the mathematical sophistry and refinements concerning the infinite divisibility of matter, yet we are furnished with one from the doctrine of gravity. For if the power of gravitation between bodies be as the squares of their distances, and if at any given distance, it is not material whether the distance assumed be one yard or 100,000 miles, the power of their tendency towards each other is equal to any given number; at half that distance the power will be four times as much, and so on. Hence it must follow, that if the distance be divisible into infinite parts, that power of attraction will increase beyond all calculation. But this is not true in fact; for we know, when bodies come into actual contact with the earth by this power, their resistance to separation is easily overcome by an excessively finite power. Therefore, as at a contact, which is the end of distance, the power of attraction is finite, the spaces they traverse before they meet cannot be infinitely divisible; and no demonstration can be true, which implies a flat contradiction.

---

* The word *atoms* means small elements which cannot be cut or divided.

To assume, as a first principle in philosophy, that matter is infinitely divisible, is to assert, that matter has no beginning of substance; that there are no limits between matter and nothing. To assert, that there cannot be such a thing as a unit of matter, is to maintain, that matter has an infinite property. To say, that any portion of matter is finite, and at the same time that its component parts are infinite, is philosophy baffling with itself, and maintaining contradictions.

Time is made up of a perpetual succession of years and days, and is measured by hours, and minutes, and seconds: now, there would be just as much truth in asserting, that a moment of time, by subdivision may be lengthened out into a thousand ages, as there is in saying, that a grain of sand may be rarefied and enlarged into the bulk of a planet. Nothing can be more shocking to reason than eternal time; infinite divisibility is not less absurd.

Concretes, in which form matter is made subject to the senses, can no more be without atoms, or first principles, than numbers in arithmetic can be without units.

Having shown you the absurdity of supposing matter to be infinitely divisible, I shall now endeavour to give you some idea of the parts actually separate, and give you instances where you will find the parts so small and so numerous, as almost to surpass imagination.

Though plants, by their exhalations, lose so great a quantity of their substance, yet we cannot affirm, that the part assigned for propagating their smell, suffers much by this sensible decay. It appears, that the fragrant parts are endued with such prodigious divisibility, that they can furnish sufficient for the purpose, and with very little loss for a long course of time. Every body knows, that a grain of musk will make itself smell in a manner not very agreeable, for a space of twenty years, in an apartment where fresh air is admitted every day.

A grain of leaf gold will cover fifty square inches, and contains two millions of visible parts. But the gold which covers the silver wire, used in making gold lace, is spread over a surface twelve times as great. In making this wire, it is usual to gild strongly a cylindrical bar of silver, and afterwards to form it into wire, by drawing it successively through holes of different magnitudes, formed in plates of steel. By this means, the surface is prodigiously augmented; notwithstanding which it remains gilt, so as to preserve a uniform appearance, even when examined by the microscope. It has been calculated, that a single grain of gold would cover a surface thirty yards square.

FURTHER CONSIDERATIONS ON ATOMS.

Atoms must originally subsist in numbers, which cannot be increased or diminished; it is equally impossible for many to become one, as it is for a unit to be multiplied: so that the original number of atoms is as invariable as their sizes. An atom may as soon be reduced to nothing, as two can be made to coalesce into an unchangeable substance.

Number is the fundamental basis of motion and changes in matter. An atom cannot act upon itself, nor communicate action, impressed intimately to its substance though ever so large; but a concrete can and does impress its own inward parts, in a mechanical proportion to the power which acts upon its surface. If any body, though ever so small, has numerical parts, their cohesion must be mechanical; and nature can do nothing, but what under certain circumstances it can undo again. To this purpose Sir *Isaac Newton* closes a fine inquiry into the nature, laws, and constitution of matter. "All these things considered, it seems," says he "probable, that God in the beginning created matter in solid, massive, hard, impenetrable, moveable particles, incomparably harder than any of the porous bodies compounded of them: nay, so hard as never to wear or break in peices; no human power being able to divide what God made one at the creation. While these particles continue entire, they may compose bodies of one and the same texture in all ages; but should they wear away, or break in pieces, the nature of things depending on them would be changed," &c.

Such an atom or unit of matter must have dimensions; it constitutes its own space wherever it is, and occupies it. The idea of a solid atom and extension are inseparable; but that of division and a solid are irreconcileable.

Solidity among concretes is only comparative and imperfect; but it is as great a contradiction in terms to apply division to a proper solid, an atom, as it is to apply it to the space of a body abstractedly or mathematically considered, which is nothing. Though two things may be separate one from another, yet one thing and nothing are equally indivisible.

Dimension in bodies has proportion, and may be considered as less or more; and the extension of a large body may be a proportion for several smaller bodies; but the idea of parts and divisions has no connection with extension, and as little with the matter which occupies it, unless it be a concrete. Division is an idea which properly belongs to number; there is, therefore, no connection between matter having dimensions, and its consisting of parts; or one body is not many in respect of another of the same kind, because its substance is larger.

Therefore every atom or unit of matter has extension, which we may suppose to have mensurable proportions; but the atom itself is incapable of division, because it does not consist of parts. As its substance is without parts, it is without pores or interstices, it cannot be pervaded. It occupies its own space, and nothing else can be said to be or exist where it is. Pores only belong to concretes, or atoms forming a mass by adhering to one another.

Atoms are absolutely inert, and can neither conciliate motion to themselves, nor continue in motion longer than they are impelled. Thy are, however, perfectly mobile. For as they cannot move themselves, or alter their place, so they have in themselves not the least resistance to motion, or to be continually altering their station while they are impelled.

Atoms can have no attracting or impelling powers. 1. To suppose them to have such, is contrary to the *vis inertiæ ;* for if the first principles of matter attract each other, they must have a *vis motus*, a power of moving towards one another.

2. If there be such a power as attraction, it must either be a material or spiritual one. If it be a material one, an atom cannot be possessed thereof. It cannot move itself. It consists of no parts; it cannot therefore detach subtiler parts, to draw its fellows into contact. If it be a spiritual operation, then it is no property of matter. If the first principles of motion in matter be universally spiritual, then there is no need of mechanical agency in matter. If mechanical agents are employed in the operations in nature, it is absurd in philosophers to use unmechanical principles, where mechanical ones will answer the purpose.

If attraction be a principle, or primary property of matter, how is it that it can be weakened, suspended for a time, and even quite destroyed? If it be a power determining matter to matter, as a real essential property, how comes it to be overpowered and destroyed by matter, acting mechanically upon matter, since such action must always be consistent with, and subservient to, its original properties? Now, the force of fire will suspend the strongest attractions. From all we can understand of the units of which matter is composed, a mass of them lying together, unacted upon by a mechanical material agency, would neither attract, gravitate, nor take place of one another.

Matter hath a capacity of motion, not an ability to move; neither doth any matter act but so far as it is acted upon. The trumpet hath a capacity of sounding, but never acts till it is sounded; of itself it is dead and silent, and would, if left to itself remain so forever. To invest matter with any innate powers, (call them by what names, attraction, &c. you please) is as contrary to the real nature of matter, as to suppose that all trumpets are born with lips and lungs, and breath of their own. A stringed instrument has the capacity of sending forth all possible harmony; but it must first be acted upon either by the vibratory motion of the air, or immediately by the hand of the master. Such then is the mobility of matter; it is a capacity of being moved and acted upon, but no motive faculty of any kind with itself.

The operations in nature, as far as we can trace them, are carried on mechanically; and though our senses are limited as to the minutiæ of that mechanism, yet we are certain that matter does exist in such forms, as will assure us the mechanism may go on further than we can describe. All the properties of matter fit it to act and be acted upon in a natural way.

Though matter is not infinitely divisible, yet it is certain its primordial atoms are indefinitely small, so as to be far beyond the reach of our senses. At the same time our organs are machines, so exquisitely constructed, as to be subject to the impressions of the smallest.

But though the units of matter are so small as to elude our senses, it does not therefore follow, that we can have no certain knowledge of them or of their properties. From the knowledge of concretes, we may clearly discover what are the general and invariable properties of their constituent atoms. Every concrete possesses two kinds of properties: the first are such as are inseparable from it as matter, and belong to every atom of which a concrete is composed; the second are such as are produced from matter variously combined in concretes; the latter are not the original properties, though naturally produced by them. You are therefore to be cautious, lest you should ascribe the different properties which matter acquires in a concrete form, unto the original atoms themselves: thus, for instance, you might as justly conclude, that the units of which ivory are composed, are white, as that they are elastic.

OF INERTIA AND GRAVITY.

As I am now going to consider properties that relate to motion, it will be necessary to proceed with care and circumspection. "That matter at rest will persevere for ever in that state, unless it be compelled by some cause to move," is one of the ideas philosophers have included in the word inertia; and as far as this idea goes, without considering it as a primary property of matter, on account of its relation to motion, it may be admitted as true.

To the other idea, which is often included in that of inertia, there are many objections, and I think you will hesitate long ere you adopt it. By this we are taught, "that any particle of matter, which has been once put in motion, will move on for ever with the same velocity, unless stopped or resisted." I do not know that any thing can be started more unphilosophical than this notion; a notion which gives to matter an eternal power of changing its place, in consequence of its being once disquieted. But leaving every other consideration out of the place, as we do not know what motion is in itself, it would be a question, whether the inertia, here supposed, flowed from the nature of matter or of motion; a question that may be ranked among those that are indeterminable. This alone will exclude inertia, in the second sense of the word, from the number of properties.

In the Lectures on Mechanics it is shown, that the notion of the *vis inertiæ*, in the second sense, is repugnant to sound reasoning, and unsupported by any experimental evidence; and, to lessen the effect of prejudice, it is there also shown, that some of the first mathematicians have confessed, that this *vis inertiæ*, considered in the sense usually attributed thereto, must be abandoned.

The resistance observed in matter, arises from that certain determination, which all the parts of matter are under, from their situation and connection in the general system; and in consequence of which, they require a force to turn them out of that way, which is appointed to them by the established laws of nature. How far any parcel of matter would resist, if it could be taken independent of the present frame of nature, and what force would be requisite in such a case, to move any given quantity of matter in all directions indifferently, we cannot tell, because we cannot place any matter in such a state to make a trial.

I must own it appears to me exceedingly clear, "that no particle of matter can move a single instant, without the presence of the moving cause." In this proposition, motion is indeed considered, not as a mere modification, but as something real, and which is always foreign to matter; so much so, that matter continually obeys an active cause, which flows originally from a class of beings totally distinct from that substance, which has impenetrability, extent, figure, divisibility, hardness, and inertia in the first sense, for its characteristic properties: a class of beings, which are not the object of any of our five senses, except by this property of motion, which is perceived by us when it is communicated to matter; but of which we can form no conception, while we have only our five senses, or, in other words, while we are destitute of a sense analagous to the cause of motion.

Neither can I perceive any objection to this notion, as it is only the extension of another, that I admit in the fullest and most absolute manner, namely, that the first cause of motion neither is nor can be in matter; consequently, motion cannot be admitted as a property of matter.

Nor do I conceive how any one could possibly conceive that motion was essential to matter! How can any thing which has degrees, which may be divided by communication, be considered as an essential property? If so, all reasoning on these subjects is at an end. That, and that only, can be termed an essential property, which is inseparable, even by the imagination, from the subject to which it is attributed. Every thing else is only phenomenon.

I see that matter, in a general sense, follows the laws of motion; but then these laws belong only to a communicated modification, for they also may be communicated, which is contrary to all notions of an essential or primary property. I feel as intuitively as I do any other axiom, that matter is essentially impenetrable, extended, figured, divisible, and that the atoms thereof may be hard, that every particle at rest perseveres therein till something puts it in motion. But I perceive also, with the same degree of evidence, that the idea of motion may be separated from that of matter, as easily as that of any other evident modification. If I must then allow, that motion is essential to matter, in order to have the nature of man and the universe explained to me, I would rather choose to be ignorant. And I exhort you to avoid such instructors; for knowledge you cannot obtain from them. Let *Dinarzade*, in the Arabian Tales, amuse herself with fictions: but do you seek for truth.

Let us proceed, and by considering one of the known laws of the motions which prevail in the universe, we shall perceive what further suppositions are necessary to the propositions we are combating. The law to be considered is that of gravity, from which *Newton* has spread so much light over the physical system of the universe. Now among philosophers there are those to be found, who consider gravity as an essential property of matter. We will endeavour to discover what can be meant by such a strange assertion.

Gravity is that general phenomenon, or that law, in the operations of nature, whereby bodies approach each other; and which acts, as far as observations have hitherto determined, *in the direct ratio of the masses, but inversely as the squares of the distances*. By it matter is grouped into masses of different kinds; by it, and a simple rectilinear motion, which is perpetuated, the celestial orbs revolve in their orbits. This is the law that some would have us consider as an essential property of matter.

But who can conceive that a body can act where it is not, act without any thing intermediate? To suppose that two distant bodies should, without the assistance of any intermediate substance, produce motion in each other, is to invest matter with a power of beginning motion itself; a position false and dangerous, and which has been turned to the purposes of atheism, by a modern philosopher of France. What two particles of matter are at 100,000 leagues, or at the 100,000,000th part of an inch from each other, without any material communication between them; and yet, on account of the one the other is moved! Again, without any thing happening to one of the particles, let the other be placed at half the distance at which it was before, and they will move towards each other four times quicker! What magic power determines them? What! only because the distance is lessened, which is a mere non-entity, when there is no intermediate agent, the tendency increases, and that accurately in a certain ratio! Let us shut our books of speculative philosophy, if they all hold this language; for it is worse than unintelligible.

It is not easy to comprehend how those philosophers, who reject a spiritual principle, an immaterial soul in man, because they cannot conceive that a reciprocal action can take place between two substances which are not of the same nature," can, nevertheless, digest, and allow a reciprocal action between the particles of the moon and those of the earth, without any thing intermediate, but the magical power of the words, "gravity is an essential property of matter!"

If each particle of matter was even possessed of intelligence, and could determine itself by motives, still it would be necessary to inform them of the surrounding bodies, to acquaint them with their mass, their relative positions and distances; in a word, of every thing that causes a particle to move towards a certain point with a certain velocity. Who are the *aides de camp* that give this information? For such there must necessarily be.

Till there has been a serious and sound answer given to this question, consider gravity, and in general whatever else is included under the notions of attraction and repulsion, only as appearances or phenomena.

It was thus they were considered by that great man who has instructed us. *Newton* never considered gravity or its laws but as facts. He asserts clearly, that he only used the words attraction and repulsion, to express effects of causes more remote, which general effects explained particular subordinate effects. And he declared at the same time, that he conceived these general effects might be produced by impulsion, and tried to explain them as well as the particular attractions and repulsions, perceived in certain phenomena, by means of a universal elastic fluid, that he called ether, ascending always to a cause foreign to matter, for a first source of motion.

Remember, that though it may be difficult to find a mechanical cause of gravity, which shall be altogether satisfactory, it is not difficult, as you have seen, to show the absolute necessity for such a cause.

So long as you keep within the limits of physics, you must account for the motions of nature, by referring them to mechanical causes; and where this cannot be done, you must consider them only as appearances, till you shall have some further light by experience. Be not amused with names and qualities, which contradict the known laws of mechanism, and are used to supersede the agency of the elements.

OF ATTRACTION AND REPULSION.

The universe appears to be one great machine, fitted and disposed to perform all the operations which are carried on throughout the whole. No one part of it should be considered as acting, without being acted upon; and no individual can be considered as an agent, without being a patient. It is unphilosophical to say, matter in general, or any portion, has essential or separate properties, by which one part acts upon another. It is the essential property of no one wheel in a machine to move its fellow, though, in consequence of its being placed in the station it is fitted for, it acts upon its fellow, because it is acted upon.

It is exactly the same with the whole system of nature. You cannot take up any parcel of matter, and say thereof, This has essential, separate properties, which empower it to be a natural agent. A philosopher should consider it as a concrete, with a certain disposition of its parts, liable to be acted upon by the more subtile parts of the machine. And it may as justly be asserted, that it is the essential property of animal substances to live, as that it is the essential property of the loadstone to attract.

The visible system of this world, created, disposed, and set in motion by the power of God, acts as a machine does, a connection and communication being preserved between all the distant parts. And you will find that a mechanical agency takes place, to produce all the phenomena that surround you, and is concerned in the formation, growth, and support of animals and vegetables: the more you consider and observe the whole system of nature, the more clearly you will perceive, that *impulse* is the only known *material* cause of motion.

There are appearances, which have been hastily considered as sources of motion, without any adequate cause; these are generally known by the names of attraction and repulsion, and are distinguished into several kinds; that of bodies falling to the earth, is called the attraction of gravity, or, simply, gravity; that of the small parts of bodies to each other, is called the attraction of cohesion, and so of other species.

Here I must again caution you not to let words supply the place of real agents, or causes, where no causes have been discovered. Names may be used, as expressive of appearances, or to distinguish one appearance from another, but no further; thus the motion of small bodies approaching each other, as that of iron to the magnet, or that of bodies falling to the earth, may with propriety be distinguished from each other, by peculiar names, as magnetic attraction, attraction of gravity, &c. &c. But if you use these words for another purpose, and say, attraction is the cause of cohesion, the use is perverted, and becomes very exceptionable; they are then no longer the names of things perceived,—of facts, but they become names of imaginary and unknown things.

Thus, when it is said, that the parts of bodies cohere by attraction, what idea have you of the thing signified by the word attraction? If you say, bodies descend to the earth by their gravity, what idea have you of this gravity? If you say, they gravitate by attraction, what do you mean by the word attraction different from the fact?

We are under a necessity of giving a name to every known fact, whether cause or effect; but we ought not to give a name to a cause, while it is unknown; for this would be to use words without ideas, to perplex the mind, and to retard the progress of knowledge, by substituting an appearance for a reality.

If you examine the writers on this part of philosophy, you will find their notions vague, crude, and contradictory; sometimes the word attraction has no fixed meaning; sometimes it is cause, sometimes effect; you will find their answers to every question concerning it unsatisfactory. If you ask, where attraction is seated, for instance, whether it be in the earth, or in the stone that falls down to it, or in both, or in neither, but in some substance exterior to both, you can obtain no answer. If you inquire, whether it be a material or an immaterial force, you are still left in the dark.

Many and greater are the difficulties and objections which still remain against the use of this word; but for a full account of these, and of the contradictory manner in which it has been used by various writers, I must refer you to the Rev. Mr. *William Jones's* Essay on the First Principles of Natural Philosophy, and shall only give you some instances where it has been applied improperly to explain phenomena.

Aqua fortis will dissolve iron. If you ask why? the answer is ready, because the particles of iron are more attracted by those of aqua fortis, than either the particles of iron are by each other, or the particles of aqua fortis are by each other. By this attraction, the particles of iron and aqua fortis are separated from each other, and by the same attraction they are mixed and blended together.

But then, this aqua fortis will not dissolve gold; and why? The answer is ready, for words are easily put together. There is no attraction between gold and aqua fortis. If you ask, why aqua fortis, which will not dissolve gold, will dissolve silver? the answer is ready, and the same as before; aqua fortis either does not attract gold at all, or else the mutual attraction of the particles of gold amongst one another, is greater than the force with which they are attracted by the aqua fortis. Ask again, why aqua regia, which will dissolve gold, does not dissolve silver? you are again answered, that aqua regia attracts gold, but does not attract silver.

To evince how unsatisfactory a method this is of accounting for dissolution, you need only ask those who maintain it, what they mean by the word attraction? Put the word drawing instead of it, and you will find what they say may amuse but not instruct. Thus inquire why aqua fortis dissolves iron, or draws the particles of a mass of iron asunder? you are answered, because aqua fortis attracts them more than they are attracted by each other; which is just the same as to say, that aqua fortis draws them asunder with a greater force than they draw one another. Now this is only to assign the fact (phenomenon) as a reason for itself: it is saying, aqua fortis will draw iron asunder, because it has a sufficient force to draw it asunder. This account is therefore very unsatisfactory; for I wanted not to be informed that aqua fortis has such a force, for that my senses informed me; I wanted not a name for a fact; I wanted to know what the force is by which the effect is produced.

It might be inquired, what attraction that is which dissolves hard bodies in fluids? Is it the attraction of cohesion? If they say no, but quite a different species; I do not see how they get rid of the charge of introducing so many different attractions into nature, and thereby rendering their explanations no better than the Aristotelian one, by occult qualities, inscrutable principles, indeterminate powers. If they say, by the attraction of cohesion, they are evidently mistaken, for the bodies do not cohere so strongly as before. And it may be here observed, to the credit of *Aristotle* and his numerous followers, that though they supplied their ignorance by terms of art, yet they never assumed any previous maxim, which, while received, entirely prevented discovering the falsity and absurdity of their occult qualities.

Lord *Bacon* considered all nature "as filled with an aerial or fiery nature," and denied that the pores or cavities of tangible bodies admit of a vacuum, but that they contain either air or a subtile spirit proper to their nature and disposition. He treats this spirit as a material cause, rejecting all solutions as pretended, unmeaning, and unphilosophical, which are deduced from virtues and qualities in matter, with which the schools in his times did very much abound. "Whatsoever is invisible," says he, "either in respect of the fineness of the body itself, or the smallness of the parts, is but little inquired; and yet these be the things that govern nature principally, and without which you cannot make a true analysis and indication of her proceedings. The spirits or pneumaticals that are in all tangible bodies, are scarce known; sometimes they take them for a vacuum, whereas they are the most active of bodies; sometimes they take them for air, from which they differ as much as wine from water; sometimes they will have them to be natural heat, whereas some of them are cold; and sometimes they will have them to be the virtues and qualities of tangible parts which they see, whereas they are things by themselves: and when they come to plants and living creatures, they call them souls; and such superficial speculations they have; like prospectives that show things inward, when they are but paintings. Neither is this a question of words, but infinitely material in nature. As to the motions corporeal within the inclosures of bodies, whereby the effects pass between the spirits and the tangible parts, which are rarefaction, colliquation, concoction, maturation, &c. they are not at all handled, but they are put off by the names of virtues and nature, &c. and such other words."

This reasoning will be illustrated by considering another instance, where the word, attraction, and its magic powers, have been substituted in the room of the true agents to explain effects; thus leading men to conceive falsely of nature, and to affirm what is contrary to experience.*

Immerge a glass tube into water, or any other fluid except mercury; the fluid will be raised to a certain height within the tube above the surface of the fluid in the vessel, and its elevation in several tubes of different sizes will be reciprocally as the diameters of their bores.

The fluid is said to be drawn up by the tendency it has from the principle of attraction, till the surface is loaded with as great a weight as that tendency can support; from hence we are told, that a right notion is obtained of the ascent of sap in vegetables.

If the sap ascend in vegetables on the same principles that water rises in a capillary tube, in those of the same diameter the sap should be raised to the same height, but no higher. Now, take one, for instance, in which water will be raised two inches: in a tube of a plant a heavy viscous juice will issue plentifully at the height of two feet, and that from vessels considerably larger than the orifice of a capillary tube.

The same quantity of liquor will be suspended in a capillary tube, when its lower orifice is lifted out of the water, as was raised in it while immersed under the surface. If the sap ascended in a vegetable on the same principle, it ought to remain there, and be sustained by the attractive power of the vessels, when the orifices of them are exposed to the air. The contrary appears from experiment: if a branch of the birch be taken off, or a young tree be cut away, and held in the same perpendicular posture in which it grew, the vessels will bleed copiously from the bottom, as when a limb is cut off from an animal; and the like will happen to many other plants. Again, water will never run out at the upper orifice of a capillary tube, be it ever so short; but the stump of a vine will send up its sap into a tube cemented upon it, to the height of 20 feet and upwards: the juices are not therefore merely attracted by the plant, but also forcibly propelled into its vessels.

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* *Jones's* Essay on the First Principles of Natural Philosophy, p. 172.

In a capillary tube, it makes no difference whether there be air on the surface of the water or not, for the experiment will succeed equally well in vacuo: but it is not so with the vegetable; no plant will take up its nourishment, no seed will germinate, as long as the air is absent.

The capillary tube will not raise water to a greater height at one part of the year than another; but the rays of the sun, or the heat of an artificial fire, are so necessary to the growth of herbs, that in their season for taking sap, they are, in their stature, and in their other qualities, wholly influenced by the sun's heat. When the sun is at its greatest exaltation in summer, the whole vegetable creation is in its greatest glory and beauty; as the sun declines, the vegetative motion languishes. But what is all this to the ascent of water in a capillary tube? It is to apply a principle to explain a phenomenon, which in no one thing accords therewith. Thus it will always happen, when men connect together things so foreign to each other, and account for a hundred experiments by one that hath not itself been accounted for.

Other effects have been attributed to attraction and repulsion, which, on more accurate investigation, have been found to depend on hydrostatic principles. Among other instances, a repulsive power has been deduced from the appearances of the globules of rain which lie on the leaves of colewort, whose mobility and lustre have often engaged your attention: on a close inspection, the lustre is found to be produced by a copious reflexion of light from their flattened inferior parts. It has also been observed, that when a drop rolls along a leaf which has been wetted, its brightness disappears, and the green leaf, which before was scarce discernible, becomes very visible. From these facts it is inferred, that the globule does not touch the plant, but is suspended at some distance in the air by the force of a repulsive power, because there could not be any copious reflexion of white light from its under surface, unless a real interval subsisted between that surface and the plant. The Rev. Mr. *Bennet*, of Wirksworth, has shown, that there is no occasion in this instance to have recourse to any imaginary powers; for the drops of water rolling over the leaves of colewort, are prevented from adhering to the surface by a blue powder, which covers the leaves of that and various other plants; for if the powder be wiped off, the water will adhere. The leaves of honeysuckles and barberry-leaves are blue on one side only, and the water adheres to the green side, and not to the blue. If the surface of a pond be struck with a stick, numerous globules of water roll over without breaking, while they pass several yards from the place where the water was struck. In these cases of apparent repulsion, every drop of water surrounded with air becomes globular by the cohesion of its own particles; and unless its weight or contact with other surfaces be sufficient to overcome its cohesion, and displace the air, both of the drops and the surface which it falls upon, it retains its form and cannot unite with them.

The difficulty of overcoming this cohesion is rendered evident by experiments on soap-bubbles, which will roll over or rebound from a carpet, though they are filled with nothing but smoke, which renders them heavier than when blown with clear air; pins may be thrust through them, and even the small end of a tobacco-pipe, so as to blow a smaller bubble within the larger, without its immediately breaking; but if a bubble fall upon a plane smooth surface, it instantly breaks.

Mr. *Banks* has also shown, that the experiments made with clean bubbles of glass, or pieces of cork left to swim in water, &c. are no proofs of any attractive force. You may make the experiments in any vessel of glass or earthen-ware, five or six inches in diameter.

If a clean cork be wet, and placed about one inch from the side of the containing vessel, it will approach the side with an accelerated motion.

If two corks be placed about an inch from each other, and at a sufficient distance from the side, they approach each other with an accelerated motion.

Pour water into the vessel, till it is rather higher than the brim; place the cork close by the side, and it recedes with a retarded motion.

Sink a piece of metal in the centre of the vessel, so that the top thereof may be above the surface of the water; the cork placed at a proper distance will approach the piece of metal.

Raise the water until the metal is covered, and the cork will remain at rest at any distance from the metal.

In all these experiments, except the third, the water which surrounds the balls is elevated, as is also the water by the side of the containing vessel.

Pieces of dry cork, or painted balls, placed gently on water and near together, will approach each other; but if one be placed near the side of the containing vessel, adjoining to which the water is elevated, it will recede.

That these phenomena do not depend on attraction, is clear, 1. Because in the third experiment they recede from the side to which they adhered in the first. 2. Because the balls, around which the fluid is depressed, universally recede from those around which it is elevated. 3. Because they are not attracted by bodies exceedingly near, when those bodies are perfectly covered with water. 4. Because assigning attraction as a cause is contrary to all sound reasoning in physics.

It is no doubt better with Mr. *Bank's Gravesande*, &c. to explain these phenomena on hydrostatical principles. Every body that swims impresses the fluid with a force equal to its own weight; the fluid re-acts, and presses the supported body with the same force; the sides of the body are also pressed by the surrounding fluid with forces which are as the depth. If the encompassing water be elevated round the body, the pressure will still be the same, or equal on opposite sides, so that without force the body cannot move; but if it be placed so near the side, that its elevated water joins the water elevated at the side of the vessel or by another ball, the pressure on that side is diminished, while that on the other side, remaining the same, will cause it to approach the side or other ball.

In the same manner, if the fluid be depressed when the cavities meet, the pressure on that side will be diminished, and the bodies of consequence approach each other on the side of the vessel.*

The recess in the third experiment arises from the pressure being superior on that side next the glass, when the fluid is elevated above it, and the elevation round the ball joins the declining surface near the edge; for, when the elevated fluid surrounding the ball, is caused to join the declining surface of the supporting fluid, it must then gravitate and press in all directions with more force, as there is more of it elevated above the surface of the supporting water.†

In these cases,‡ the attraction and repulsion is accounted for by the action of intervening fluids. Perhaps the elasticity of the air, electricity, and all other elastic fluids, may be explained in the same manner, that is, by supposing the mixture of a system of fluids, some of which are capable of permeating glass and other solids; as light, fire, magnetism, &c. so that when air, for instance, is condensed in a vessel, the finer fluids are forced through the sides, and suffer the particles of air to approach nearer together.

Or, if the electric fluid be forced upon one surface of glass, a finer fluid, or system of finer fluids, which constitute its elasticity, is pressed out and rarefies the other uninsulated surface. This hypothesis of Mr. *Bennet* does not differ much from another of the wonderful *Stagirite*, and perhaps is not far from the truth: arguing against those who insisted on a vacuum as necessary to motion, he says, "that a plenum is capable of being so affected as to consist very well with the motion of bodies in it; for its parts may yield mutually to one another, and give an easy passage to bodies, though the space in which they move is full of matter. Of this we have instances in the circumgyrations of continuous bodies, and also of fluids; nor does it follow that there is a vacuum, because some masses of matter may be compressed into lesser dimensions; for if water should be forced into a smaller space, the air it contains will be forced out of it. *Aristotle* might have added, that if air be compressed, the fire is forced out thereof.

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* When the circles of surrounding water meet, the pressure on the exterior sides forces them to form one circle, as two bubbles or drops of water unite and form one larger, and thus brings the corks, &c. together.

† If one cork raises, and the other depresses the water, the water, in endeavouring to restore the level, comes between and pushes them asunder.

‡ *Bennet*, Manchester Transactions, vol, iii. p. 123.

Dr. *Hamilton* has endeavoured to account for a very remarkable phenomenon by attraction; but it does not appear that attraction has any thing to do with it. If a tube six feet long, perfectly clean and dry, be filled with pure quicksilver purged of its air by being boiled in the tube, upon inverting it, the quicksilver will not fall to the level, as in the common barometer, but the whole column will be sustained, till it is made to fall by some shock or stroke. How far this experiment may be carried, we cannot say; it has succeeded to the height of 75 inches.*

Now, says Dr. *Hamilton*, the pressure of the atmosphere can only support about 29 or 30 inches of mercury; and therefore, some other agent (attraction is meant) must have sustained the rest, by making its parts adhere to the glass.

This cannot be the case; for, 1. Quicksilver does not adhere to glass in the manner water appears to do. 2. Quicksilver tends towards quicksilver with more force than quicksilver tends towards glass, and is therefore depressed in capillary tubes. 3. Mr. *de Luc* has shown, that glass and mercury are inclined to separate, and that the barometer is liable to an error on this account, which in accurate experiments must be corrected; therefore if a column of this fluid be extended within a glass tube, it will contract itself and collapse into the cistern, unless some other cause should interfere.

Every body that is left at rest in a free space, or where it has liberty of motion, will not remain at rest, but will be moved by the principle of gravity till it meets with some impediment, unless the cause of gravity should be suspended, or some superior force act against it.

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* *Hamilton's* Four Introductory Lectures, p. 56.
Jones's Physiological Disquisitions, p. 249.

From the foregoing circumstances, which are demonstrable by experiment, it is clear, 1. That the column is not suspended by attraction. 2. That it does not remain at rest from the inertia of matter. Consequently it is supported by some medium; and as the air is not adequate to the effect, you must have recourse to another medium more subtile and powerful than common air.

Now in a large sense the pressure of the atmosphere is the pressure of air and fire. Sometimes these two act against each other, as in the common barometer; sometimes they conspire together, as in the case of this extraordinary column.

When the tube is open at both ends, and the atmosphere has access to each, the air then acts against itself, and the mercury falls by its gravity into the cistern.

When the upper end is closed, the air then acts at one end and not at the other; and this is the reason why the mercury is sustained in the tube till it is a counterpoise to the atmosphere.

On the same principle you may account for the higher or longer column. When the more subtile medium which can penetrate the mercury has possession of the top of the cavity in the upper part of the tube, it acts at each end; and being thus a counter-balance to itself, nothing remains but the pressure of the air, which supports a column equal in weight to itself.

But when that medium is excluded at top by the perfect contact between the mercury and the glass, and cannot pass readily through the substance of the glass, it acts only on the open end in conjunction with the air, and thus produces a much greater effect of the same kind than the air can do when it acts by itself.

Thus the extraordinary column is found to act on the same principle with the ordinary column, the one being from the pressure of the air only, the other from the pressure of air and fire together; as they are sustained on the same principle, so do they also fall or subside on the same principle. If air insinuate itself at the lower and open end of the tube, it ascends to the top, and by its elasticity and expansion depresses the mercury; if more enter, it is more depressed; if a continual stream of air be admitted, it keeps ascending to the top till it has expelled the mercury.

Now, this is what actually happens in the other case, for when once the more subtile element is admitted to the top by reason of the smallest separation of the mercury from the glass, it follows rapidly in a stream till it has got full possession, and reduced the column to a counterpoise with common air. To prevent this, it is necessary that the contact must be rendered as absolute as possible by thoroughly purging the mercury from its air.

This experiment puts us in possession of a principle, which in its application will be found almost as extensive as philosophy itself. The pressure of the atmosphere, as demonstrated and explained by the Torricellian tube, enables you to give a satisfactory account of many phenomena, about which words had been multiplied for ages to little purpose. Here we have another force which comes into our aid, and will carry us through higher and more subtile phenomena, where air is inadequate; cohesion with such a medium as this is no longer difficult; and as to gravity,* if a column of so great a weight does not fall because the medium is not admitted to fetch it down, I can see no difference between that and the cause of gravity.

OF GRAVITY CONSIDERED AS A FACT.

Gravity is that force by which all bodies, when left to themselves, fall towards the centre of the earth.

The same force which causes them to fall when they are not supported, causes them to press on those obstacles which retain and prevent them from falling; thus a stone weighs or presses upon the hand which supports it, and falls in a line perpendicular to the horizon as soon as the hand lets it go.

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* See the sentiments of Mr. *Huygens*, Dr. *Wallis*, and Dr. *Jurin*, Phil. Trans. abridged by *Lowthorp*, vol. ii. p. 25, &c. vol. v. p. 191.

It was always known that bodies, left to themselves, descended towards the earth, and pressed the hand which prevented them from falling; but as there were some bodies whose weight was scarce sensible, and which rose in the air, as feathers, smoke, flame, exhalations, &c. *Aristotle* said that there were two kinds of appetence *(appetentia)* possessed by different bodies, one by which they tended to the centre of the earth, which he supposed to be the centre of the universe; the other, by which certain bodies were driven from this centre.

Aristotle taught also, that all bodies fell in different mediums with velocities proportional to their masses; but *Galileo* showed, that the resistance of the mediums was the only cause of any difference in the descent of bodies, and that *in an unresisting medium all bodies would descend with the same velocity.*

In the air, indeed, a piece of lead will fall faster than a feather, but this is owing to the different resistances they meet with from the air; for in an exhausted receiver, you have seen, that if they be both let fall from the top at the same instant, they will both come to the bottom together, and having fallen from the same height in the same time, they must have moved with equal velocities; consequently the forces whereby bodies descend, must, at equal distances from the earth, be as the quantities of ponderable particles in the descending bodies. For, if a certain force be necessary to carry down a certain quantity of matter with a certain swiftness, then double the force will be required to carry down double the quantity of gravitating matter with the same velocity, and a triple force to carry down a triple quantity of matter; so that the weights of different bodies at equal distances from the centre of the earth, or the forces with which they descend freely, or endeavour to descend, are to each other as their quantities of ponderable matter.

That the gravity of bodies is proportionable to their quantities of gravitating matter, and does not depend upon the figure or the texture of their parts, is evident from experiments on the motion of pendulums: for when the lengths of the pendulums are equal, bodies of very different bulks, and different internal and external texture, perform their vibrations in times exactly equal in equal arcs, keeping always pace together, and acquiring always equal velocities at the corresponding points of those arcs, unless so far as the resistance of the air may act upon them unequally.

Now, the quantity of motion in each is the product of its weight by its velocity; and, as the velocity is the same, it will be in proportion to the quantity of gravitating matter in each. But the force producing motion is in proportion to the quantity of motion produced; consequently gravity, which is the producing force, acts in proportion to the quantities of gravitating matter.

The force of gravity above the surface of the earth decreases in the same proportion that the squares of the distances from the centre increases; so that if a body at the surface of the earth, which is about 4000 miles distant from the centre, weigh four pounds, and fall through sixteen feet in a second of time, it will at double that distance weigh but one pound, and will fall through but four feet in a second of time.

The force of gravity, when considered as acting differently on bodies at different distances from the centre, is called the accelerating force, and is measured by the velocity it is able to generate in a given time.

Though gravity, strictly speaking, decreases in the manner above-mentioned, yet, where the distances from the earth's surface are inconsiderable compared with the earth's radius, the force of gravity may be considered as equal at all such distances. Thus, for instance, the gravity of a body at the distance of half a mile from the earth, may be considered as equal to the gravity thereof, at the distance of a quarter of a mile, or at the surface itself; for the difference is so small, that if it be rejected, it will not occasion any sensible error in the calculations.

Gravity does not vary; it acts equally on all bodies at all times, whether they be in motion or at rest, and thus uniformly accelerates the motion of a falling body; that is, the velocity thereof will be increased, and the increments will be equal in equal times.

A few considerations will show you, that a force constantly and equally acting, must produce a uniform acceleration of velocity; for if you suppose the time of descent to be divided into a number of equal parts indefinitely small, in each of these, gravity must make equal impressions on the body to carry it downwards, and must consequently by each impulse generate a velocity in the falling body equal to the former. Now, as all the velocities are in the same direction, the last acquired must be still added to the former; that is, the velocity given in the first portion of time, will be doubled in the second, tripled in the third, quadrupled in the fourth, and so on continually through the several portions of time; and therefore the velocity of the body will be uniformly accelerated.

To be a little more particular, let us suppose that a body begins to move with a velocity constant and gradually increasing, in such a manner as would carry it through a mile in a minute; at the end of this space, it will have acquired a velocity sufficient to carry it through two miles the next minute, though it should then receive no new impulse from the cause by which it was accelerated: but if the same accelerating cause continue, it will carry the body a mile farther, on which account it will have run through four miles at the end of two minutes; and then it will have acquired such a velocity, as is sufficient to carry it through a double space in as much more time, or eight miles in two minutes, even though the accelerating force should act upon it no more.

But this force still continuing to operate in a uniform manner, will again in an equal time produce an equal effect; and so by carrying it a mile farther, cause it to move through five miles in the third minute; for the velocity already acquired, and the velocity still acquiring, will each have its complete effect.

Hence we learn, that if the body should move one mile the first minute, it will move three miles the second minute, five the third, seven the fourth, nine the fifth, and so on in proportion.

So that the spaces described by a uniformly accelerated motion, are always as the odd numbers 1, 3, 5, 7, &c. and consequently the whole spaces are as the squares of the times, or of the last acquired velocities: for the continued addition of the odd numbers will yield the squares of all numbers from unity upwards. Thus 1 is the first odd number, and 1 is the square of 1; 3, the next number, added to 1, makes 4, which is the square of 2; 5 added to 4 makes 9, the square of 3, and so on for ever. Now the times and velocities proceed evenly and constantly, as 1, 2, 3, 4, &c. but the spaces described in each, as 1, 3, 5, 7, &c. it therefore follows, that the spaces described

In 1 minute will be	1	equal square of 1
In 2 minutes	1×3	2
In 3 minutes	1×3×5	3
In 4 minutes	1×3×5×7	4

Wherefore the spaces described in different times by a falling body, are to each other as the squares of the times from the beginning of the fall, or as the squares of the velocities acquired at the end of those times.

As the motion of a body falling from a state of rest is uniformly accelerated, so likewise the motion of a body thrown directly upwards is uniformly retarded; for the same force of gravity, which conspires with the motion of a falling body, acts in direct opposition to the motion of such as ascend; therefore it will retard the one in the same manner that it accelerated the other.

Whence it follows, that a body thrown directly upwards, will continue to ascend for a time, equal to that in which, by falling from a state of rest, it would acquire the same velocity with which it was thrown up. For since the action of gravity is constant and uniform, in whatever time it generates any velocity in a falling body, in the very same time it must destroy that velocity in a rising body.

Hence, if a body be thrown directly upwards with a velocity equal to that which it acquired by falling from any height, it will ascend to the same height before it loses all its velocity.

To determine the absolute force of gravity at the surface of the earth, it is necessary to know through what space a body will fall in a given time; and, by very accurate experiments on the pendulum, it is found, that a body near the surface of the earth, falling from a state of rest, will descend through the space of about 193 inches in a a second of time; consequently, the force of gravity acting on a body only for one second of time, will give it such a velocity, as being continued uniform, would carry it through thirty-two feet two inches in one second, or very nearly at the rate of twenty-two miles in an hour.

Here, however, I must observe to you, that bodies do not descend in all climates at the rate of 193 inches in a second of time, but only in such places as are in or near the latitude of forty-nine degrees. In places more distant from the equator, the descent is quicker, and slower in those nearer thereto. For it has appeared, from a great number of observations, that the force of gravity is less under the equator, and that it continually increases as you approach towards the poles, where it is greatest.

This difference arises chiefly from the rotation of the earth round its axis, by which all bodies on the surface acquire a centrifugal force,* and would fly off if they were not retained by their gravity.

Now, as all these bodies revolve uniformly round the earth's axis in twenty-four hours, either in the equator, or in circles parallel to it, their velocities, and consequently their centrifugal forces, must be as the peripheries of the circles they describe; and therefore, as the equator is the greatest of all those circles, and others grow less and less as they are more and more distant from the equator, the centrifugal force of bodies must be greatest in the equator, and be continually diminished towards the poles, till at last it vanishes at the polar points. As this force acts in opposition to gravity, it must of course diminish it, and this diminution must be greatest in the equator, and grow less and less in approaching towards the poles.

There is also another reason why the force of gravity should be more diminished by the centrifugal force at the equator, than in any other place, which is this: The centrifugal force always causes a body to tend from the centre of the circle in which it moves; and therefore it must act in direct opposition to the force of gravity at the equator, whose centre is the same with that of the earth: but as the centres of all the circles parallel to the equator, are at several distances from the centre of the earth, in every such circle the centrifugal force acts obliquely against the force of gravity, and the more so, as the circle is nearer to the pole, and therefore cannot diminish it so much as it does at the equator.

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* By the centrifugal force is meant, that force which we see makes bodies, lying loosely on a wheel or globe, fly off when it revolves swiftly round its axle; and this arises from their tendency to move on in a right line.

Sir *Isaac Newton* has proved, by a most ingenious method of computing, that bodies at the equator lose $\frac{1}{230}$th part of the weight which they would have if they were placed at the poles; and has shown, that the equatorial diameter of the earth exceeds the polar diameter, or the axis round which the earth turns, by about thirty-four English miles and $\frac{1}{5}$th*; for if the equatorial parts of the earth, where the waters are lightest, were not so much higher than the polar parts, where they are heaviest, the seas about the poles would subside, and rising at the equator, would overflow the lands thereabouts.

This law of gravitation is not confined to the matter of which our earth is formed, but is found to extend to all bodies that come any way under our observation. Sir *Isaac Newton* has shown, that the moon gravitates towards the earth, and is retained in her orbit merely by that force. And since the revolution of the moons round Jupiter and Saturn, and of the primary planets round the sun, are phenomena or effects of the same kind with the revolution of our moon round the earth, he concludes, by the second rule of philosophizing, that all these effects must proceed from like causes, and therefore that the moons of Jupiter and Saturn gravitate towards their primaries, and that all the primary planets gravitate towards the sun. He has also showed, that if one body attract another with any force, with the same force does that other body attract it; so that the earth must gravitate towards the moon, the sun towards the planets, and they all towards each other. He has proved likewise, that the attractive forces of these great bodies act according to one universal and invariable law, which is, that every two of them attract each other with forces that are directly as their quantities of gravitating matter, and inversely as the square of the distance between their centres.

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* This spheroidal figure of the earth also contributes to produce an increase of gravity from the equator to the poles; for, since the surface of the earth at the poles is nearer the centre of gravity than at any other place, the gravity will on this account alone be greatest at the poles.

A. Edit.

Sir *Isaac Newton's* great discovery consists in his having proved, that the well-known power which we call gravity, acting throughout the solar system, according to the law above-mentioned, preserves the planets and comets in their orbits round the sun; and that this force is fully sufficient to account for all the irregularities of the lunar motions, for the retrogression of the equinoctial points, and for the tides in our seas, whose waters gravitate towards the moon.

All this he has done by showing, from mathematical calculations, that this force, acting on those bodies, must necessarily produce such effects and appearances, as exactly correspond with those which, the best observations assure us, do really take place in nature.

REFLECTIONS ON GRAVITY AS A LAW.[*]

In forming a general law for the explanation of phenomena, we must take care, that it be founded upon actual principles, which should be clearly seen; otherwise, whatever follows in consequence of the general law is no more than conjecture. But supposing the law to be properly founded on unquestionable principles, yet the law is not to be extended beyond those natural appearances that by it are explained: these, with respect to gravity, are the gravitation of the distant bodies of the system.

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[*] *Hutton's* Dissertations on different Subjects in Natural Philosophy, p. 327, &c.

To apply the laws, by which the gravitation of bodies has been generalized, for the purpose of explaining other phenomena, without good reason for such analogy, would be to depart from the evidence of science.

It is therefore of considerable importance to know the nature and extent of those laws, which are to be employed in generalizing gravity; for, by admitting such a law in any degree more general than it truly is, or by applying it improperly to a subject in which it has not been investigated, instead of making any further progress in our knowledge of the material system, we might introduce inconsistency in nature by this system of our own.

Gravity is that action by which a body feels heavy when supported by our hand, and by which, when unsupported, it falls to the ground.

Hence it appears, that gravity is a principle of action, which is relative in its nature, when considered as weight in a body; for, in this case, there is also another action required, by which that weight must be supported, and it is only when unsupported, or not counteracted in its intention, that gravity, as an active power, appears to produce an effect peculiar to itself. Thus gravity appears to be a principle of action of two kinds; it is a resisting power, when the action of another power is opposed to the weight of a body; it is a moving power, when a body is suffered to fall, or when it acts by its own weight.

Weight in bodies, when it is not actually producing motion, may be called a pressing power, seeing it is then continually soliciting to motion, with a certain force, though not actually producing that effect.

So far as the volumes of bodies are considered as regularly connected, and corresponding to gravitation, the pressing power of gravity would appear to be various in its intensity or force; for no regular proportion is observed as always taking place between that force and the volume of bodies; bodies having not only their volumes changed, while their weight remains the same, but also different substances in equal volumes, are found to have various weights, or pressing powers.

With regard, however, to different bodies of the same, or similar substances, it is otherwise; for in this case, there is an invariable rule observed in the proportion of their volume and weight, which last is always the same in equal or similar conditions, whatever be their figure, and is always in proportion to the quantity or magnitude of that substance.

But as from this it does not necessarily follow, that the quantity of matter should be in proportion to the pressing power, or tendency which the body has to move, it cannot hence be concluded, that the weight of bodies is in proportion to their quantities of matter.

Hence it will appear evident, that this received maxim, that in bodies their weight and matter is the same, is an assertion destitute of truth and utility. That it is contrary to the actual state of things is evident, because bodies have other moving and resisting powers besides those of gravitation. That it has no utility will appear from considering the import of the maxim, that the weight of bodies is in proportion to their matter; so far as matter is not known otherwise than by its weight, this maxim adds nothing to our knowledge.

The expression therefore of gravitation being in proportion to the solid matter of a body, must be rejected, as either having no distinct or proper meaning, or as liable to give a wrong idea of that power constituting ponderability, but which does not necessarily belong to all matter, or to every external thing.

That it is necessary to correct this maxim of gravitation being in proportion to the solid matter in a body, will appear by considering, that this position is only a supposition, and that this supposition rests upon another, namely, that physical bodies have a proportion of vacuity in their substance.

Now we have no evidence of any vacuity in nature, nor that there is any such porosity as implies a vacuity or want of matter; nor ought it to be admitted without better proof of its existence than bare supposition or conjecture. Therefore the proposition grounded on this idea, can be considered only as an hypothesis, and must not be admitted as a principle, in order to make another truth appear.

"Any one who knows that resin consists chiefly of phlogiston and acid, and that it contains very little earth and water; although he had never weighed resin, might conclude therefore, that its specific gravity must be much less than that of a stone or metal, even when the pores in these bodies are equal. Those philosophers who consider gravity as acting equally upon all kinds of matter, impute the difference of specific gravity of bodies to the different densities of bodies, and thus a cubic inch of gold is supposed by them to contain about sixteen times more matter than a cubic inch of resin. It might be expected, that these philosophers, who are strong advocates for experimental proof, would have supported their hypothesis by experiments, directly proving that resin is sixteen times more porous than gold. But nothing has been done to this purpose. Some observations have been indeed made with respect to the inertia of matter; but they only prove, that inertia relates to the gravity of bodies, not to their quantity of matter."

Notwithstanding the universality of gravity, which is continually operating in every part of the universe, yet various phenomena clearly prove, that there are powers which act in a direction contrary to that of gravity. As some of these have been already noticed, and others will be mentioned hereafter, I shall only point out a few in this place. Dr. *Halley* allowed, that there was, in a certain degree, a *conatus* against gravity from those steams and exhalations which arise within the bowels of the earth; and such a power seems also necessary for the elevation of vapours.*

The remarkable phenomenon of the perpendicular growth and position of plants and trees, must be attributed to the constant agency of some force, external to the plant or tree itself; for if the ground, on which a vegetable is planted, be inclined to the horizon at any angle whatever, the plant will, however, soon obtain a direction perpendicular to the horizon, and will continue to increase in that direction.

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* *Atwood* on Rectilinear and Rotatory Motion, p. 178.

Another phenomenon may be here mentioned, namely, if a bar of metal be fixed in a vertical direction, and heat be by any means applied to the middle of it, this heat will be communicated to the upper part of the bar, when the temperature of the lower part is scarcely altered.

Light must be acted upon by a power in a direction contrary to the principles of gravitation; for as all bodies near the earth gravitate towards the earth, so do bodies in the heavens gravitate towards the sun. But a flood of matter is sent off every moment from his orb, in a direction opposite to that of the gravitating body, and this in so large a quantity, that no space, however small, could be assumed within the solar system, where the point of a needle could be opposed to the sun, without stopping some thousands, or millions, of the particles thus sent from him.

Still, gravity may be considered as the combining cement, the operative spring of the mighty frame: urged by this principle, the rivers circulate with a never-failing current; it confines the ocean within its bounds, and the planets within their orbits; thus the same force which determines the fall of a stone, is one of the ruling principles of the heavenly motions; thus every where you find a wonderful mechanism, whose simplicity and energy give unceasing tokens of the profound wisdom of its Author!

MIND AND MATTER ESSENTIALLY DIFFERENT.

Nothing in the actions of matter can induce you to think, that they proceed from any sense, perception, intelligence, or will; or that sense or will can be essential to matter; or that they are naturally involved or complicated with the actions of matter: for our ideas of the actions of matter are perfect and complete, though it were supposed, that sense, perception, intelligence, or will, existed in no created beings but ourselves.

Since we cannot doubt of the existence of sense, or perception, intelligence, and will, they must be the actions, operations, or properties of some kind of being, distinct from what is commonly called matter. There is nothing in the actions of motion, resistance, or elasticity, that raises in our minds any idea of sense, perception, intelligence, or will; otherwise we could not conceive any machine, for example, a mill, without at the same time conceiving that it may have some degree of sense, or perception, or intelligence, or will.

There is nothing then in the idea of the actions of matter, by which the parts of it can form themselves into any kind of regular system, with any view to serve any purpose or end; neither is there any thing in any system of matter that we know, which necessarily supposes an existence of that system, or without which we cannot imagine matter to exist. We cannot conceive any thing essential to matter, whereby such a quantity of matter, for example, must exist in that part of space, where the sun now exists, that it should contain such a proportion of resisting matter, and such another proportion of light; that it should be of a globular figure, &c. or why one part of matter should be collected and placed in such order, as to form an animal; another part in such another order, as to form a vegetable or plant. If there be nothing in the actions of matter to do this, then it must be done by something different from matter.

We have no idea of substances; we have as little knowledge of the substance of material beings, as of intelligent ones; we have no more idea of the thing in which the power of resisting, or of moving, or of reacting, subsists; than we have of the being in which intelligence subsists: but we may have ideas of the actions or operations of intelligence, as we have of the actions of matter, or as we have of motion or resistance.

The essential or characteristic distinction between the material agent and the intelligent agent is this: the material agents act always uniformly, and in all directions; they have no power in themselves to increase their force of action, or to determine it to one direction more than to another; all alteration in their action, or in the direction of them, is made by something external, which for that reason is called an efficient cause; they have no will, purpose, view, or design in their actions: but the intelligent being determines and directs its own actions, by the purpose, design, or view which it has; and therefore its actions are said to be determined or directed by final causes, and this direction by final causes is called the will, therefore in all actions of intelligent beings, which are likewise called moral actions, the intention, purpose or will, is principally to be considered. This is the guiding principle in morality, policy, and religion.

The actions of intelligent beings cannot be the object of mathematical inquiry; for quantity, and the ratios of quantities, are the sole object of mathematics; but there can be nothing of quantity in design, intention, or will; therefore any inquiry into the actions of an intelligent agent must be on different principles from what are used in an inquiry into the actions of matter: but frequently our ideas arise from the complicated actions of intelligent and material agents, in which case a mixture of mathematical and metaphysical principles become necessary in our inquiries.*

" To perceive, to compare, to desire, to will, to feel pleasure and displeasure, require a simple substance, which must represent to itself things which are distant and separated, collect things which are scattered, and compare things which are different. All that is spread over the wide space of the corporeal world presses itself here together, as it were, into a point, to make out a whole; and what is past is in the present moment brought into contact with that which is to come. Here is neither extension, colour, motion, rest, space, nor time, but an active being which represents to itself extension, and colour, motion and rest, space and time, combines, separates, compares, selects, and possesses a thousand other capacities which have no relation to extension or motion, attraction or repulsion. Pleasure and displeasure, desire and aversion, hope and fear, are no change of place of little atoms. Modesty, benevolence, philanthropy, the charm of friendship, and the sublime emotions of piety, are something more than the agitation of the blood, and the beating of the arteries, with which they are usually accompanied ; nor can they ever be confounded together but by ignorance, folly, and infidelity."

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* *Colden's* Principles of Action in Matter, p. 158.

LECTURE XXV.

THE OPINIONS OF THE ANCIENTS CONCERNING MATTER, AND OF MATERIALISM.

HAVING given you an account of the properties that are essential to matter, and of the force of gravitation so universally acting thereon, I shall now endeavour to give you some idea of the opinions of the ancients on matter ;* these are not only highly interesting in themselves, but even throw much light on the physics of the moderns.

They considered the visibility of matter not a necessary consequence of its creation, but to arise from its formation. The smallest particles of matter in solution are invisible ; thus a mineral water strongly impregnated with iron, is as clear and transparent as any other water, though the particles of the metal are copiously distributed through the whole body of the fluid. As soon as they begin to concrete into masses by the addition of an astringent, they show themselves, the water becomes turbid, and by degrees turns as black as ink: by the reverse of this operation, ink, with the addition of a strong acid, will be turned into a colourless water. The sky, though retaining a very large quantity of water, preserves its clearness so long as the moisture is in a state of solution; but as soon as the atoms of water are assembled together in masses, by means of some change in the temperature of the air, the sky is overcast, and becomes dark and cloudy. The visibility, therefore, of matter commences when it concretes and assumes a form; so long as it is formless, it is also invisible. The chaos, in its primeval state, is said to have been without form, or, according to the Greek version, invisible; and the Scripture, referring elsewhere to this condition of the world, tells us, that things which are now seen were not made of things that do appear; in other words, visible forms were composed of invisible atoms.

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* See the works of Mr. *Harris*, particularly the Philosophical Arrangements, and the Hermes ; *Sydenham's* Plato ; *Jones's* Physiological Disquisitions ; *Petvin* on Mind ; *Monboddo's* Ancient Metaphysics ; *R. Clarke's* Series of Letters ; *Berrington's* Immaterialism Delineated ; *Cudworth's* Intellectual System, &c.

Though matter is no object of our knowledge but in its formed state, we are nevertheless obliged to presuppose a kind of first matter, out of which all the visible forms are raised, and into which they return at their dissolution.

The ancient philosophers carried their speculations on this subject very far, supposing the first matter to be homogeneous, and accommodated to the formation of all sorts of bodies indifferently.

Matter they conceived to be that elementary constituent in composite substance, which appertains in common to them all, without distinguishing them from one another. Every thing generated or made, whether by nature or art, is generated out of something else, and this something else is called its subject or matter; such is iron to the saw, such is timber to the boat.

Thus matter was considered as a homogeneous being, the common basis or element out of which all bodies were formed; it therefore implies a privation of all form, and a capacity to every form, as is the brass to the statue, the marble to the pillar, the timber to the ship, or any one secondary matter to any one peculiar form; so is the first and original matter to all forms in general, possessing no quality whatever except that of being passive in receiving any sort of form; a notion framed by rejecting every bound that can be set to substance, and every positive attribute except the bare capacity of receiving some form or other.

Such is that singular being which those philosophers, says Mr. *Harris*, who are immerged in sensible objects, know not well how to admit, though they cannot well do without it; a being which flies the perception of every sense; and which is at best even to intellect but a negative object, no otherwise comprehensible than either by analogy or abstraction.*

This invisible and formless being, the universal recipient of all forms, was by ancient poets and mythologist pourtrayed by Saturn, who signified this hidden and secret state of matter, out of which all visible forms are generated, and into which they sink again; whence he is reported to have devoured his own children. The decay of forms being the work of time, Saturn had also the name of Kronos. He is fabled to have been married to Ops, because matter when united to form becomes visible; and Ops is called the mother of the gods, because their gods were the elements in a formed state, and were no objects of worship to them till they became visible. The same idea you will find under the allegory of Proteus, or original matter the receptacle of all forms, who is said to be discovered by Eidothea his daughter, by her leading him forth out of capacity into actual form.

It is well observed by *Plato*, that while we are in this state of existence, slumbering and as it were dreaming, we cannot see the truth of things except by shadows, as we see them in our sleep; and it is a certain truth, that we do not know the substance or essence of any of the works in nature, but only some of their qualities and properties; for example, all that we know of any animal or vegetable is, that it has such or such qualities, and by these we define it; but what is the substance of it, and what is the hidden essence producing all those visible qualities, we cannot tell.

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* Abstraction makes it stand under the same character to the touch, as darkness stands to the sight, silence to the hearing. We cannot be said to see the one, or hear the other; and yet, without the help of those two senses, we could have no comprehension of those two negations, or sensible privations.

It is the same with body; it would seem as if this were only the outward cover, that there was some such primary matter as has been mentioned common to all visible things, and becoming the subject of the natural or sublunary elements. The conception of an original matter, and which can only be denominated from its capacity or aptitude to receive all forms, appears essential to any rational theory of the constitution of things.

To this principle we must add form, which draws matter as it were out of its chaotic state, and impresses it with a distinctive character, and renders it an object of the senses; not that there ever was actually any matter without body, or body without quality, but they may be so separated by the mind for the better contemplation of the well-ordered generation of things.

The first form that matter assumes is extension, by which its parts become contiguous, that is, joining to one another, and having one common boundary which is threefold, length, breadth, and depth; matter triply extended, is what may be called pure and original body; extension enters into the primary conception of body; but yet extension, though the inseparable quality of all bodies, is itself preceded by something as its source or principle of eduction, without which it would not exist. Thus a point or unity is the essential constituent of a line, a line of a superficies, and a superficies of a solid; so that unity is the essential principle of extension, which is an effect arising from the arrangement of units or points, as number is supposed to flow from the succession or conjunction of units.

Body, even under this character, is still too vague for scientific contemplation; its extension must be bounded, but the bound or limit of body is figure, which becomes the next form after extension.

But matter thus extended and bounded by figure is still something vague and indefinite, scarce an object for the natural philosopher; it is only body mathematical; but this not being sufficient for the purposes of nature, it must be invested with other forms; not only the external form must be duly bounded, but the internal parts must be duly organized, that is, the materials must be properly adjusted, disposed, and arranged, which give rise to body physical.

These three, extension, figure, organization, were considered as sufficient to characterize physical body; figure having respect to its external, organization to its internal, extension being common to both.

From a variation in these universal and primary forms, may arise most of those secondary forms, usually called qualities sensible, being the proper objects of our several sensations; such are roughness and smoothness, hardness and softness, with the tribes of colours, savours, and odours.

Thus matter, the elementary constituent of every composite substance, is a chaotic mass, void of every property, but capable of becoming the basis or recipient of every form; it is therefore itself formless, imperceptible, and passive. But of this universal and common subject, body is produced by the introduction of forms or qualities, which invest and characterize particular portions of it; as in the works of human art, out of one common mass the skill of the statuary educes those beauteous forms, in which a Venus or a Hercules loves to charm and astonish us.

The form or essence of every natural substance, or in other words, its system of internal qualities, extends itself outwardly every way from within; and as it must necessarily stop somewhere, so according to the different points at which it stops in its evolution, it communicates to each substance a different and peculiar figure. Hence the true character of every natural and specific figure, which is not to be considered as a mere surface, but the bound to which the internal essence or form every way extends itself, and at which it terminates.

Hence of all the external qualities there is none so characteristic as figure; it is a kind of universal signature, by which nature makes known to us the several species of her productions; the primary and obvious test by which we pronounce this a vegetable, and that an animal; this an oak, that a lion.

If you proceed from natural subjects to works of art, you will find that figure is almost all that art is able to communicate; it is to this that the painter arrives by addition, the sculptor by detraction, the founder by fusion, and the stucco artist by moulding. Even when you contemplate the tools of art, you will find the figure a principal circumstance. It is from this that the saw divides, the hammer drives, and the pincers extract; and it is from their figure, not materials, that they derive their character and name.

All natural and artificial things partake of form, and partake also of matter; form is derived to them from mind; superficial form is derived to the works and performances of art, from the minds of human artists; internal and essential form is derived to the works and operations of nature from the Divine Mind. For though the common subject matter of all external forms or beings is in itself boundless and void of all form, yet it is every where bounded and impressed with form by the Great Cause of all things, from whom proceeds every form, the bound of every distinct being, and the whole of its essence or essential nature. The ancients allowed the necessity of a spiritual agency to animate and produce form, that it might be conjoined to the Divine Being, and subjected to the Divine Providence; hence matter received that specific form, which is most agreeable to the order and beauty of the universe. Through the whole universe, we see matter bearing the marks of mind, and every where impressed with the prototypes of genus and species; we see all animals continually seeking by natural instinct the welfare of their own being, the continuance of their kind, the preservation of their offspring; and we feel ourselves, in the rational part of our nature, charmed with the sight of beauty and smitten with the love of it; we feel our minds urged on to inquiries after truth as it were by a presensation of its supreme beauty, and a preconception of its being to us a good; impulses so strong, that however often we may be diverted from such inquiry by the necessities of the body, by fancy, or passion, or the desire of imaginary good, yet, when left to the exercise of its own faculties, unhindered and undisturbed, the mind is always in pursuit of it, tracing it through the natural and necessary connexion of ideas; and when it has found it, is never driven away from this attachment so long as it turns the inward eye thereto. Thus our view of all the forms of outward nature, our experience of the disposition of all animals, and the consciousness of our mental feelings, sentiments, and actions, conspire to evince to us the spiritual and divine agency that runs through the whole.

From form and figure, the ancients proceeded to another and higher form, the life, or animating principle, which was conjoined in many instances to matter: these are forms which characterize not by visible qualities, but by their respective powers, operations, and energies. As is the piper's art to the pipe, the harper's to the harp, so is the soul of the lion to the body leonine, the soul of man to the body human.*

It is to this diversity of spiritual power in these, that the diversity of the organization in the corporeal world has reference. That strong and nervous leg, so well armed with tearing fangs, how perfectly is it correspondent to the fierce instincts of the lion! Had it been adorned, like the human arm, with fingers instead of fangs, the natural energies of a lion had been all of them defeated. That more delicate structure of an arm, terminating in fingers so nicely diversified, how perfectly does it correspond to the pregnant invention of the human soul! Had these fingers been fangs, what had become of poor Art, that by her operations procures us so many elegancies and utilities? It is here we behold the harmony between the visible world and the invisible, between the active and the passive, between the living and the lifeless. The whole variety in bodies, as well natural as artificial, is solely referable to the previous variety in these their animating essences; it is for the sake of these they exist; it is by these they are employed; and without them they would be as useless as the shoe without a foot.

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* Those, says the *stagirite*, who adopt the notion of placing any soul in any body, talk as absurdly, as if a person was to say the carpenter's art might enter into a musician's pipe: now it is necessary every art should use its proper instruments, and every soul its proper body.

As nothing can become known by that which it has not, so it would be absurd to describe these spiritual powers by any visible or other qualities, the objects of our sensations. The sculptor's art is not figure, but it is that through which figure is imparted to something else. The harper's art is not sound, but it is that through which sounds are called forth from something else. They are of themselves no objects either of the ear or eye; but their nature is understood in this, that were they never to exert their proper energies on their proper subjects, the marble would remain for ever shapeless, the harp would remain for ever silent.

It is the same in natural being; the living essence of a natural being is neither its organization, nor its figure, nor any other of those inferior forms, which make up the system of its visible qualities; but it is the power, which not being that organization, nor that figure, nor those qualities, is yet able to produce, preserve, and employ them. It is the power, which departing, the body ceases to live, and the members soon pass into putrefaction and decay. It is in mind that forms exist, before matter can receive them; it is from mind, when they adorn matter, that they primarily proceed; so that whether we contemplate the works of art, or the more excellent operations in nature, all that we look at as beautiful, or listen to as harmonious, is the genuine effluence or emanation of mind.

These transcendant objects are of an origin so truly mental, that nothing but mind or intellect can recognize or comprehend them. Hence, if this intellective faculty be wanting, as it is in inferior animals, or be unhappily debased, as too often happens to our own species; though their sensitive organs may be exquisite to a degree, yet are such to such objects, as if they had no organs at all. "Eyes have they, and see not."

From this view of things, we see that means do not lead to ends, but ends lead to means; that it was not the organization of the sheep's body which produced the gentle instincts of the sheep; nor that of the lion's body, which produced the ferocious instincts of the lion; but because in the divine economy of the whole such spiritual principles necessarily exist, they are clothed with such bodies as are agreeable to their respective natures, and fit for their proper business in this world.

But still all the different capabilities of mind and matter are derived from the Almighty Being, who is the source of all capacity, as well as of all power; who hath distributed amongst all animals, both the irrational and the rational, such capacities, abilities, and energies, as may fit them for being salutary and serviceable; and who has impressed on matter such distinct characters of his own beauty as the subject will bear; as in the glory of the heavens, in the symmetry of form, in the harmony of sounds, and in the elegance of colours, in the elaborate texture of the smallest leaf, and in the infinitely fine mechanism of such insects and minims of nature, as are scarce visible to the eye of the clearest discernment.

From the foregoing principles, which throw light on all the objects of mind, make them visible as it were to the mind's eye, and at the same time pour light into that eye to see those objects, the ancients concluded the Divine Being to be Goodness itself and Truth itself; and that these were continually diffused into all things, in proportion to their several capacities. Under these characters, the most obvious to be perceived, they considered him as the cause, both final and formal, of all the good which is enjoyed by beings who are capable of any enjoyment, and of all that beauty and truth which is enjoyed by those who have a sense of beauty and truth.

" But they even go further than this. The sages of antiquity affirm, with great appearance of truth and strength of argument, that we have a principle of existence within us, infinitely more intimate, more valuable and durable, than this expanse of heaven and earth, this inclosure of flesh and blood, or any species of life and health that can be subject to mortality; that this principle of existence is of divine original, incorruptible, unperishable, a fount that must flow for ever, a flame that cannot be extinguished; that the virtues are its treasures, its enjoyment, its beauty, and illumination; that these constitute the man, all that can properly or inherently be denominated himself, and all that deserves his regard or attention."

"Thus far stretches the doctrine of Pagan philosophy; and conformably thereto, its votaries rejected the world, with all its enjoyments, and even boasted a superiority over poverty, calamity, pain, sickness, and death."

"But if to this we add something of that Redeeming Word, which brought life and immortality to the fulness of their light, and the weight of their glory; all that is temporal fades and vanishes away in the comparison with any article that is eternal: the goods of this world are no longer to be relished, nor its evils to be feared; the present is sunk and lost in the greatness of the future; and possession is cast behind us, in reaching after the immensity of that for which we hope."

"Nothing becomes considerable, nothing of estimation, save so far as it conduces to the purchase or acquisition of some article or degree of goodness; some of those benevolences, those charities, those affections, those elegancies of elevated humanity, which the great Apostle to the Gentiles prefers even to the graces of faith and hope; and which he says "cannot fail," but must endure for ever, when faith is swallowed up of vision, and hope of enjoyment; when every will shall be subdued to the Will of Good to all; when our Saviour will take in hand the resigned chordage of our hearts, and will tune them, as so many instruments, to the song of his own sentiments, and will touch them with the fingers of his own feelings. Then the happiness of each shall multiply, and overflow in the wishes and participation of the happiness of all."

OF MATERIALISM.

From contemplating the beautiful opinions of the ancients, I unwillingly turn to the unpleasant task of pointing out the errors of some modern philosophers, who, from false views of the nature of matter, and from ignorance of the principles of mind, have adopted the dire and gloomy system of materialism. If this system be true, if our soul be mortal, reason is a dream, and virtue loses all the splendor which makes it godlike in our eyes. If you delight in friendship, if you respect virtue, and are charmed with beauty and perfection, reject with horror the system of materialism. It is a system that will checquer every scene of joy with despair. A breath, a beat of the pulse, deprives you of all that is glorious and valuable. The god-admiring being putrefies, moulders, and becomes dust.*

In refuting these dangerous principles, I shall chiefly follow that excellent guide of philosophers, M. *de Luc.*† In the first volume of the work here cited, you will find a full and satisfactory refutation of the political and theological errors of modern philosophers, even so far back as the year 1774. You will find him pointing out to those of France, the consequences of the wretched system they had adopted! Unhappy men, heedless of the voice of benevolence, and the calls of reason, they have verified his predictions, and proved at the same time the weakness and malignancy of their principles.

A modern materialist among ourselves has gone so far as to say, "That he rather thinks the whole of man is some uniform composition, and that the property of perception, as well as the other powers that are termed mental, is the result of such an organical structure as that of the brain; and, consequently, that the whole man becomes extinct at death, and that we have no hopes of surviving the grave, but what is derived from the scheme of revelation."

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* *Mendelssohn's* Phædon, p. 96.

† M. *de Luc's* Lettres Physiques et Morales sur L'Histoire de la Terre et de l'Homme, vol. i.

Many observations occur on this passage; a few only will suffice here. I think we may infer therefrom, that the author thereof considers the soul to be, 1. Of the same nature with the body. 2. That it is not immortal. 3. That he derives his hopes of surviving the grave, from the general resurrection of the dead. 4. That there is no such existence as a soul between the hour of death and the general resurrection, when the man resumes the same organical structure of the brain. Now the revelation of Christ considers the soul as of a distinct nature from the body, and as existing after death. From revelation, therefore, this author can derive no hope of surviving after the grave, if the soul becomes extinct with the body.

But further, the mystery opened in the gospel, relates to quite another kind of life in the soul, than that which is implied or consists in its bare immortality. For immortality considered in itself, as a physical necessity of its continuing in life for ever, may as well be a curse as a blessing, and have heaven or hell for its portion. But this is not the life and immortality that the gospel preaches, It leaves this natural immortality, which belongs as well to the spirits of darkness as to men, as wholly untouched as it does the original of the alphabet.

The immortality of the gospel is not so called on account of its eternal duration, but because the soul is thereby eternally freed from those evils and errors which would prevent it from enjoying the light and life of heaven. But though the natural immortality of the mind is never expressly mentioned or asserted in the gospel, yet is it fully proved therein, because unavoidably supposed, and necessarily implied in and by the plain and open doctrine of the gospel.

In the system of Dr. *Hartley*, the soul was rendered purely passive; the memory, the judgment, the will, and the principle of all voluntary motions, are placed in the physical properties of the organs; thus reducing all the intellectual and actual faculties to motions of the brain, of which the soul is only a spectator, though, by a continual illusion, she attributes them to herself. But a more modern materialist has thought it would be simpler to make the scene its own spectator.

In order to effect this, he endeavours to render those ridiculous who assert, that the soul and body have no property in common, though they have a mutual action on each other, and are capable of an intimate communication. The ridicule thrown upon this opinion may divert the reader's attention; but it gives no strength to the arguments of the materialist. For we have no objection to allow him, that thinking substances and physical substances have very great relations, and that they have no repugnance, but a certain correspondence the one to the other. The body being a visible agent in all we do, has indeed made some weak heads imagine, that we are nothing else but body; as, from the same want of thought, some have concluded that there is nothing besides the material world, because nothing else is obvious to their eyes.

The man of matter, penetrated with the idea that he should reconcile all the world to materialism, if he could exalt matter; and conceiving that what was supposed to be despicable and abject therein consisted in its impenetrability, and its inertia; he has endeavoured to deprive it of these properties, but in vain; for matter, even coming out of his own hands, is as impenetrable and inert as that of *Newton*.

It is rather singular in our materialist to begin his work with expressing his veneration for the great man I have just named, proposing to adopt his rules of philosophizing, and complaining of their having been fatally neglected in regard to the most general principles of human knowledge. And yet, at the same time, contradicting the foundations of his system. Sir *Isaac Newton* continually speaks of solidity, pores, impenetrability, inertia, divisibility, mass. But our materialist wishes to destroy these notions, because they render matter contemptible.

It is affirmed, says the materialist, that matter is a substance necessarily solid and impenetrable, and in itself destitute of all powers, as of attraction and repulsion;

and that it possesses a *vis inertiæ*, and is totally indifferent to motion or rest. That the vulgar should have formed such opinions and acquiesce in them, he does not wonder, because there are common errors which must necessarily lead them to form such a judgment; but it is extraordinary, he thinks, that philosophers should have been duped by appearances so void of all real foundation.

The materialist endeavours to prove, that atoms are neither hard, nor solid, nor impenetrable; that their solidity only depends on a certain power, &c. But M. *de Luc* has clearly shown, that all his reasoning on this head is founded on a mistake, is occasioned by his confounding impenetrability with hardness, although the notions of these have no relation to each other. By impenetrability, *Newton*, and every other sound philosopher, means the simple idea necessarily connected with extension, namely, that no two particles or bodies can exist at the same time in the same place. And we shall soon show you, that the matter of our materialist is as impenetrable as our own; his argument bears only against the hardness of atoms, or first elements; but it lessens not the certitude thereof. For, as an actual infinite division is a contradiction, it follows, that the atoms must be undivided; and this is all that is here meant by hardness. The materialist says, that without a power, the parts of atoms would separate themselves, and be dispersed; but I say, that without a power, they can neither be separated nor dispersed; and you can be under no doubt, which of these propositions is most conformable to the Newtonian rules of philosophy.

Again, because the materialist has thought proper to divest matter of solidity and inertia, is it therefore entitled to the powers of sensation and thought? Because he chooses to define matter *a substance possessed of the property of extension, and of powers of attraction and repulsion*, are we to conclude that there is no longer any reason to suppose in man two distinct substances?

"If there be any truth in late discoveries in philosophy (says this man) resistance is in most cases caused by something of a quite different nature from any thing material or solid, *viz.* by a power of repulsion, acting at a distance from the body to which it has been supposed to belong; and in no case whatever can it be proved, that resistance is occasioned by any thing else." He relates those experiments which show, that there is a considerable difficulty to bring bodies to a real contact; those on the dilatations and condensations of bodies, which prove that their parts are not so close as they might be; and conceiving that the particles can never be brought into real contact, he concludes with saying, "That there is nothing real in matter but attractions and repulsions; and that these opposite powers relate only to certain points of space, points mathematical, without solidity, and which can only be considered as the place to which the power is referred."

I must confess with M. *de Luc*, that if any one can comprehend what these powers are which have extension, and which are still only mathematical points, and which attract and repel each other, and can consider such an account as a solution of the difficulties concerning matter, he has either a sense more or less than I have. *Newton* had seen these repulsions, or the resistance that is found in bringing certain bodies into perfect contact; but he considered them simply as the effect of an elastic fluid surrounding them, of which air and fire afford us palpable examples. He also declared expressly, that he considered energy without substance as an absurdity. So far was he from thinking that the resistance made by bodies in approaching each other was invincible, that on the contrary he proved by experiment, and he explained by his elastic fluid, that after the repulsion was conquered, the effect of the proximity was a strong adhesion of the bodies, produced by the exterior pressure of the same elastic fluid.

Not to insist, that the powers assumed by our materialist are entirely hypothetical and contrary to the general agency in nature, and therefore inadmissible, it will be easy to prove, that even his own account of matter establishes principles contrary to his wretched scheme, and you will find his matter as inert as ours; for the idea of impenetrability only supposes that two extended substances cannot be in the same place at the same time. Now the materialist must give the same property to his sphere of powers, because he still considers extension as necessary to his matter; his spheres therefore must be impenetrable, one cannot be in the place of the other at the same time, for this alone is the physical idea of impenetrability, which has nothing in common with hardness. In whatever manner we conceive matter, it may be compressed, divided, subdivided to infinity, and yet continue impenetrable.

It is the same with inertness: we shall find his particles possessed thereof as well as ours; for what is inertia, but matter continuing at rest until some cause puts it in motion? Can the materialist refuse this property to his sphere of powers? They have relation to space, they occupy place, and are moved; consequently, all the true notions of motion are as applicable to these, as to any other conception of matter; it is as inert as ours, and after all his endeavours to separate the ideas of impenetrability and inertness from matter, he has still made matter impenetrable and inert. They are essential properties of that substance which composes the physical world, which no effort of the imagination can separate from these substances.

But the matter of this man is not only inert and impenetrable, but it is also possessed, by his own confession, of properties quite of a different nature from any thing material; so that, after endeavouring to prove, that matter was the only substance entitled to existence, that it was the cause of all appearances, yet, to account for the various phenomena, he is obliged to give us something of a quite different nature from any thing material; this immaterial something is the power of repulsion, the property of all matter. Is such a power really different from every thing material? or can matter act by immaterial powers? So rooted is the aversion of our materialist to mind, such is his pneumatophobia, that he will suppose or adopt any thing sooner than admit the agency of mind.

He tells us, we conceive spirit and body to be possessed of no common property. We ask him where he finds any thing common between perception and the powers of attraction and repulsion, that is, between the sentient and corporeal powers? Nay, what is there common between the attractive and repulsive powers themselves? We insist, that sensation and the powers of attraction and repulsion, as likewise these two respectively, are as opposite and as far removed from the possession of any common property as those very powers which, as immaterialists, we attribute to matter and spirit.

It is mind that perceives the difference not only between black and white, bitter and sweet, but (which no sense is equal to) the difference between black and bitter, white and sweet, and the various tribes of heterogeneous attributes. Nor does it show this supremacy in these recognitions only; but likewise when at one and the same view it recognizes objects of sense and intellect united, as in the case of syllogisms made of propositions particular and universal.

This joint recognition of things multiform, contrary, and heterogeneous, and that by the same faculty, and in the same undivided instant, proves in the strongest manner that the mind is immaterial; for body is incapable of such a simple and perfect unity.

But leaving these, and many other considerations, let us see whether the matter of the materialist, such as he imagined himself, will seem fitter for becoming a percipient being. To him indeed it appears easy, for since the thing that we call matter consists only in powers, there is nothing to prevent its having the powers of perception; but a little consideration will show, that the difficulty is more unsurmountable in his system than in any other.

For, as long as we considered a substance to which the powers of attraction and repulsion were attributed, there remained this obscure idea, "that a substance might be endowed with many kinds of powers, and it would be necessary to discuss this question; but in the present case there is no room for discussion; if a power can be conceived without substance, it will be itself the whole being. A power of attraction is a power of attraction, and can be nothing else. This class of powers forms, according to our materialist, one part of matter; the remainder is formed of powers of repulsion, which can be nothing else but powers of repulsion. If therefore he wants power of perception, he must make it, his matter has no such power.

"In what kind of principles, simple or compounded, does he suppose even the powers of attraction and repulsion to reside?"

"Does he think the same numerical substance can at the same time possess the attractive and repellent powers; *i. e.* that the same body can be at once attractive and repulsive? By repulsion all absolute approach or contact is impeded: by attraction, bodies, when at a distance from each other, tend to mutual approach."

"Now as he declares these powers to be essential to all matter, all matter is ever tending to approach, and ever receding from all matter; A and B always going to, and at the same instant, always going from each other!"[*]

But why does the materialist suppose the physical world to be composed only of small spheres of attraction and repulsion? It is this: on examining the various phenomena one by one, he thinks, that at the end of his analysis, he finds nothing remaining but attractions and repulsions without substance. The elements are therefore only attraction and repulsion, when the phenomena indicate nothing but attraction and repulsion; but when the phenomena indicate sensibility and self-consciousness, what are the elements? Are they still only attraction and repulsion? Is there no faculty of perception in the ingredients?

Our materialist feels no force in the objection: with him it is no more "than if it were said, that no harmony could result from a harpsichord, because the single notes, separately taken, can make no harmony." The man who can thus answer, does not understand the nature of the proof; for what is harmony? it is the union of two or more sounds, of which the whole is agreeable to the ear. It is therefore of the essence of harmony not to be in each of its elements, as it is of number not to be in unity.

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[*] *Berrington's* Immaterialism Delineated, p. 229.

Mr. *Berrington* pursues this still further: harmony in music arises from a number of modulated sounds; proportion also or symmetry in achitecture arises from the appropriate arrangement of materials; but each sound taken separately is void of harmony, as each stone in a building is divested of proportion. In the external objects themselves, what have you, but detached and insulated sound, detached and insulated stones only rising in a definite order of succession and co-existence? What is it then that gives existence to the charms of harmony and proportion? That being alone which, gifted with percipient and comparing powers, can take in such various tones and various parts, can compare them together, and therein perceive accord and proportion. This exalted power through the wide stretch of nature is alone capable of giving existence to such unsubstantial forms. Harmony and symmetry are mere effects of comparison, all their reality is derived from man.

But if the brain were the sole substance on which such impressions are formed, harmony would be eternally excluded from the material universe; for on that supposition a thousand distinct vibrations in the nervous system could no more give it reality, than it could be raised by the sound itself. Were each particular sounding particle to act on the ear, or each part in an edifice on the eye, and proceed no further, what would be the effect? Evidently only this; the most regular and finished structure would remain as a heap of sand, and the airs of an *Arne* be as unaffecting as the whistling of the wind. Just so must it be in the material system, because no centre of union can exist therein, and consequently no comparison or just perception; therefore no harmony or proportion can take place: so that the materialist's own examples, adduced to prove, that perception may belong to a whole system, and not to its component parts, directly make against him, and support the immaterial hypothesis.

Is there nothing real in life, says our materialist, because each particle has not life? It is not more easy to conceive the nature of life, then of perception. The life of the human body in one point of view may be considered only as an arrangement of organs put in action by the elements; but even in this lowest view, like harmony, life is the result of the whole, and is not to be found in the component elements. In order that a watch may measure time, it is not necessary that each of its particles or each of its parts should also measure time. In a word, these are only properties attached to the idea of composition; or to speak more exactly, they are results not properties, last effects not primary causes, which are evidently different.

If the materialist had properly considered the examples he used, he would have found therein a full confutation of his own principles; as an instance, he affirms, "it may just as well be said, that sound cannot consist of a vibration of the air, because no sound could result from the motion of a single particle of that elastic fluid, as that perception cannot belong to a whole system and not to the component parts." No one, I believe, ever asserted, that the individual particles of air were not concerned in causing sound, for each particle of the fluid which transmits the sounds transmits a part, but too feeble to be perceived by our organs; it is from the number of these small impressions that one results sufficient to be perceived. Is not the sentiment of light, which can only take place from a pencil of rays, produced, because each particle has the faculty of exciting this sentiment?

Much is usually urged in favour of the unity of the being that perceives, and of the absolute impropriety of attributing thereto any parts according to our idea of parts in matter. This reasoning he considers as proving no more against the divisibility of mind, than if we concluded, "because a sphere was one thing, that it was therefore indivisible. It is true, he adds, that it is impossible to divide one sphere into two spheres, but it may be divided into parts, so as to be no longer a sphere. In the same manner, the system of intelligence, that we denominate the soul of man, cannot be divided into two souls; but it may be divided and dissolved, so as no longer to form a system of intelligence, a soul."

After this comparison, in which he triumphs much, he adds, "If any one can define the unity of conscience in a more favourable manner for the immateriality of the soul, he should be glad to hear it, and pay it a proper attention." As the definition of this unity has been often and so well given; as every idea of parts affirmed or denied with respect to this oneness of the mind, is as absurd as to apply thereto colour or taste, it will be sufficient to answer his argument in a manner somewhat similar to his own, "that if any one can prove, that self-consciousness depends on a spherical, cubical, pyramidical, or any other material form, you may admit this faculty as depending on a physical whole."

Not tired with comparisons, our materialist elsewhere compares the unity of the soul to that of a collective number of moral beings, as of states and societies, which, though composed of several members, have a species of unity. Here, as in the comparison he drew from sound, the materialist is really pleading our cause, and this instance is still far less to his purpose than the former; for as the unity of man can only be perceived and compared to itself, if this unity was a compound, each of the parts must have the property of the whole, that is, self-consciousness. For the collective moral beings, to which the materialist compares us, have a common unity, because each of the members has a unity of its own.

The unity of consciousness is inseparable from every mode of perception; whenever I perceive, or whenever I think, a sentiment arises within me, which tells me, that it is I that perceive, and I that think. Thus to feel is to be conscious; consciousness tells me that I, who now am, am the same person who was yesterday, the day before, and so on; it connects the past with the present, and can only reside in a spiritual undivided substance.

Having finished my remarks on the notions of a modern materialist, I have only to regret the occasion of them, and to hope that ere long the fictions, sophistry, and errors which encompass his mind, may be removed, and that he may return to the faith once delivered to the saints. In the mean while, on our parts, let us contemplate him with the greatest compassion, remembering that discernment is alone clear and unerring in the Supreme Mind; all whose ideas are pure and perfect, whose power is constant energy, and whose eye is intellectual light itself, unobstructed, unbroken, and unclouded by any of its objects. But the eye of man's reason, how short-sighted and weak it is; through what a medium of false colourings, and with what interrupted glances it discerns the few objects to which it is directed; and how frequently and how greatly man's judgment errs, is well known to every fair mind, who has had experience of her particular and private self; and has conversed with truth and right reason intimately as with friends, yet modestly as a disciple with his teachers, and simply as a child with his natural parents.

OF THE UNITY OF THE PERCIPIENT BEING.

As the unity of the percipient being is the argument at which materialists in general aim all their forces, knowing that, if this be allowed, their scheme falls to the ground of itself; knowing, that it is as it were an *egis* impenetrable against all their monstrous endeavours to form the soul of man by an assemblage of material parts; I shall, therefore, endeavour to place it before you in different points of view, and in different manners; thus the arguments which might escape your attention under one form, may have their due weight under another.

Let a man lose a leg or an arm, or an eye or an ear, he stills continues the same man, and holds his rank upon the list of beings as he did before; whatever can be separated from him, he may look upon as a possession, an instrument, or an organ of conveyance. But that alone, which remains after all imaginable separation, is properly himself.

Besides, our organs have their separate offices, not interchangeable with one another. The eye which sees cannot hear, and the ear which hears can never see; but these being numerically distinct, if they were the perceptive substance, it would follow, that what sees is a different thing or substance from that which hears. Therefore, they can only be channels of conveyance to some one individual thing; for no man can doubt that it is the same himself which sees, and hears, and receives all the perceptions; and we cannot conceive this self divisible, because what might be taken away upon division, would not be him, for he cannot be parted from himself.

Compound bodies consist of parts, having the same nature and primary proportion with themselves; nor is it conceivable, that any assortment of unsolid or immoveable parts should form a solid and moveable body. For if composition prevailed in mind, every self must contain a number of little selves, every mind many little minds, and every sentient principle a multitude of sentient principles. But this is a proposal that will not bear mentioning, for who would not be shocked to hear talk of half or quarter of a man's self?

To avoid this absurdity, some have asserted, that self, mind, or a perceptive being, may be produced by a combination of unperceptive principles. Not to urge, that in this case a creation may be effected by compounding, let us remark, that upon such a combination, the parts considered simply, cannot have, nor are they pretended to have, any other properties than they possessed before; neither can they club to take their several shares of a perception; for perception has not parts without parts, and therefore cannot be received piece-meal; but the perceptivity resides in the whole compound jointly. Now it seems very hard to understand how a collection of distinct substances, for such every compound must be, can perceive what is not perceived by each of them. I can conceive a whole camp to hear the evening gun, because every man in the camp hears it; but I cannot comprehend how they can all hear a sound that escapes every single person; nor can I, for my life, distinguish between their all hearing it, and every one of them hearing it. I can no more comprehend how an imperfect whisper, heard by twenty persons, shall become an audible voice, than how they can all hear a sound heard by none of them singly.

If the mind be a compound, then upon some of the parts being slipped away, and others substituted in their room, though it might still remain a mind, it must be a different one from what it was before; and as *Lucretius* tells us, the mind grows and decays with the body, every man would have a different self in his childhood, his maturity, and his old age!

But it may be said, mind is not so much a collection of particular atoms as a figure, or harmony, resulting from the order wherein they lie, and may therefore continue the same, although some or all of the atoms be shifted. For if you place twelve shillings in a circle, change your shillings as often as you please, you do not alter the figure, which still remains a circle. But what then becomes of our own existence? For form has none, but is only a modification, or particular manner of existence in body; and harmony has none, being nothing more than the concordance, or mutual congruity of sounds, as perceived by mind.

I cannot conceive it possible, that perceptivity should be annexed to a system, by which must be understood a composition of matter: it must reside in something that is numerically one and undivided. For suppose an object to strike upon any one component part of the system, if it raises a complete perception there, the rest are useless. If the perception does not begin till this part has communicated the impulse to some other, then is that other the perceptive substance, and the first only an organ of conveyance; but if both receive it equally, then does the faculty belong not to the system, but to the several parts; nor can each take its respective share of the perception, for, as we observed before, it is inconceivable that a perception should be received piece-meal, or made up of what is no perception.

This matter will appear plainer, if you consider the nature of judgment and comparison, where both terms of the one, and both branches of the other, must be apprehended together, in order to determine between them. If this man knows the other properties of gold, and that man knows what ductility is; they can never know the more for this, either jointly or separately, whether gold be ductile or not: so if one man be ever so well acquainted with St. Peter's at Rome, and the other with St. Paul's at London, they can by this knowledge never tell which is the larger or handsomer, or make any other comparison by virtue of this knowledge. But you say, one may communicate this knowledge to the other: very true; but then each has the idea of both before him in his mind, and the judgment is the act of each individually, not of both jointly. Now the case is the same with the percipient system; let the idea of an elephant be impressed upon the particle A, and that of a mouse upon the particle B, they can never know, either jointly or separately, which is the larger creature, nor can a judgment be formed, until the ideas of both coincide in one and the same individual, It is unintelligible to talk of any impulse arising solely from the composition; so neither can any system have a power of beginning motion, where there was none in the parts. A hundred men may certainly lift a weight that would crush any one of them; but a thousand would never be able to stir it, unless each man had some strength of his own, independent of the rest. Whence we may justly conclude, perceptivity and activity to the primary qualities, essential to the substances possessing them, inseparable therefrom, belonging to individuals, and not producible by any combination, or organical structure whatsoever, of imperceptive and inactive ingredients.*

The importance of the subject is so great, its connexion with your best interests so intimate, that you will, I hope, excuse my having placed it in various lights; and permit me to conclude, by summing up the arguments, so as to exhibit the reasoning under a form still different from those that have preceded.

Whether matter be indefinitely divisible or not, if it be capable of any degree of perception, such perception must either be naturally inherent, or arise from some peculiar modification: now, as no two particles of matter can exist in the same place, (for then neither part would exist in any place, as each would occupy the place of the other) the particles, however harmoniously modified, or closely united, are absolutely distinct from each other, since their coherence can only consist in neighbourhood and contiguity, and not in corporation.

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* *Tucker's* Light of Nature Pursued, vol, ii. part 1, p. 67.

If, then, the parts so distinct have any inherent perception, they must have a perception as distinct from each other as the particles; and if infinitely divisible, there is such a confusion of perceptions as is too absurd for any thing but a jest.

But if matter be reducible to atoms, and every atom supposed to perceive, I would ask, how atoms can be organized, so as to see, hear, smell, &c. And if organization be necessary to perception in matter, either such perception arises entirely new on the organization, or the organization only gives a liberty of action to the perception that was prior and distinctly latent in every part.

But if, on the former supposition, such perception be solely produced by the organization or modification, you must remember, that organization and modification, however exquisite or mechanical, are still no other than a mode of form or figure, the most extraneous and incidental of any property of matter; whereas perception is the most simple and absolute of any thing we know, and by which alone we know all that we do know. Such a hypothesis, therefore, carries in itself the most palpable contradiction and confutation, as it makes what is simple, absolute, and invariable, to be produced by what is compound, precarious, and changeable.

But if it be alledged, that by the organization the parts become so loving and neighbourly, as by sharing the perception of each other, to make one amicable union of the whole, each part must still retain its proper right to its portion of perception; and if, upon any accident, a member of the system be lopped off, a piece of such united perception being gone, we have only a piece of perception. And thus perception, the most simple of all units, must be daily and hourly divided by the perpetual flux of matter.

Thus, if the intelligence within us was corporeal, what one part knows would be unknown to the others; we could not compare nor unite our ideas, so as to make propositions, nor these propositions, so as to form arguments, nor these arguments, so as to form one continued discourse. All the operations of comparing, uniting, and compounding our ideas, suppose one individual, indivisible, self-conscious, spiritual being. Moreover, we hear, see, taste, touch, and smell, by five different organs; and these sensations are as different as the organs themselves; yet they are all united in the same substance which compares them, judges them, and distinguishes between them. The change of form, bulk, and place in organized or unorganized bodies, does not alter the intimate essence of things, and therefore cannot add to them a new and dissimilar property. Intelligence is a perfection, non-intelligence a defect; so that it is absolutely impossible that what is unintelligent and insensible before organization, can become intelligent and self-conscious by organization; because organization does not alter the nature of things. A mass of numberless subtile, invisible, and unintelligent polygons, spheres, cylinders, pyramids, or any other small solids, cannot become intelligent and self-conscious by organical structure. Thus the poet:

Where can this soul then take its birth?
Not sure from matter, from dull clods of earth;
But from a living spirit lodg'd within,
Which governs all the bodily machine;
Just as th' almighty universal soul
Directs and animates the whole,
Cease then to wonder how th' immortal mind
Can live, when from the body quite disjoin'd;
But rather wonder if she e'er could die,
So fram'd, so fashion'd for eternity;
Self mov'd, nor form'd of parts together tyed,
Which time can dissipate, and force divide;
For beings of this make can never die
Whose pow'rs within themselves, and their own essence lie."

Having thus refuted the gloomy tenets of materialism, let us finish this lecture with a different theme; let the consideration of the wonderful powers of the soul make us attempt a higher strain, a hymn of praise to the Lord our Saviour, the joy of every soul, and the desire of all nations. For, if in the verdure of the field, and the azure of the sky, the unlettered rustic admire thy creative power, how blind must that man be, who, contemplating his own living structure, and moral frame, discerns not thy forming hand! How various and complicated the machinery of the one, how extensive and noble the powers of the other.

How sublime are all the faculties of the mind, thoughts that wing infinity; apprehensions that reach through eternity; a fancy that creates; an imagination that contains a universe; wishes that a world hath not wherewithal to satisfy; desires that know neither end nor bound! It is endued by thee with divine prerogatives, invested with spiritual powers, and enabled ardently to aspire after the felicities of heaven.

Even while man pursues happiness here as his chief aim, thou blendest self-love into the social direction; thou infusest the generous principle, which makes him feel for sorrows not his own; thy divine hand formed the connecting tye, and by sympathy linked man to man, that all might tend towards mutual association; thy wisdom hath erected within him a throne for virtue; but thou hast not decked her with beauty only, but hast thrown around her the awful effulgence of divine authority; and yet thou hast left him a field open for free and generous action, in which, performing a glorious course, he may attain the high reward by thee allotted; thou entertainest him with all the glories of the material world, and enablest him to penetrate into the recesses of nature. It is thou that givest to all bodies their forms and their motions, to all spirits their reason and their virtues; it is from thee that every thing flows, it is in thee that every thing exists, it is by thee that every thing lives, and to thee alone should all things be referred. We see thee existing in wisdom and in benevolence both supreme. As spots in the sun's bright orb, so in the universal plan scattered evils are lost in the blaze of superabundant goodness. Even by the research of human reason, weak as it is, those seeming evils continually diminish and fly away; and objects supposed superfluous or noxious assume a beneficial aspect. How much more to thine all-penetrating eye must all appear excellent and fair! It must be so, we cannot doubt; neither imperfection nor malice dwells with thee; thou appointest as salutary what we lament as painful. Even the follies and vices of men administer to thy wise designs; and as at the beginning of days, thou sawest, so thou seest and pronouncest still, that every thing which thou hast made is good.

Now we see through a glass darkly, and know but in part, but the time cometh when we shall know even as we are known. We shall then see things in that sacred uncreated light, which permits no error, which admits of no obscurity. Then, O Almighty God, all that thou art now doing, all that thou hast done, will be found harmoniously beautiful, and divinely worthy of thee, its great and sacred Lord. Then will thy creation and government appear sublimely wise to the purified eye of man; and what now bewilders and perplexes us, what cannot now be comprehended by us, will be seen in all its unerring rectitude, will blaze forth upon us in the clearness of its holy splendor, and we shall join with angels and archangels in declaring that heaven and earth are full of thy glory.*

LECTURE XXVI.

ON MECHANICS.

ONE of the first arts men had occasion to practice was that of *mechanics ;* it was therefore studied in the earliest ages of the world. Possessing very little theory, what knowledge they acquired was more the result of experience than of reasoning, having neither any methods of working, nor instruments to work with, but what they must first invent; their progress was slow, their performance rude and imperfect.

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* Lord *Kains.*—*Commyns's* Sermons.

We hear, therefore, of no considerable mechanical inventions for a long series of ages; the first we find mentioned are in the book of Genesis; there we have an account of ships on the Mediterranean, even as early as the days of *Jacob*. In the time of *Saul*, 1070 years before Christ, we find the Philistines bringing 30,000 chariots into the field; in 1030 before Christ, *Ammon* built tall and long ships with sails on the Red Sea and the Mediterranean; about 800 years before Christ, *Uzziah* made engines to be used on the towers and bulwarks, to shoot arrows and great stones.

Corn-mills were of early invention, for in Deuteronomy we find it was not lawful for any man to pledge the nether or the upper millstone. In Jeremiah we read of the potter's wheel. The works of *Archimedes*, who lived about 200 years before Christ, would alone afford sufficient matter for a volume; some of his discoveries appear so much above the reach of man, that many among the moderns have found it more easy to doubt their existence than to imitate them. His name still occupies the foremost ranks in the science of mathematics; by his mechanical knowledge, he alone, for three successive years, supported the tottering towers of Syracuse, and baffled the attempts of the Romans. *Marcellus* admired the superior science of *Archimedes*: he instructed the Romans to respect his safety, but the impatience of a soldier proved fatal to his life.

It is not easy, at this distance of time, to conceive the nature of their machines, or to say how they reared their bulky towers 150 feet in height, and 60 in circumference, with a battering ram at the bottom sufficient to beat down walls, with a drawbridge in the middle to be let down upon the wall of the attacked city; and at the top a body of men, who being placed above the besieged, harrassed them without danger.

It would be endless to enumerate the instances of their mechanical abilities. Italy is filled with monuments, and the ruins of monuments, which aid us in comprehending the resources and genius of the ancient Romans. The stones which are laid upon the tops of the pyramids of Egypt, each of which is as large as a small house, create wonder even in a modern mechanic, and teach him to reverence the superior arts of antiquity.

"Ignorant of the numerous arts which depend on mechanical knowledge, man would exist as in a desert, comfortless and unsocial, little superior in enjoyment to the lion and the tyger, but much their inferior in strength and safety." Aided by these the fields are cultivated, the wilderness becomes a garden, strength is given to the fortress, and elegance to the palace; and man beholds himself far removed from those animals, to which while in a state of nature he seems to have been nearly allied.

But if the exertions of our mental powers in giving rise to the arts are thus worthy of attention, it is surely proper to devote a few minutes in surveying the works of the Divine mechanic; here, whichsoever way you turn yourselves, you will perceive the clearest footsteps of Omniscience.

Consider then your Creator determining the precise number of substances; allotting properties and capacities, fitted to complete the grand design he had in view; forming the plan of a world which was to last for ages, with infinite varieties and successive changes; where every particle must have its appointed station, every motion its determinate direction and velocity; calculating at one glance all the combinations they will run into, the species of compound they will produce, their secondary operations, and mutual affections.

How stupendous must be that wisdom which directed infinite power, and by which every thing was established in number, weight, and measure! God knew the exact quantity of invisible force, whereon the different kinds of attraction depends which, if it had been greater or less, would have been productive of infinite disorder.

He proportioned the elements, that none of them might either predominate or fall deficient, and contrived springs for mingling them together to concur in performing their several tasks; he appointed the degree of influence in the sun and moon, the inequalities of the earth, the rise of exhalations, the varieties of soils, and other causes, which bring on the change of seasons, the vicissitudes of the weather, and various other dispositions of the air, causing the earth to yield her increase in due season, neither redundant nor wanting.

He contrived the curious structure of vegetables, the more admirable organization of animals, where every vessel, gland, fibre, &c. perform its appropriate office for the growth and preservation of the whole. He adapted the contexture of his plants to the wants of his living creatures, so that each species has its proper food, its nests, and places of harbour, finding uses in that which to others is unserviceable.

He ranged the elements in such order, as to carry on what we call the course of nature; to produce minerals and fossils below, vapours, clouds, dews, and rains above; to insinuate themselves into the seed, to make it germinate; into the plant, to make it bear fruits and seeds again; into the fœtus, to bring it to maturity; and into the perfect animal, causing it to fructify and renew the species. He gave various instincts to brutes, and appetites to man, urging both to effect purposes they do not think of themselves.

He allotted the several provinces to the causes of destruction, as well as those of formation and preservation; he maketh the storms his ministers, directing them what to overthrow and what to spare; he commanded the earthquakes how far to lay waste and where to stop, the lightning whom to strike and whom to pass over. Blight, famine, and pestilence have their limits and directions, in what quarters, and to what extent, to spread their havoc; and all this is performed by the intervention of second causes, so accurately arranged, so wonderfully contrived, and exacly adjusted, as never to disturb that order and that succession he has established.

Nor is the Creator's wisdom less conspicuous in the moral than the natural world; he has left much in the power of free agents, and many things to their choice and management, yet he directs their choice by such unseen springs as leads them to execute his purposes; he has distributed various constitutions, talents, endowments, passions, and desires among men, so that some are fitted as well in inclination as in ability for every office wanted in society; and thus all the conveniences depending on human industry are supplied.

Commerce, agriculture, and the mechanic arts, want not hands to carry them on; nor policy, learning, and science, able heads to improve them; the jarring interests and opposite views of private persons serve to balance each other, and are made to produce order by their proper commixture, out of that which separately would tend to confusion. He knows how and when to raise up peculiar characters that may found or overthrow empires, and erect new kingdoms upon the ruins of the old; he provides for the establishment, the security, and general welfare of nations and individuals; nor are his cares confined to this sublunary stage, for there is every reason to conclude, that we are here preparing for another state of existence; and we may reasonably presume, that there is a connection of interests between the visible and invisible world, to adjust which requires a more stupendous wisdom than any thing falling under our notice can exhibit, though that is enough to excite our wonder, and exceed our comprehension.

God is indeed incomprehensible in all his attributes; and if we endeavour, with the scanty line of human reason, to fathom the depths of Omniscience or Omnipotence, we shall be lost in darkness, and overwhelmed with difficulties. You may, however, fix your eye upon a prospect clearly discernible to the mental eye; God performing by second causes all the mighty works you see performed, and able to perform whatever we can comprehend as possible to be executed.*

You can consider him as giving existence to substances, solidity to matter, perceptivity to spirit, and understanding to man; limiting the ocean, spreading out the earth as a garment, and stretching forth the vast expanse of heaven; rolling the planets in their orbits, fixing the golden sun, and appointing the stars their stations; causing gravitation between large bodies, cohesion between small; supplying us with air to breathe, water to drink, clothes to put on, and innumerable objects all around to employ and entertain us; commanding the issues of life and death, and having the future condition of spirits at his disposal. The contemplation of these, and of a multitude of other things that a little thought will easily suggest to you, proves that his preserving providence, his animating spirit, and efficacious influence, are universally exerted; will give you the fullest idea of Omnipotence we are capable of attaining, and convince you, that it is from the Lord alone that men and angels derive all their present powers and future hopes, and that in him they live, and move, and have their being.

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* *Tucker's* Light of Nature.

"Eternal and Supreme Lord, all nature is the work of thy hands, and is created into a temple in which thy glory is displayed! Here things visible and invisible are as thou commandest, are just what thou biddest! To this blessed power we must all apply for help; every thing that sustains you, all that comforts you, from whatever secret cause it may seem to come, proceeds from heaven. The riches of the wealthy are alike the gift of God, with the narrow morsel that supports the beggar at your gate. Things animate and inanimate, rich and poor, men on earth and angels in heaven, are all supported by Divine Goodness; he openeth his hand, and filleth all things living with plenteousness.

"Thus, on every side you find a scene unfolding that is replete with life, energy, and use; a universe of progressive relations, from which inferior beings may derive lessons of admiration and humility; in which superior intelligences discover still more astonishing signs of art and magnificence; and on which, pleased with his own work, God looks down with the parental regard of love and complacence."*

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* *Comyn's* Sermons.

OF MOTION.*

We are led by an instinctive principle of the mind, to consider every change which we observe in the state of things, as an effect, indicating the existence, characterizing the kind, and determining the degree of its cause.

The kind and the degree of the cause are therefore inferred from the observed kind and degree of the change, which we consider as its effect.

The appearances of the material world exhibited in the change of motion which we observe, are called *mechanical appearances ;* and the causes to which we ascribe them, are called *mechanical causes.*

The general object of mechanics is the principles and effects of motion, and the equilibrium, which it investigates in order to understand the mechanical appearances of the universe, and apply the effects of motion to the improvements of the arts, and the general purposes of life.

Motion is change of place : change is a generic idea including many species ; motion, as a sensible idea, is a species of that genus. Motion is one species of change, but there are also many others.

Change is therefore a necessary part in the definition of motion ; it marks the genus of the thing defined. Motion is a change ; but as there are many species of change, which of those species is motion ? The answer is, change of place. This marks the species, and distinguishes it from change of colour, figure, &c.

We cannot conceive any actual motion, without combining together these three ideas, a *being* which moves, a *place* in which that being is, and the *change* of that place.

In motion is therefore supposed the successive presence of the thing moved in different parts of space ;

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* Professor *Robinson's* Outlines of Mechanical Philosophy.

therefore in our ideas of motion are involved the ideas of *space* and *time.*

In treating of motion, we have therefore to consider, 1. The power or force which causes the motion. 2. The substance moved. 3. The space passed over by the moving body. 4. The time or duration of the motion.

Space is the place of every existence; it is distinguished into absolute and relative, or place and situation.

We do not perceive the *absolute* place of any object. A person in the cabin of a ship does not think that the table changes its place, while it remains fastened to the same part of the deck.

We acquire our notion of time by means of a succession of events.

Time is unbounded, continuous, unchangeable in the order of its parts, homogeneous and indefinitely divisible.

There is so great analogy between the affections of time and space, that in most languages the same words are used to express the affections of both. Hence it is that time may be expressed by lines, and measured by motion.

The boundaries between the successive portions of time may be called *instants*, and very small portions of it may be called *moments.*

Time may be distinguished into absolute and relative.

We perceive not the *absolute* time of any event, but only its *season*, in relation to some other event.

All things are placed in *space*, in the order of situation.

All events happen in *time*, in the order of succession.

No motion is instantaneous.

In the passage of a moving point, from one part of space to another, it passes through all the intermediate points of its path.

Absolute motion is the change of absolute place.

Relative motion is the change of situation.

The absolute and relative motions of a body may be different, and even contrary.

The relative motions of bodies are the differences of their absolute motion.

The real and absolute motions of bodies are detected by means of observations made on their relative motions.

Motion is susceptible of varieties in *quantity* and in *direction.*

Our senses do not testify the absolute motion or absolute rest of any body. When one body removes from another, this may be discerned by the senses; but whether any body keeps the same part of absolute space, we do not perceive by our senses. When one body seems to remove from another, we can infer with certainty that there is absolute motion; but whether in the one, or the other, or partly in both, is not discernible by sense.

A man sitting in a barge in motion is relatively at rest, that is, with respect to the parts of the barge; but absolutely in motion, being removed with the vessel from one part of space to another. On the contrary, the bargeman who fixes a staff in the ground, and gives motion to the barge by walking along the gun-wale, is absolutely at rest, for the staff against which he leans is fixed; but relatively in motion, since, with respect to the barge, he walks from one end to the other. But if the earth be supposed in motion, the absolute motion of the barge and its contents will be compounded of its relative motion together with the absolute motion of the earth.

That affection of motion, by which we determine the quantity, is called *velocity*. This term expresses the *rate* at which the time increases, compared with the rate at which the space increases; or, velocity is that property of motion, by which the moving body runs through a certain space in a determined time. We cannot gain a distinct idea of velocity, without combining together those of space and time: thus, in the case of a traveller, you know the quickness of his pace is measured by the space he has gone over, and the time he has employed in going over it.

The *direction* of a motion is the position of the line along which it is performed. When the motion is curvilineal, the direction at any instant is the position of the tangent to that point of the curve where the moving point is situate.

The term *matter* is used to express that substance of which all things that affect our corporeal senses are composed.

Bodies are penetrated by other bodies, without any removal of their parts, in consequence of their porosity; and therefore under the same visible extension there may be different quantities of matter.

The *visible extension* occupied by any body is called its *volume* or *bulk.*

The *quantity of matter* in any body is estimated by its *weight*, and this is often termed the *mass*.

Thus a pound of gold and a pound of cork are of equal mass, as they contain the same quantity of ponderating materials; yet they are of very unequal volume.

Density is a term which, in strictness of language, expresses the vicinity of particles; but, in common language, it expresses the quantity of gravitating matter contained in a certain magnitude or volume. Thus one body is said to be more dense than another when it contains more gravitating matter in the same space; hence, as this quantity of matter in any body is proportional to its weight, that body is the most dense whose specific gravity is the greatest.

Bodies of the same density contain equal quantities of matter in the same volume.

If the volume of two bodies be equal, their densities are as their weight; if the density of one be double that of the other, its weight is also double.

If two bodies have the same density, their weights are to each other as their volumes; *ex. gr.* if the volume of A be double that of B, and their densities equal, A will be twice the weight of B.

Hence *the quantity of matter in bodies is said to be in a compound ratio of their densities and magnitudes.*

And the *densities of bodies to be in a compound ratio of their mass directly, and of their volume inversely.*

All changes of motion are considered as indications and characteristics of active causes.

That which causes a change in the state of motion of any body, or which puts it in motion, is called *force*.

The sources of force are various.

1. The mind, by the muscles, &c. is enabled to communicate motion to quiescent substances; to accelerate, retard, and alter the direction in which bodies have before been put in motion. This power is confined within certain limits, by which the strength of animals is determined.

2. Motion is communicated or destroyed by the impact of bodies either solid or fluid.

Thus the wind, by the force of its impact against the sails, sets a ship in motion, which is still increased by the impulses of the waves in the same direction; and retarded by them, if they roll in a direction opposite to that of the ship.

3. Gravity, electricity, magnetism, &c. are sources of force.

When powers are considered as residing in a body, and the changes of motion in another body as produced by their exertion, the first body is said to *act* on the second.

The exertions of mechanical forces are differently termed, according to the reference we make of the effect.

If the effect of a force exerted by A be considered in relation to the change produced in B, A is said to act on B; but the same effect is sometimes considered in communication of the former state of A, and it is considered as the effect of an endeavour in A to retain this state. In this case A is said to *re-act* against the power exerted by B.

Action and *re-action* may be thus distinguished: if A move when B ceases to act, the change in the state of B, from that which would have taken place in the absence of A, is ascribed to the action of A; but if A do not move, the same change in the state of B is ascribed to the re-action of A. Thus a weight is conceived to act on the string which supports it, while the string is conceived to re-act on the weight. Where I have used the word re-action, some have employed the term resistance, which, as both bodies may be said to act, is not only inconvenient, but obscure and equivocal.

When the changes of motion in a body, B, are always observed to be directed towards another body, A, A is said to attract B, and the power conceived as residing in A, is called an attractive force in A. In like manner, when the changes in motion in B are observed to be always directed from A, A is said to repel B, and the force is called the repulsive force of A. These are metaphysical expressions, and have led into many mistakes. If we cannot ascribe the motions of B to immediate impulsion, it is safer to say there is a tendency in B to or from A, of which A is the occasion, but not the cause.

Mechanical forces are not objects of our contemplation, and therefore we cannot compare them, nor compare their proportions with the proportions of any other quantities; we only compare some sensible quantities, which we conceive to be their proper measures.

The only quantity which we perceive in mechanical phenomena, are the phenomena themselves; and the term force, in all our comparisons, means nothing but some or any quantities that are proportional to the observed phenomena.

The direction of the observed change in motion is assigned to the force; thus we say the force of gravity acts downwards.

The action of force is inferred, when a force which is known to act produces no change of motion; thus the forces of magnetism and cohesion are inferred from our observing a heavy body suspended by a loadstone or a string. The known force is conceived to be opposed by another force, of which the existence and agency are detected, and the kind and quantity by these means determined.

OF SIMPLE, REAL, UNIFORM, RECTILINEAL MOTION.

I consider as simple that motion which is produced by one force acting instantaneously on the body, and causing it to describe a right line with a uniform velocity, that is, describing equal spaces in equal times.

It will be shown hereafter that there is no single motion but what may be considered as resulting from various forces, tending jointly to carry the moving body to the same point; and reciprocally that there is no rectilineal uniform motion so compounded, but what may be considered as produced by the simple action of one force.

Three circumstances are to be considered in simple motion.

1. The velocity with which the body moves. 2. The force consequent to its motion. 3. The laws to which it is subject.

Here it may be proper to observe, agreeably to a distinction of *Aristotle*, that there are two kinds of quantity, one which may be called *proper*, and the other *improper*.

Proper quantity, is that which is measured by its own kind, without taking in any quantity of a different kind as a measure of it; thus a line is measured by known lines, as a foot, inches, &c.

Improper quantity is that which cannot be measured by its own kind, but to which we assign a measure by the means of some proper quantity that is related to it; thus velocity of motion, when considered by itself, cannot be measured. We may perceive one body, indeed, to move faster, another slower; but we can have no distinct idea of a proportion or ratio between their velocities, without taking in some other quantity of another kind to measure them by; in this case the measure is time. Having once assigned this measure, we can, and not till then, conceive one quantity to be double, triple, &c. that of another. The measure of an improper quantity ought always to be included in the definition thereof; for it is giving it a measure that makes it a proper subject of mathematical reasoning.

The velocity, as I have already observed, is that property of motion by which the moving body describes a certain space in a determined time.

A body moving has so much more velocity as it describes a greater space in a given time, or as it takes up less time in describing the same space; that is, the velocity of a moving body increases in proportion as the space is augmented, or the time diminished. Mathematicians express this by saying, that in uniform motion, the velocity is directly as the space, inversely as the time; and consequently that the velocity is expressed by dividing the numbers, indicating the space by those that denote the time.

Thus if one body, *a*, moves through a space of twelve feet in two minutes, and another, *b*, through but six feet in the same time, the velocity of *a* is double that of *b*; the velocity of *a* is expressed by 6, that of *b* by 3.

The velocities in equal times are therefore as the spaces described in these times.

But the velocities required to describe equal spaces are reciprocally as the times employed.

The velocities of two bodies, which describe spaces that are in the same ratio as the times, are respectively equal.

The velocities of two bodies describing spaces which are reciprocally as the times, are to each other directly as the squares of the spaces, or reciprocally as the squares of the times.

The spaces described by a body are as the product of the velocity by the time, or in the proportion of the times and velocities jointly.

The spaces described with equal velocities are as the times.

The spaces described in equal times are as the velocities.

The spaces described in times reciprocally proportional to the velocities, are respectively equal.

The spaces described in times proportional to the velocities, are to each other as the squares of the velocities, or as the squares of the times.

The times are as the spaces directly, and as the velocities inversely.

The times, therefore, employed to describe equal spaces are reciprocally as the velocities.

The times employed to describe unequal spaces with equal velocities, are respectively as the spaces.

The times employed to pass through spaces which are proportional to the velocities, are respectively equal.

The times employed in describing spaces which are in the inverse ratio of the velocities, are to each other respectively as the squares of the spaces, or reciprocally as the squares of the velocities.

Velocity is distinguished into relative and absolute.

Absolute velocity is that which we have just been describing, and is measured by dividing the space by the time the moving body takes in describing that space. Thus, to know the velocity of a body that moves through fifteen required spaces in three seconds, divide 15 by 3, and the quotient gives the velocity, that is, the space described in each second.

Relative velocity is that with which two or more bodies approach or separate from each other.

1. To render the explanation of this subject more simple, I shall consider, first, two bodies moving on the same line, *plate* 1, *fig*. 1, *Mechanics;* for this purpose, let us suppose two bodies at the extremities of the line A B, one moving from A towards C, the other moving from B towards C, and with the velocities expressed by the lines A C, B C. Now it is evident, that they will approach each other with the sum of these velocities, and consequently that their relative velocity will be equal to the sum of their absolute velocities.

2. Again, suppose two bodies, *plate* 1, *fig*. 2, placed at the middle of the line D C, the one moving from A to D, the other from B to C. It is here also plain, that they will separate from each other with the velocities expressed by A D, B C, and consequently that their re-

lative velocities will be equal to the sum of their absolute velocities.

3. Let the bodies be placed on the line A C, *plate* 1, *fig*. 3, and moving in the same direction; A moving from A to C, with a velocity denoted by A B; the body, B, moving also towards C, with a velocity expressed by B C. In this case, they will approach each other only with the excess of the velocity of A above that of B.

In the same manner, the relative velocity will be equal to the difference of the absolute velocities, when the line, A C, is prolonged towards C, and the body B is supposed to move quicker than A; for in this case they evidently separate from each other only in proportion as the absolute velocity of B exceeds that of A.

The relative velocity is therefore the sum of the absolute velocities when the bodies are moved in opposite directions, or their difference when they move in the same direction.

If the bodies move on different lines, their relative velocities will be susceptible of great variation: for the particular cases, I must refer you to M. *de la Caille*, *Keil*, and other writers on mechanics.

I must beg you however to notice, that when any quantity is said to be given, it is meant, that the relation of it to some fixed quantity of the same sort, considered as a standard, is known. In like manner, when any quantity is sought, it is required to find its relation to some fixed standard of the same kind.

All forces of acceleration are referred to gravity, that is, to the force whereby the earth accelerates adjacent bodies towards its centre.

Time is referred to the earth's revolution round its axis, whether it be considered as divided into hours, minutes, or otherwise.

These two being definite standards, common to all ages and climes, it would appear, that standards of space and velocity are also determinable; but many difficulties have prevented the determination from being carried into practice; so that different nations, and sometimes even the same nation, make use of different measures, to which they refer all other spaces; and even in the same nation, these measures, as well from the imperfection of the materials of which they are formed, as from various accidents, have been found in a series of ages liable to alteration.

Though gravity, and the revolution of the earth round its axis, are standards of force and time, so plainly pointed out by nature, as to have found universal reception among mankind; yet, as the application of them requires skill in the theory, and dexterity in the practice of mechanics, these measures have not been adopted as general and invariable standards; for men find it necessary to weigh and to measure, before they become philosophers; and in more improved times, philosophy is of too little consequence to alter what has been established by centuries of continual use.

OF THE MOMENTUM, FORCE, OR QUANTITY OF MOTION.

Though the force of a body in motion increases with its velocity, yet its force or quantity of motion cannot be determined by considering the velocity alone; the mass or quantity of matter moved must also be taken into the account.

The force impressed on a body to put it into motion, is distributed uniformly to all the parts of this body, and impresses on each a velocity similar to that in the body; which can neither move with more nor less velocity than its parts. The velocity is only, then, an expression denoting the force resident in each part: for if one body contain a quantity of ponderable matter ten times greater than another, then may the heavier be divided into ten bodies, each equal in quantity of such matter to the lighter; and whatever force be required to produce a certain velocity in the lighter body, ten of these forces will be necessary to impel the ten bodies through the same space in the same time; so that the velocities of all the bodies shall be equal at the end of the motion: and it is the same as to the velocity produced, whether the bodies be separated or united; the ten forces still acting upon them.

We must, therefore, take in the mass, as well as the velocity, in order to judge of the force which animates a moving body; its mass, for the number of parts possessing a certain velocity or given force; its velocity, to know the intensity of the force resident in each of its parts.

Thus, let the body *b*, weighing one pound, have six degrees of velocity, whilst another body, *a*, of four pounds weight, has also six degrees of velocity; the body, *a*, may be considered as composed of four other bodies, each equal to *b*, and having the same velocity; consequently the body, *a*, has four times the momentum or quantity of motion of *b*. Multiply the mass, *b*, equal 1, by its velocity, and the product is 6; but multiplying the mass, *a*, of four pounds, by its velocity, 6, the product is 24; expressing the quantity of motion estimated by multiplying the quantity of matter by the velocity, and the momentum, or force, is considered as in a ratio compounded of the quantity of matter and the velocity.

As the quantity of motion in bodies is as the product of the mass by the velocity, it follows, that the velocity is as the quotient of the motion divided by the mass, and their mass is as the motion divided by the velocity.

$4 \times 6 = 24$ the motion;
$\frac{24}{4} = 6$ the velocity;
$\frac{24}{6} = 4$ the mass.

In computations relating to forces, motions, times, &c. the proportions of the quantities to each other are considered, and not the quantities themselves. In such computations therefore, when any two quantities are compared together we may substitute any other two quantities, which have the same proportion to each other, as numbers, lines, &c.

1. It follows, from what has been said on the quantity of motion, that if the velocity of a body increase

whilst its mass remains the same, its force will be augmented as the velocity is increased; or if several bodies, of the same mass, move with different degrees of velocity, their forces will be as the velocities with which they move.

2. If several bodies move with the same velocity, their forces will be as their masses; *i. e.* they will have as much more force as they have more mass, being possessed of a greater number of parts animated with the same force.

3. That a force being given, it will produce less motion in a body, in proportion as that body contains a greater number of parts.

4. That the forces will be equal in two bodies, when the velocities are reciprocally as the masses; *i. e.* when the velocity of that which has the least mass, exceeds the velocity of that body with the greatest mass, as many times as the mass of this exceeds that of the former.

It follows from what has been said, that a small body may have as much *momentum*, or force, as a large one, provided their velocities be reciprocally proportional to their masses. This is one of the reasons why, since the invention of gunpowder, the battering rams of the ancients have been disused: for these machines were large and ponderous, not easily moved; and when moved, acted with only a small velocity.

The effect of mechanical engines depends in a great degree upon this principle; for when a small weight is to balance a great one, we must contrive to increase its velocity just as much as it wants of matter, and then it will balance the large weight, if they act in contrary directions; for their momenta, being equal and contrary, destroy each other.

The battering rams of the ancients consisted of very large beams of wood, terminated by solid bodies of iron or brass. Such a mass being suspended as a pendulum, and driven partly by its gravity and partly by the impulse of men, against the walls of a fortification, exerted a force which, in some respects, exceeded the utmost efforts of our battering cannon, though in other respects it was probably inferior to modern ordnance.

To compare the effects of a battering ram, (the metal extremity of which suppose equal in magnitude to a twenty-four pounder,) with that of a cannon-ball of twenty-four pounds weight; in order that the two bodies may have the same effect in cutting a wall, or making a breach in it, the weight of the ram must exceed that of the ball in the proportion of about $\overline{1700}^2$ to the square of the velocity with which the battering ram could be made to impinge against a wall, expressed in feet. If this may be estimated at about ten feet in a second, the proportion of the weights will be that of about 2,890,000 to 100, or 28,900 to 1; the weight of the battering ram must therefore be equal to 346 ton. In this case, the battering ram and the cannon-ball moving with the velocity of 10 feet and 1700 feet respectively in a second, would have the same effect in penetrating the substance of an opposed obstacle. But it is probable, that the weight of the *aries* never amounted to so much as is above described, and consequently the effect of a cannon-ball to cut down walls, to make a breach in them, must exceed those of the ancient battering rams. But the motion of these, or the impetus whereby they communicated a shock to the whole building, was far greater than the utmost efforts of cannon-balls: for if the weight of the battering ram were no more than 170 times the cannon-ball, each moving with its respective velocity, the momenta of both would be equal: but as it is certain they were above 170 times the weight of our heaviest cannon-balls, it follows, that their impetus to shake or overturn the walls, was far superior to that which is exerted by the modern artillery: and as the strength of fortifications will be generally proportioned to the means which can be used for their demolition, the military walls of the moderns have been constructed with less attention to their solidity and massy weight than those of the ancients.

OF THE LAWS OF MOTION.

Law I. Every body perseveres in a state of rest or of uniform motion in a straight line, unless in so far as by some force impressed it is obliged to change that state.

This proposition was invented by *Descartes*, who asserts, that a body once put in motion by the impulse of another body, will continue always to be moved in a straight line, till its motion be stopped or altered by something extrinsic to it. And, in this respect, rest and motion are said to be governed by the same law; because, as the body continues at rest till it be moved by something extrinsic to it, so it also continues in motion till it be stopped in the same way.

This law, as it is generally understood, has been much objected to, and is, I believe, now given up by the best mathematicians and philosophers both here and abroad. They have considered it not only as unreasonable, but as unnecessary; there being no occasion to consider motion as the cause of its own continuation; for motion is only an effect, and must, like all other effects, be referred to its proper cause. The nature of motion will be more fully treated of hereafter: I shall now only lay before you the sentiments of some able men on the present law.

Dr. *Horsley*, Bishop of St. David's, among others, confesses it cannot be defended, and " believes, with the author of the Ancient Metaphysics, that some active principle is necessary for the continuance, as well as for the beginning of motion. I know," says he, " that many Newtonians will not allow this: I believe they are misled, as I myself have been formerly misled, by the expression, *a state of motion*. Motion is a change, a continuation of motion is a further change; a further change is a repeated effect, a repeated effect requires a repeating cause. State implies the contrary of change; and motion being change, a state of motion is a contradiction in terms."

The property of bodies to persevere in a state of rest or motion, which is announced by this law, is usually termed their *vis inertiæ*, and sometimes, in case of a moving body, their *vis insita*, or inherent force. " These terms, and the ideas conveyed by them," says Mr. *Robinson*, professor of philosophy in Edinburgh, "are inaccurate, and should be used only to express the necessity we are under of employing force in order to produce a change in motion.

M. *Prony*, in his Architecture Hydraulique, shows, that these terms are equivocal, and only serve to perplex and confuse the mind; that ignorant of the nature of force, and the nature of motion, we should confine ourselves to the effects alone.

Mr. *Berrington*. "This *vis inertiæ*, or power of resistance, is generally said to be something passive; something which resists action, but does not itself act. However, either to resist is really to act, or else it is a word void of all meaning, and therefore ought to be exploded from a science which is not to be amused with empty words.

Law II. Every motion, or change of motion, in any body, must be proportional to, and in the direction of the force impressed.

This seems nothing more, than that any change should be proportional to the cause producing such change; and, considering motion as an effect, it will always be found, that a body receives its motion in the same direction with the cause that acts upon it. If the causes of motions be various, and in different directions, the body acted upon must take an oblique or compound direction, resolvable into two or more simple directions; and hence it will follow, that the cause of a curvilineal motion cannot possibly be simple, but must arise from the joint effect of different causes concurring at the same instant to act upon the body.

Law III. Re-action is always equal to action, and contrary thereto; or, in other words, the actions of two bodies on each other are always equal, and are exerted in opposite directions.

Action and re-action are correlatives; one cannot exist without the other: resistance, or re-action, is a necessary condition to action: even action becomes re-action by a mere change of circumstances. In other words, *a* acting on *b*, as it determines *b*, is said to act;

as it resists the re-action of *b*, itself re-acts: and consequently every body that acts, is at the same instant both active and re-active.

If you press with your finger one scale of a balance, to keep it in equilibrio with a weight in the other scale, you will find that the scale pressed by the finger acts against the finger with a force equal to that with which the other scale endeavours to descend. Thus, also, if a man in one boat, *a*, draws another boat, *b*, they will approach each other with equal quantities of motion. If the cord be cut in the middle, the part next *a* will fly back towards *a*, and that part nearest *b* will go to *b*; which would not happen if the cord was only drawn towards *a*.

When a horse or horses are drawing a boat or barge against the stream up the river, the rope which connects them to the boat, draws the horses as much as they draw the boat. In other words, whatever motion the horses communicate to the barge or boat, they lose an equal quantity themselves: for suppose them to draw with a force equal to 100, and the force necessary to keep the cord tight be equal to 50, they will not be able to draw with any more than the remaining force 50; for the same force of muscles and sinews which they exert in order to drag the boat, would, if they were freed from the incumbrance, enable them to go much farther than they can go, in the same time, while connected with the boat.

When a cannon is discharged, the rarefied powder acts equally on the ball and on the breach of the gun; for the rarefied air, expanding itself every way, will equally press the cannon backwards, and the ball forwards, communicating the same quantity of motion to each, though the velocities are very different. If the ball weigh 10lb. and the cannon and carriage 10,000lb. the velocity of the ball will be 1000 times greater than that of the cannon, but the quantity of motion equal.

The re-action of the water on the oar occasions a boat to advance, and communicates to it as much motion as it received; the fishes perform with their tails and fins, and aquatic birds with their feet, what the waterman effects with his oar, the mutual action and re-action impelling them along.

Birds support themselves, and pass through long tracts of air, notwithstanding that the weight of their bodies much exceeds that of an equal volume of the fluid in which they move. If their wings strike and push the air towards the earth, its re-action supports their bodies; if the air be pushed towards the east, its re-action drives the bird towards the west. Birds which fly far, as the swallow, the falcon, &c. have generally small bodies and large wings, proper to act on a large quantity of air; while those whose flight is short, and for a little time, have smaller wings, and strike the air oftener than the others. If you compare the muscles of a man's arm with those in the wing of a bird, you will find, that the most robust and agile man would never be able to move wings proportioned to the weight of his body, with a velocity capable of supporting himself in the air.

Daniel Bernouilli proposes a new way to navigate vessels, founded on this principle of action and re-action. He requires a tube to be fixed to the stern of a vessel, open at both ends, and kept constantly filled with water, by means of a pump. The water running out of the tube would act against that in which the vessel lay; the re-action of this would push the vessel forward without sail or oar. *Jacquier* tried the experiment on a small scale with success.

OF COMPOUND MOTION.

By compound motion, I mean that which is produced by the simultaneous action of several powers acting in different directions.

The composition is not in the motion, but in the powers or forces by which it is produced.

General law of compound motion. Any body solicited to move by the simultaneous action of different forces acting upon it in different directions, will take a mean direction between what each power tends to communicate to it, and will move with a velocity proportionable to the force acting efficaciously : for, as the body cannot move in more directions than one at the same time, it must move in a direction resulting from the combination of the acting powers; and the intensity of the force it receives, will be as the intensity of the force subsisting in each, independent of their oppositions.

1. Thus, if two forces act at the same time on any body, and in the same direction, the body will move with a velocity equal to the sum of the velocities of the two forces acting upon it.

2. If the two forces be equal, and act in exactly contrary directions, the body will not move; as the equal and opposite forces of the acting bodies destroy each other.

3. If the forces be unequal, but acting in opposite directions, the body will move in the direction of the stronger, with a velocity equal to the difference of the velocities in the two given bodies.

PROPOSITION. *A body acted upon at the same time by two forces, whose directions and intensities are represented by the sides of a parallelogram, will describe the diagonal in the same time it would have described either of the sides, if it had been impelled by one force alone.*

Let the body at *a*, *plate* 1, *fig*. 4, be impelled by a force acting on it in the direction *a c*, which would cause it to move from *a* to *c*. At the same instant let it be impelled towards *b*, by a power that would carry it from *a* to *b*, in the same time, and acting in the direction *a b*. Complete the parallelogram, and draw the diagonal *a d*, and this line will represent the direction and distance the body will move in the same time, when acted upon by both conjointly. For let us suppose a trough, or tube, equal to *a b* in length, in which a ball, *a*, can move freely; and that in the same time that the ball is moving uniformly from *a* to *b*, the tube is also moved uniformly from *a* to *c*, but so as to be always parallel to *a b*, and with its extremities to describe the lines *a c*, *b d*. But the ball has moved from *a* to *b* in the tube, in the same time that the tube has descended to *c d*. Hence it is evident, that when the tube, *a b*, coincides with the line *c d*, the ball will be at the extremity, *d*, of that line, and that it has arrived in the same time it would have described either side. It is obvious, that it has also described the diagonal; for by assuming smaller forces, and forming the parallelograms *a e f g*, *a h i k*, &c. it will be found at every interval in the diagonal of the parallelogram. Its motion is also uniform, being left immediately to itself after the simultaneous action of the two forces. The composition of motion, and the doctrine of projectiles, depend on this proposition.

The motions along *a b*, *a c*, may be called the *simple* or *constituent* motions; the motion along *a d*, is called the *compound*, or the *resulting* motion.

When a body is kept at rest by three forces, they will be as the three sides of a triangle parallel to the directions in which they act.

For they are represented by the diagonal and two sides of the parallelogram forming a triangle, two of whose sides lie in the direction of two of the forces, and the side parallel to the third force; and if the lines be drawn parallel to these directions, they will form a similar triangle, and therefore the proportion of the sides will be the same.

It follows, therefore, that the three forces will be as the respective sides of a triangle perpendicular to which they act, because such a triangle will be similar to the other. The three forces must be all directed to the same point in the body, otherwise they will give it a rotatory motion.

The converse of this proposition is true. A body acted upon by three forces, proportional to the three sides of a triangle parallel to which they act, will be at rest.

I shall now illustrate this reasoning by a pleasing and evident experiment. Here is, *plate* 2, *fig.* 1, *(mechanics)* a square frame, the sides of which are divided into equal parts, and strings are strained from side to side, through the equal divisions, forming a number of small squares: a string is also stretched from one corner, A, to the other, F, of the frame, to represent a diagonal of the square.

At H is fixed a pin, to which one end of a silk string is tied, passing over a pulley at C. A brass perforated weight, M, is suspended at the other end of the string. The pulley is guided by two straight parallel wires, stretched between H and I, and may be drawn from H to I, by a string which passes over another pulley at I.

By drawing the string I, the pullies are moved from H to I, the weight is raised from F, and the centre thereof describes the diagonal of the frame. Now the weight is acted upon by two forces; one determining it to move from F to O, in the same time that the other moves from O to A. By obeying both these forces, it describes, as you see, the diagonal line, AF.

OF THE COMPOSITION AND RESOLUTION OF FORCES.

A particle of matter may be urged by different forces at once, and acquire a motion in which they combine their influence.

As the motion which it thus acquires might have been produced by the action of a single force, those forces, by whose joint action this motion is really produced, are conceived to have composed the force which would alone have produced the same motion. This is called the *composition of forces*; and the single force, which would have produced the same motion, is called the *resulting force*, the *compound*, or, *equivalent force*.

For similar reasons, any motion may be considered as resulting from the joint action of two or more forces. This is called the *resolution of forces*; and the forces which are conceived to have produced the motion, are called *simple*, or *constituent forces*.

Thus, as the lines *a b*, *a c*, *plate* 1, *fig*. 4, represent the effects of the two forces, when acting separately; and *a d* their effect, when acting conjointly; we may consider *a b*, *a c*, *a d*, as representing the forces themselves: for we estimate force only by its effect; and may therefore consider the line *a d*, as a single force, which may either be divided into two, or considered as the result of two forces, represented by *a b*, *a c*.

Therefore, in general, any body moving with a force represented by the diagonal of a parallelogram, may be considered as compounded of the forces represented by the sides; for the motion will be the same as if it had been impelled by two forces proportionable to those sides; and as upon the diagonal an infinite number of parallelograms may be constructed, any single force may be considered as compounded of more forces, and thus resolved into numberless pairs of forces, acting in the direction of the sides of those parallelograms, and proportional thereto.

The practice of reducing compound forces to simple, and that of finding two or more forces equivalent to one, is called *the composition and resolution of forces.*

As the two sides, *a b*, *b d*, of the triangle *a b d*, *plate* 1, *fig.* 4, are greater than *a d*, it is plain, that the sum of the two forces, acting successively, is greater than the result from both, when acting together in different directions; therefore, when we substitute the force *a d*, for the two forces *a b*, *b d*, it is not because it is equal to the other two, but because it produces the same effect as they do, when acting in the given directions.

The composing forces and their effect are necessarily in the same plane, for the diagonal is always in the plane of its parallelogram.

As a body subjected to the action of two simultaneous forces, necessarily describes the diagonal of a parallelogram, whose adjacent sides represent the directions and intensities of these powers, while the diagonal represents the action of these two powers on the moving bodies; you may readily find the line a body will describe, when you know the direction and intensity of the forces by which it is actuated.

It is easy to determine by these principles not only the effect of two forces acting at the same time upon any body, but also that of any number of forces. In order to do this, first find the effect of two forces, compare this first effect with another force, from this comparison the second effect will be discovered; this may be used instead of the three powers already examined, to compare with one of the remaining forces, and proceeding in this manner you will obtain the general and last result.

As the two forces P, *p*, *plate* 1, *fig.* 5, are supposed such, that acting separately, P would have carried the body from G to D, while the other would have carried it from G to A, it is evident, that as P is to *p*, so is G D to G A.

As G is a body acted on by the two forces P, *p*, and as the spaces, G D, G A, are passed over in the same time by these forces, it follows that G D, G A, represent the respective velocities, as well as the spaces of the two forces P, *p*.

If, with the directions P G, *p* G, you construct the parallelogram P G *p* F, the diagonal, G B, prolonged towards F, will be the diagonal, FG, of the smaller parallelogram; this diagonal will therefore represent a force capable of carrying G to B, in the same time that P would have carried it to D, or *p* to A.

The force represented by PG will have the same effect as the two forces P, *p*, acting at the same time on G, and the single force, F G, may be substituted for the united forces PG, *p* G; and reciprocally for the single force, FG, you may substitute any other two forces, as P G, *p* G, provided they be such that the spaces G D, G A, that each acting separately would have caused the body, G, to describe, be the sides of a parallelogram, of which the space, G B, described in the same time by the force, F G, is the diagonal.

From what has been said, you will find the following problem very easy, and where any case occurs, be able to apply and readily solve it.

Problem. To substitute for a given force, F, two forces, P, *p*, which shall produce the same effect, *plate* 1, *fig.* 6.

This problem has two cases; 1. When the direction of the forces, P, *p*, is given. 2. When the forces are expressed by lines of a given length.

Case 1. Let the line, F C, express the given force, make at C an angle, *p* C P, equal to the given angle of direction, including F C between the legs of the angle; from F draw the lines F *p*, F P, parallel to the lines P C, *p* C, and thus you will form a parallelogram, whose sides, P C, *p* C, will represent the required forces.

Case 2. In which the required forces are represented by the lines, *p* C, C P, and F C, the given force: the sum of these two lines must exceed the given force F C, or the problem is impossible. Form a triangle of the three given lines, and draw C P parallel to *p* F, and F P parallel to *p* C, and you will obtain the parallelogram, whose sides, *p c*, P C, determine the direction of the required force.

It will be easy now to see why the forces *p* C, P C, whose sum exceeds that of their effect F C, have not a greater effect when acting at the same time on the body C, than would be produced by a single force, F C, acting in the direction F C.

Let fall the perpendiculars, *p* S, P T, on the line F C, and complete the parallelograms H S, T K; now we know, by what has been already said, that the force, *p* C, may be resolved into two forces *p* H, *p* S, which, acting at the same time on the body C, are capable of producing the same effect as the force *p* C. In the same manner the force, P C, may be resolved into P K, P T.

The body, C, may therefore be considered as acted upon by these four forces, but the direction of two of these H C, K C, are equal and opposite; so that there only remains the forces *p* H, P K, or their equals S C, T C, to act on the body.

This shows you, that it is the obliquity in the direction of the forces that destroys a part of their absolute effect; the larger or more obtuse the directing angles are, the greater part is lost, and as the angle increases, the velocity along the diagonal will decrease. If the directing angles become infinitely acute, the two angles will become one straight line.

When the forces act in opposite directions, the velocity will be equal to the difference of the velocities arising from the impression of each force when acting singly. Thus, in the parallelograms, BACD, *plate* 1, *fig.*

7, whose sides, AB, AC, form different angles, whose diagonals are represented by the dotted lines AD, AD, AD, it is evident, that as the angle, BAC, increases, the diagonal grows shorter, till at last the angle and with it the diagonal vanishes, and the two lines become one line; and the forces now acting in contrary directions, their velocity will be in the direction of the stronger and proportioned to its excess above the smaller one.

In the three parallelograms *plate* 1, *fig.* 8, whose sides, AB, AC, form different angles, as that angle becomes more acute, the diagonal, AD, becomes longer, till at last the angle vanishing by the coincidence of the sides, the diagonal becomes equal to the sum of both the sides, and the velocity of C, equal to the sum of the velocities wherewith the body would move, were each of the forces acting upon it in the same direction.

In *plate* 1, *fig.* 6, the lines, PC, *p* C, represent the absolute forces of the powers, P, *p*, and the straight lines, T C, S C, their relative force or real effects on the body C; from whence we may conclude, that the effect of powers is only produced by their relative forces, and in the direction of those relative forces.

Hence then you perceive, that if a particle of matter is urged at once by two forces, which would singly produce motions in the same or in opposite directions, the motion which the particle will acquire, is the sum or the difference of the motions which they would produce singly.

A particle urged by equal and opposite forces will continue in its former state; and those forces which produce no change of motion by their joint action, are to be considered as equal and opposite. Such forces are said to be in equilibrio, to balance and destroy each other, and the particle is said to be in equilibrio; when two forces balance each other, they are considered as equal and opposite.

Any number of forces acting on a particle of matter, will be balanced by a force equal and opposite to their equivalents.

If any number of forces be in equilibrio and be considered in parcels, the equivalent of these forces would be in equilibrio.

If a number of forces be in equilibrio, and be estimated in one direction, or reduced to that direction, the forces would be in equilibrio.

Problem. Three forces concuring in a point with directions that are not in the same plane, to find the direction and expression of one force equivalent thereto. In *plate* 1, *fig*. 9, let *p* C, P C, Q C, be the three given forces acting at the same time upon C, but situate in different planes.

Now it is clear, 1. That as two lines which meet are in the same plane, by drawing P D parallel to *p* C, and *p* D parallel to P C, you will form a parallelogram *p* D P C, in the plane of which the forces P C, *p c* exert their action. In the same manner we obtain the parallelogram C P A Q, for the plane in which P C, Q C exert their forces, and the parallelogram, C Q E *p*, for the forces, Q C, *p* C, so that by drawing E B parallel to *p* D, and D B parallel to *p* E, and joining A B, a parallelopiped is formed.

2. The forces, P C, Q C, may be reduced to one A C, which is one of the two diagonal planes of the parallelopiped; and lastly, the forces, A C, *p c*, are represented by the length and position of B C, which is a diagonal of the parallelopiped.

The same mode will answer for a greater number of forces acting in different planes on the same body.

The whole theory of the composition and resolution of the forces may be deduced from the following principle: *two forces acting at the same time on a body, in directions which are oblique to each other, do not move the body by that part of their force, which on account of their obliquity, is opposite and contrary, but by what remains after the opposite forces are deducted.*

If ever so many forces, acting against one another, be kept in equilibrio by these actions, they may all be reduced to equal and opposite forces.

For any two forces may, by composition, be reduced to one force acting in the same plane; and this last force and any other may likewise be reduced to one force acting in the plane of these; and so on, till they all be reduced at last to the action of two equal and opposite forces.

From the general tenor of what has been explained, I hope you have perceived, that, strictly speaking, there is no such thing in the abtract as an oblique force, or a force producing a motion in a single body not in the direction of that force. The motion produced in every body is supposed to be in the direction of the force that produces it: indeed the direction of the motion produced is what marks out the direction of the force producing it, and the quantity of this motion is the measure of that force. But one force, as you have seen, may produce a motion in two bodies acting against each other, neither of which motions, taken separately, shall be in the direction of the original force producing that motion; thus one body may strike the surface of another in a direction oblique to that surface, and then supposing them hard, their motions, considered separately, will neither be in the direction of the original force, nor of the surface struck.

Instances of compound motion in nature are innumerable: I shall only enumerate a few, with a view of engaging you to reflect on the subject.

A fish, by striking the water with its tail, advances forward in a mean direction between the two impulses.

Birds who wish to turn, or change their direction, strike the air oftener with one wing than with the other. You may observe the same, perhaps more readily, by attending to the motions of butterflies: the irregularity of the motions of these insects, is entirely owing to the irregular action of their wings. Snakes generally advance by oblique and opposite directions.

The various manœuvres of watermen, the motions of flame, smoke, &c. that the least breath agitates; rain, hail, &c. that fall more or less obliquely, according to the impulses by which they are actuated; the accidents that often happen in jumping out of a carriage, in which the lateral motion is not so great as that which we suppose we have given ourselves: these, and innumerable other instances, are proofs of the extensive application of this useful principle: which you will find brought forward on many occasions, in the course of these lectures.

That the action of oblique forces may be more strongly impressed on your minds, I shall exhibit two or three experiments, which you may extend at leisure. To this purpose here is a round table, to the edge of which you may fix any number of pullies, varying their direction at pleasure. By this I shall now illustrate the following proposition.

Two forces acting at once on a body in the direction of, and in the quantity proportional to, the sides of a parallelogram, will be kept at rest, or counterpoised, by a force opposed to them in the direction of, and proportioned to, the diagonal.

I place a circular paper upon the round table, *plate* 3, *fig.* 5, so that its centre may coincide with that of the table. Upon this paper a triangle, ABC, is delineated, whose sides are to one another as 2, 3, and 4. Draw C E parallel to AB, and continue AC towards D.

I take three strings, which are joined in one point by a knot; and placing the point over C, I stretch the strings over C D, C E, C B, and place the pullies T, T, T, to coincide with the direction of the strings: then putting the strings over their respective pullies, at the end of the thread, C D, I suspend a weight of four pounds, to C E three pounds, and to C B two pounds. These weights will remain in equilibrio while the knot remains over C; but if it be removed out of that point, they will not be at rest.

It is evident from this experiment, as well as from what I have before mentioned to you, that power is always lost in the composition of forces; for here a weight of three pounds, and another of two, only counterpoise a weight of four; consequently no motion produced by a composition of forces can be lasting, unless those forces be perpetually renewed.*

With the same apparatus you may perform a variety of experiments, with three, four, or five forces acting at once.

To illustrate further the nature and application of this article, let AB, *plate* 1, *fig*. 10, represent an artificial kite, kept up by the wind blowing in the direction, WC; by drawing the string AI BIH, fixed at A and B, the kite will gain such a position, that HI produced will pass through the centre of gravity at C.

Draw CO perpendicular to BA, and DO perpendicular to the horizon, HO. Then OC is the direction the force of the wind that acts against the kite; and the force by which it is kept up is as the square of the sine of the angle ACW, or COD. Now, if DO represent the given weight of the kite, CO will be the the force of the wind acting against it, and CD the force pulling at the string. The tail, EF, which generally has a small weight at E, keeps the head of the kite always towards the wind.

As the direction of the string always passes through C, the angle ACH, and consequently HCO, will be the same at all altitudes, and the kite can never ascend so high as to make the angle CHO equal to ACH. Hence it follows, that the smaller the angle HCO is made, the higher will she rise: the greater the wind, or the lighter the kite, the higher also will it rise.

The wings of a bird are so constructed, that in striking downwards they expand to their greatest, and become almost two planes, being somewhat concave on the under side. These planes are not then horizontal, the back part, K, *plate* 1, *fig*. 11, being higher than the fore-part, DFG; but in moving the wings upward, to fetch a new stroke, they go with the edge, DFG, foremost, and the wings contract and become more concave. Their bodies are specifically lighter than that of men and beasts; their bones and feathers extremely porous, hollow, and light; the muscles which move the wing downwards are exceedingly large, and have been estimated in some instances to be not less than a sixth part of the weight of the whole body. When a bird is on the ground, and intends to fly, he takes a large leap, and stretching his wings right from the body, strikes them downwards with great force, by which they are put into an oblique position; and the resistance of the air acting strongly against them from the stroke, impels them and the bird in a direction perpendicular to their planes; which is in an oblique direction, partly upwards, and partly horizontally forward. That part of the force tending upwards is destroyed by the weight of the bird; the horizontal force serves to carry him forward. The stroke being over, he moves his wings; which being contracted, and turning their edges upwards, meet with very little resistance from the air. When they are sufficiently elevated, he takes a second stroke downwards, and the impulse of the air again moves him forwards; and so from one stroke to another, which are only as so many leaps taken in the air. When the bird wants to turn to the right or left, he strikes strongly with the opposite wing, which impels him to the contrary side. The tail acts like the rudder of a ship; except only that it moves them upwards or downwards, instead of sidewise. If the bird wants to rise, he puts the tail in the position I H; if to fall, in the position I L; whilst in a horizontal position, it keeps him steady. A bird can, by spreading his wings, continue to move horizontally for some time, without striking the air, because he has acquired a sufficient velocity, and his wings being parallel to the horizon, meet with but a small resistance; and when he begins to fall, he can easily steer himself upwards by his tail, till the motion he had acquired is nearly spent, when he must renew it by two or three more strokes with his wings. On alighting, he expands his wings and tail full against the air, that they may meet with all the resistance possible. The centre of gravity of a bird is rather behind the wings: to counterbalance this, they thrust out their head and neck in flying.

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* So far as forces counteract each other, *power*, in a certain sense, may be said to be *lost*: yet the conclusion drawn by the author is by no means warranted; for a motion produced by a composition of forces will be as uniform, and stand in as little need of the forces being *perpetually renewed*, as a motion produced by a single force. A. Edit.

It is impossible, as already observed, for men to fly by the strength of their arms. For their pectoral muscles are vastly too weak: in a man, they are not one-sixtieth part of the muscles of the body; in a bird, they are more than all the others put together.

Some birds are said to fly a thousand yards in a minute.

A fish swims by the help of his fins and tail; and fishes are nearly of the same specific gravity with water. The muscular force of the tail is very great: their direct motion is obtained by moving the tail from one side to the other with a vibrating motion. When going to move it, the fish turns the end oblique to the water, and moves it through it in that position. The water re-acts obliquely against the tail, and moves him partly forward and partly laterally. The lateral motion is corrected by the next stroke the contrary way, while the progressive motion is continued. They exert a very great force with their tail. By the help of the tail they also turn to one side; striking strongly with it on that side, and keeping it bent, it acts like the rudder of a ship. The fins of a fish keep it upright, especially the belly fins, which act like two feet: without these he would swim with his belly upwards, as the centre of gravity lies near the back. By contracting or expanding the fins, they also assist him in ascending and descending: by inclining his tail obliquely, and turning it a little from an erect position to one side, it helps him to rise and fall. Fish swim but slow, and soon tire; yet some are said to swim seventy or eighty yards in a minute.

Brutes swim naturally; for they are specifically lighter than water, and require but a small part of their head out for breathing. It is also easy to them; for they use their legs in swimming after the same manner as they do in walking.

Men do not swim naturally, though they are specifically lighter than water; for their heads are very large, and require to be almost out of the water for breathing, and their way of striking has no analogy to walking. Man attains the art of swimming by practice and industry. The art consists in striking the water alternately with the feet and hands; which, like oars, row him forward. When he strikes with his hands, he neither keeps the palm parallel, nor perpendicular to the horizon, but inclined; and his hands striking the water obliquely, the resistance of the water moves him partly upward and partly forward. Whilst his hands are striking, he gradually draws up his feet; and when the stroke of his hands is over, he strikes with his feet by extending his legs, and pushing the soles of his feet against the water; and while he strikes with his legs, he brings about his arms for a new stroke, and so on alternately; keeping the body somewhat oblique, that he may more easily erect his head, and keep his mouth above water.

LECTURE XXVII.

OF ACCELERATED MOTION.

THOUGH the most unlettered rustic is sensible that the fall of a stone is to be dreaded in proportion to the height from whence it descends; that if it fall from a place a foot above his head, it is not so likely to be fatal as if it fell from the house-top; yet the law of acceleration, in falling bodies, was not discovered till the time of *Galileo:* to investigate the effect of gravity on such bodies was reserved for him, who was one of the greatest ornaments of the age in which he lived.

Motion is said to be *accelerated*, when its velocity continually increases; to be *uniformly accelerated*, when its velocity increases equally in equal times.

Motion is said to be *retarded*, when its velocity continually decreases; and to be *uniformly retarded*, when its velocity decreases equally in equal times.

In our general notions of acceleration or retardation, we abstract our attention from all varieties, and they are supposed uniform: that is, we suppose that the velocities increase or diminish at the same rate with the times, or that the changes of velocity are proportioned to the times in which they are acquired.

Accelerations and retardations may be considered as quantities, and are measured by the changes of velocity, which are uniformly acquired in the same or equal times; they are therefore proportional to the changes of velocity directly, and to the time in which they are uniformly acquired, inversely.

If the velocities do not change at the same rate with the time, the acceleration or retardation is not constant.

If you suppose a body to be put in motion by a single impulse, and moving uniformly, to receive a new impulse in the same direction, its velocity will be augmented, and it will go on with the augmented velocity. If at each instant of its motion it receive a new impulse, the velocity will be continually increasing; and if this impulse be always equal, and act in equal times, the velocity will be uniformly accelerated.

The force, which at every instant gives this new impulse to a body, is called the accelerating force.

For the same reason, if a body had at first a certain velocity, and lose equal parts at each equal instant, by new impulsions acting in a direction exactly contrary to its motion, it is said to be uniformly retarded.

Every body left to itself descends with an accelerated motion, and in a direction which tends to the centre of the earth. The force causing this acceleration, and occasioning this tendency, is called *gravity*. It is a force always present; and which, constantly acting on bodies, uniformly accelerates their motion.

All that we can observe in motion, is the space described, and the time in which it is described. From observations on these, we infer the proportions of the changing forces. Uniformly accelerated or retarded motion is therefore the indication of an invariable or constant force. Hence the changes of velocity in any time, by a force varying according to any law, is the proper measure of the accumulated or whole action of the force during this time.

I shall now endeavour to explain the law of bodies falling with an accelerated motion by the force of gravity. The accelerating force or gravity decreases, as the squares of the distances increase; but as the greatest distance from which we can observe the force of falling bodies, bears no sensible proportion to the distance of the centre of the earth, we may consider gravity as acting without any decrease, or rather that its force is the same from the beginning to the end of the fall.

The nature of all accelerating forces is to act continually on the moving body, and to be successively impelling it by an infinity of small strokes. But the accelerating force, being thus repeated as often as there are instants in the given time, is proportional to the product of the weight by the last acquired velocity.

As gravity acts uniformly on all bodies at an equal distance from the centre of the earth, let us suppose the time of the descent to be divided into a number of equal parts, infinitely small; the impression of gravity will occasion the body to descend towards the earth in the first small instant. Now, if you suppose gravity to cease to act, the body will descend uniformly, in consequence of its first impression, with an infinitely small velocity; but on the contrary, if at the second instant, a new and equal impulse be added to the former; its velocity, in the second instant, will be double that of the first; in the third, by the continued impulse, three times; four times in the fourth; and so on: for the impression made in the preceding instants are not at all impaired by those that follow, but are, if I may be allowed the expression, heaped one upon another: consequently, as the body is supposed to receive a new impression every moment of its fall, the velocity increases, as the moments or time increases.

In the right-angled triangle A B C, *plate* 1, *fig.* 12, which opens wider by equal degrees from the point A towards B C, the base of the triangle expresses a motion uniformly accelerated.

The side, A B, expresses the time in which this acceleration happens, and this time is divided into four equal parts, or moments, as A 1, 1 2, 2 3, 3 B. The small lines in the triangle A 1 k, repeated at equal intervals, and increasing in length by equal degrees, denote equal accelerations of the velocity from the instant in which a body begins to fall, and the small equal lines in the square 1 m, denote an equal velocity during the time expressed by the side 1 2.

These things being kept in mind, if you attend to the diagram, it will open to you all the consequences of these positions.

The line, 1 k, will represent the velocity acquired by a falling body in the first moment of time; 2 1, the velocity acquired at the end of the second moment of time; and 3 o, the velocity at the end of the third; and so on.

If the body, during the second moment of time, should retain the velocity 1 k, which it had acquired at the end of the first, it will describe the square surface 1 2 m k; for this surface is generated by a continual repetition or motion of the line 1 k, during 1 2; as the area of the triangle, a 1 k, is described by a uniformly increasing velocity during the time A 1. But the area of the square is manifestly double the area of the triangle; whence it appears, that a body moving on, during a second moment, with the velocity acquired at the end of the first, will fall twice as far in the second moment as in the first: and the same will be universally true, that the velocity acquired at the end of any given time will carry the body twice as far in the same time.

But if the velocity continue to increase uniformly during the second moment, then the space will be as the triangle 1 2 l k, thrice as large as the triangle A 1 k.

The whole space described by the body in the two first moments will be as the area A 2 l, which is equal to four times the area A 1 k; whence it follows, that the space described by the body in its fall is as the square of the time in which it falls; for here the time is 2, the square thereof 4.

In the third moment, were the body to fall with the velocity 2 l during the time 2 3, the space described will be as the rectangle under the time and velocity, that is, as the rectangular space 2 3 n l, equal to four times A 1 k; but as the velocity is still uniform, the space will be as the area 2 3 o l, five times as great as A 1 k. And thus will the rectangle under the time and velocity be always increased by units.

As the triangles A 1 k, A 2 l, A 3 o, A B C, are all similar, as A 2 is the double of A 1, 2 l will be the double of 1 k; and as A 2 expresses the time, and 2 l the velocity, where the time is double, the velocity is double, therefore the velocity is as the time; and so on of the rest.

If the spaces described in each moment be considered separately, the space in the first moment will be as 1; in the second, as 3; in the third, as 5; in the fourth, as 7; and so on in an arithmetical progression: the common difference being 2.

Thus, I have shown you from theory, and which I shall, in the course of this lecture, illustrate by experiment,——

1. That in bodies falling freely by their own gravity, the spaces described in falling from rest, are as the squares of the times of falling.

2. That the spaces described by falling bodies are as the squares of the velocities.

3. That, therefore, the spaces described by falling bodies are in the compound ratio of the times and the velocities acquired by falling.

4. If a body fall through any space, and move afterwards with the velocity gained in falling, it will describe twice that space in the time of its falling.

The motion of a body thrown directly upwards is continually retarded, in the same manner that the motion of a falling body is accelerated: for, the action of gravity in this case, continually acts contrary to the motion of the rising body; whereas in that of falling bodies, it conspires with it.

As the power of gravity communicates to the body in every equal moment equal velocities, so the velocity of a body thrown upwards, is equally retarded in equal times; for the same force of gravity, which generates motion in a falling body, destroys it in a rising one. It follows, therefore, that in the same time and manner the same velocities are generated and destroyed.

A body thrown upwards continues to ascend till it has lost all its motion, when it will begin to descend, and the motion will then be just as much accelerated as it was before retarded; it therefore ascends during the time that a body in falling can acquire the velocity with which it was thrown up.

And the heights to which bodies projected directly upwards, with different velocities, can ascend, are to one another as the squares of their first velocities.

Therefore bodies ascending describe spaces which follow the progression of the odd numbers taken in a retrograde order.

To render this subject still clearer, let us apply the above theory to practice. Here however I must first premise, that when a body falls freely by the force of gravity, it will describe about sixteen feet and an inch in one second of time; but, for the sake of round numbers, I shall suppose it to be sixteen feet. This measure has been deduced from the motion of pendulums; between which and the rectilineal descent of heavy bodies there is so close a connexion, that neither of them can be thoroughly understood independently on the other. Now, as a body falls through 16 feet in the first second, it follows, that at the end of the next second it will have fallen 16 multiplied by 4, equal to 64 feet; the spaces being as the squares of the times, and the square of 2 is 4. By the same rule, at the end of the third second it will have fallen 144 feet, equal 16 multiplied by 9:

at the end of the fourth, 256 feet, equal 16 multiplied by 16.

Hence, if the space be given through which a body is to fall, you may find the time wherein it will finish its descent: for, let the number of feet in such a space be divided by 16, and the quotient will express the square root of the time sought, in seconds and parts of a second.

Thus, if the space be 144 feet, this divided by 16 gives 9, of which the square root is 3. The converse of this is equally true; for if the time be given, the space through which the body hath descended may be found. If you desire to learn the depth of a well, from the surface of the earth to the surface of the water, let a bullet of lead be dropped therein, and let us suppose it to strike the water in five seconds: the square of 5 is 25; which, being multiplied by 16, the product will be 400 feet for the depth of the well.

Again, were you to see a mass of burning matter, or a large red-hot stone, shot upwards from the mouth of a volcano, and could observe accurately the whole time of its flight in the air, rising and falling, which we will suppose thirty seconds, you may thence discover the height to which it arose; for, a projected body will rise and fall in equal times: therefore, we are to take half the above-mentioned time, or fifteen seconds. Now, 15 multiplied by 15, and this by 16, will give 3600 feet, or 1200 yards, which is not far from three-quarters of a mile.

But you are to observe, that in the application of this theory two things are taken for granted, before we arrive at any one of these conclusions: first, that the theory is true in practice to a mathematical exactness, of which you will soon see the proofs; secondly, that the motion is without impediment, or in a medium that gives no resistance: but this is false, as the bodies meet with great interruptions from the resistance of the air. Upon this account, the well whereof we are finding the depth, will not be so deep as may be imagined by several feet; nor the stone projected from the volcano rise so high as we have already concluded. The resistance of the air is liable to so many variations, that to find the absolute quantity of it at any time is a problem so difficult and complicated, as hardly to admit of an adequate solution.

OF MR. G. ATWOOD'S APPARATUS FOR MAKING EXPERIMENTS ON THE RECTILINEAL MOTION OF BODIES WHICH ARE ACTED ON BY CONSTANT FORCES.*

This elegant and accurate machine before you, *plate* 1, *fig*. 13, is the contrivance of Mr. *Atwood*, and renders sensible to the eye and ear, by means of a set of friction wheels, various weights, and a clock, the laws of motion uniformly accelerated or retarded, as well as those of uniform motion, and that without employing a space more than five and a half feet, which causes it to be extremely convenient and demonstrative for a course of lectures.

Mechanical experiments are of two kinds; the one relating to the quiescence of bodies, and the other to their motion.

Among the former are included those which demonstrate, or rather make evident to the senses, the equilibrium of the mechanic powers, and the corresponding proportions of the weights sustained, to the forces which sustain them, the properties of the centre of gravity, the composition and resolution of forces, &c.

By the latter, or those on motion, are shown the laws of collision, of acceleration, and the various effects of forces which communicate motion to bodies.

Of mechanical experiments it may be proper to observe to you, that those wherein an equilibrium is formed, will frequently appear coincident with the theory, although considerable errors are committed in their construction. This arises from the effects of friction, tenacity, and other causes. The case is different in experiments concerning the motion of bodies; in which, whatever care be taken to render the proportion of the forces, and the weights moved, such as is required by the theory; yet the interference of friction, which renders the former apparently more perfect than they really are, causes these to differ from the theory.

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* *Atwood's* Treatise on the Rectilineal Motion of Bodies.

If the experiments are only designed to assist the imagination, by substituting sensible objects instead of abstract and ideal quantities, an apparent agreement between the theory and experiment may be sufficient to answer this purpose, although it may be produced from an erroneous construction: such experiments cannot, however, impress the mind with that satisfactory conviction that arises from experiments accurately made.

Dr. *Desaguliers* tried the effect of falling bodies, by letting a leaden ball fall from the inner cupola of St. Paul's church, whose altitude from the ground is 272 feet. The ball descended through this space in four seconds and a half; in which time, from theory, it should have descended through 325,6 feet, which makes a difference of about one-fifth of the actual descent between the experiment and the theory. Dr. *Desaguliers* shows, in his fifth lecture, that this difference arose principally from the resistance of the air. To remedy the defects of these experiments, Mr. *Atwood* contrived his apparatus.*

Of the mass moved. In order to obtain an adequate idea of the laws that are observed in the communication of motion, and observe the effects of the moving force, the interference of all other force should be prevented. The bodies impelled should be conceived to exist in free space, and be void of gravity or weight; so that to a given substance various degrees of force may be applied. This indeed cannot be effected in bodies falling freely near the earth's surface: we cannot abstract the natural gravity or weight from any substance whatsoever; for the same substance is always impelled by the same force of gravity, which admits not of increase or diminution.

Yet, this difficulty may be obviated by balancing two equal weights, joined by a flexible line which goes over a pulley. The axle of the pulley must rest on wheels constructed for the purpose of diminishing friction.

The motive force of gravity being destroyed by the contrary and equal action of the weights, they will remain quiescent till some force is applied to them. When any impulse is communicated to them in a vertical direction, they will afterwards be observed to describe equal spaces in equal times, or will move uniformly; and the velocity communicated will be precisely the same, as if the same impulse had been impressed on a quantity of matter equal to the two bodies existing in free space without gravity, due allowance being made for the inertia of the wheels.

Thus, in this instrument, *plate* 4, *fig.* 13, which is constructed to illustrate this subject experimentally, there are two equal weights, A, B, affixed to the extremities of a very fine and flexible silk line. This line is stretched over a brass wheel, *a b c d*, moveable round a horizontal axis. The two weights, A, B, being precisely equal, and acting against each other, when the least weight is super-added to either, it will preponderate.

When the weights, A, B, are set in motion by the action of any weight, which Mr. *Atwood* called *m*, the sum of A added to B, added to *m*, would constitute the whole mass moved; but then there is to be added the inertia of the materials which must necessarily be used in the communication of motion. These materials are, 1. The large wheel, *a b c d*. 2. The four brass friction-wheels, on which the axle of the wheel *a b c d* rests: these wheels are used to prevent the loss of motion which would be occasioned by the friction of the axle, if it revolved on an immoveable surface. 3. The weight of the line: but this is too inconsiderable to have any sensible effect.

Of the resistance from the inertia of the pullies. If the whole mass of the wheels were accumulated in the circumference of the wheel *a b c d*, its inertia would be truly estimated by the quantity of matter moved. If their figures were regular, and the density distributed uniformly in each, mathematicians would furnish us with rules for finding a weight, which, being accumulated uniformly in the circumference, *a b c d*, would exert an inertia equal to that of the wheels. But as the figures are wholly irregular, recourse must be had to experiment for the discovery of such a weight.

For this purpose a weight of thirty grains was affixed to a silk line which did not weigh one-quarter of a grain; this line being wound round the wheel, the weight of thirty grains, by descending from rest, communicated motion to the wheel; and, by many trials, was observed to describe a space of thirty-eight and a half inches in three seconds. From these data we find the mass equivalent to the inertia to be two ounces and three quarters. This is a mass equivalent to the inertia of the wheel *a b c d*, and the friction-wheels together.*

The resistance to motion, therefore, arising from the wheel's inertion will be the same as if it were absolutely removed, and a mass of 2 3-4 were accumulated in the circumference of the wheel *a b c d*.

This being premised, suspend the pieces or brass boxes, A, B, by a silk line passing over the wheel *a b c d*, and make them balance each other: now, if I add any weight, *m*, to A, so that it shall descend, the exact quantity of matter moved will be ascertained, for it will be A added to B, added to *m*, added to 2 3-4 oz.

In order to avoid troublesome computations in adjusting the quantities of matter moved, and the moving force, one-quarter of an ounce is assumed as a standard weight of convenient magnitude, to which all others are referred; this weight is called *m*. Now the inertia of the wheel being 2 3-4 oz. will be denoted by 11 *m*; A and B, the pieces or boxes to which the different weights are applied, are each, together with the hooks by which they are suspended, equal 1 1-2 oz. or 6 *m*. We have a variety of different weights, *fig.* 23, some of which are equivalent to 4 *m*, others to 2 *m*, some equal *m*, and others to aliquot parts thereof.

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* Mr. *Atwood* proves in his work, † that the following formula will give the required mass $\frac{pttd}{s}=p-x$, where p signifies the weight, 30 gr.; t the time, 3 seconds; d the space described by a body in a second, 16 feet 1 inch, or 193 inches; s the space described by the body, 38.5 inches; and x the inertia sought.

That is in figures for the present case, $\frac{30\times 9\times 193}{38.5}-30$ equal to 1323 grains, or 2 3-4 oz.

† *Atwood* on Rectilineal Motion, p. 226.

If then I apply 19 *m* to each of the pieces A, B, these, with those pieces, will each be equal 25 *m*, balancing themselves, and the whole mass will be 50 *m*, which being added to 11*m*, the inertia of the wheels, the whole mass will be 61 *m*; now add *m* to both A and B, and the whole mass will be 63 *m*, perfectly in equilibrio and moveable by the least weight, added to either, setting aside the effect of friction, in the same manner precisely as if the same weight or force were applied to communicate motion to the mass 63 *m*, existing in a free space and without gravity.

Of the moving force. As the natural weight, or gravity of any given substance is constant, and the exact quantity easily estimated, we shall apply a weight as a moving force; thus, when the system consists of a mass equal 63 *m*, I apply a weight, *m*, to A, and it communicates motion to the whole system; the whole quantity of matter moved is 64 *m*; the moving force *m*, this gives us the force which accelerates the descent of A, being $\frac{m}{64}m$, or $\frac{1}{6}$ part of the accelerated force, by which the bodies descend freely to the earth.

You see by this example, that the moving force may be altered without altering the mass moved; for, suppose the three weights *m*, two of which are placed on A and one on B, be removed, then A will balance B.

Now place the weights 3 *m* on A, and the moving force will be 3 *m*, and the mass moved 64 as before, and the force which accelerates the descent of A equal $\frac{3\,m}{64\,m}$

To make the moving force 2 *m*, remove the three weights *m*, A and B will balance one another, and the whole weight will be 61 *m*; add 1-2 *m* to A and 1-2 *m* to B, and the mass moved will be 61 *m*; now place 2 *m* on A, and the mass moved is 64 as before, whereof the force of acceleration is aqual 1-32 part of the accelera-tion of gravity.

Of the space described. The body, A, descends, as you perceive, in a vertical line, along one of the scales, C C, *fig.* 13, which are each about sixty-four inches long, and is graduated into inches and tenths: these scales are so adjusted as to be vertical, and so placed that the descending weight may fall in the middle of the stage D, fixed to receive it at the end of the descent; the beginning of the descent is estimated from o, on the scale, when the bottom of A is level or even with o; the descent of A is terminated, when the bottom strikes the stage; the situation or distance of the stage may be varied at pleasure within the range of the sixty-four inches.

Of the time of motion. The time of the motion is observed by the beats of this pendulum, E, affixed to the pillar G, and which vibrates seconds. The number of seconds are shown by the index and dial-plate above.

Many mechanical devices might be applied for letting the weight, A, begin its descent at the instant of the beat of the pendulum; but it is simpler, and Mr. *Atwood* thinks better, to let the bottom of the piece or box, A, when even with o. on the scale, rest on a flat rod held in the hand horizontally, its extremity being coincident with o; by attending to the beats of the pendulum, you may, with a little practice, remove the rod which supports the box, at the instant the pendulum beats, so that the descent of A shall commence at the same instant.

Of the velocity acquired. I have only now to show you in what manner the velocity acquired by the descending weight A, at any given point of the space through which it has descended, is made evident to the senses.

The velocity of A's descent being continually accelerated, will be the same in no two points of the space described; this is occasioned by the constant action of the moving force: and since the velocity of A, at any instant, is measured by the space which would be described by it moving uniformly for a given time, with the velocity it had acquired at that instant, this measure cannot be experimentally obtained, but by removing the force which caused the acceleration of the descending body.

To effect this, here are some weights or moving forces in form of bars or flat rods, *m*, *fig.* 23, to be laid on A; here is also a circular frame, H, *fig.* 13, to be fixed to the scale at any proper height, in such manner that A will pass centrally through it; when A passes through this frame, it leaves the bar by which it had been accelerated on the circular part of the frame. After the moving force, *m*, has been intercepted at the end of the given space or time, there will be no force operating upon any part of the system to accelerate or retard its motion, and consequently the instant *m* is removed, A will proceed uniformly with the velocity it had acquired that instant, and the velocity being uniform, will be measured by the space described in any convenient number of seconds.

It may here be necessary to observe, that Mr. *Atwood* has clearly shown, that the weight of the line can have no sensible effect on the experiments, for the inequality of the motion occasioned by it does not amount to more than $\frac{1}{0.0312}$ of a second, a quantity too small to be distinguished by the senses.

The resistance of the air does not affect these experiments; for, as the greatest velocity communicated in these experiments does not exceed 26 inches in a second, and the pieces A and B being only about 1$\frac{3}{4}$ inch in diameter, the resistance of the air can never increase the time of the descent in so great a proportion as 240 to 241, and will be therefore insensible in experiment.*

The effects of friction are almost wholly removed by the axis of the wheel, *a b c d*, acting on the four friction wheels, *e*, *f*, *g*, *h*. If the weights, A and B, be balanced in perfect equilibrio, and the whole mass consist of 63 *m*, a weight of two grains, added to A or B, will communicate motion to the whole, which shows how inconsiderable the friction is; in some cases, however, particularly in experiments on retarded motion, the effects of friction become sensible, but may be very readily and exactly removed by adding rather less than two grains to the descending weight; the weight should be always less than what is sufficient to put the whole in motion.

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* *Atwood* on Rectilineal Motion, p. 314, 315.

The space which bodies describe in one second, by falling freely from rest, is 193 inches; but in the ensuing experiments, the space is taken at 192 inches, which will be productive of no error, in order to avoid fractions, which would render the use of the instrument less easy and intelligible.

Having now, I hope, sufficiently explained the instrument, I shall proceed to the construction of some experiments with it.

Let two equal weights, A and B, be suspended by a line, joining them, and going over a fixed pulley. *If any weight be added to them, it will preponderate, and in its descent will describe spaces which are as the squares of the times of falling from rest.*

The equal weights are, in the present case, each equal 26 m, and the additional weight applied as a force to communicate motion m. Then the mass moved is $m +$ 52 $m +$ the inertia of the wheels 11 m, making in all 64 m. Now I shall show you, that the preponderating weight descending from quiescence during 1, 2, 3, seconds, describes in the first second 3 inches; in two seconds 3 × 4 or 12 inches; in three seconds 3 × 9 or 27 inches; the spaces being respectively as the squares of the times of motion.

To prove this, I affix the stage to 3 on the graduated scale, bring the under surface of the piece, A, to coincide with 0, on the scale, and let it fall at a beat of the pendulum, and you will find it strike the stage when the pendulum beats again: it has done so, having passed through three inches in one second. I shall now place the stage at twelve inches, and the weight will strike it exactly at the second second; when placed at twenty-seven, the stroke of the weight will coincide with the third second.

I shall now subjoin a table of some experiments of the same kind to be repeated by yourselves, which will rivet the theory more perfectly in your mind, and render you readier in applying it to particular cases. Let A hold 36 3-4 *m*; B, 36 1-4 *m*. The spaces described, &c. will be as in the following table.

A TABLE.

Moving force.	Mass moved.	Accelerating force.	Times of motion in seconds.	Spaces described from rest estimated in inches.
			1	1
			2	4
			3	9
$\frac{1}{2}$ m	96 m	$\frac{1}{192}$	4	16
			5	25
			6	36
			7	49
			8	64

If different forces be successively applied to accelerate equal quantities of matter from quiescence, the spaces described in any given time will be in the same proportion with the forces.

If any body, equal to 64 *m*, fall freely, or be acted upon by its own natural weight, it will describe 192 inches in the first second of its fall; but if the same mass be impelled by only $\frac{1}{\ }$ of its accelerating force or 1 *m*, it will describe only a 64th part of the former space, that is, only three inches, a proportional effect.

To prove this, let A and B be each made equal to 25 *m*, then will A and B be equal to 50 *m*; to this add 11 for the inertia of the wheels, and we have 61 *m*: now put 2 *m* on A, and 1 *m* on B, and the mass moved will be 64 *m*, and the moving force 1 *m*.

Set the stage to three inches, and let the weight descend as before, and you will find it strike the stage at the first second.

If the same force impel different quantities of matter for any given time, the spaces described from rest will be inversely as the quantities of matter moved.

Let the force be *m*, and the mass 64 *m*, then the space described, during one second, will be three inches.

Let the force be *m*, and the mass 32 *m*, then the space described in the same time will be six inches.

If the force be increased or diminished in the same proportion with the mass moved, the spaces described from rest in the same time, will be equal.

Let *m*, 2 *m*, and 4 *m*, be the moving forces which impel the quantities of matter 32 *m*, 64 *m*, 128 *m*, respectively; then *m* will impel 32 *m* through six inches in a second, 2 *m* will impel 64 *m*, and 4 *m* will impel 128 *m* through the same space in the same time.

From hence you may infer, that when different quantities of matter describe equal spaces in equal times, being accelerated from quiescence, the forces must be in the same proportion with the quantity of matter* moved.

If a body be moved from quiescence during any given time, it will, at the end of that time, have acquired such a velocity, as will, if continued uniform, carry it through double the space which the body has already described to acquire that velocity.

Let the mass moved be 64 *m*, the force, *m*; but let *m*, which is applied to A, as the moving force, be one of the flat rods *m*, *fig.* 23. Place the circular frame so that A may, in descending, pass through it, and its height be such, that the instant the lower surface of A arrives at twelve inches, the rod, *m*, may be intercepted by the surface of the circular frame, and thereby be prevented from the further acceleration of the system. Let the other stage be set at thirty-six inches, that is twenty-four from the circular frame.

Now, let the weight begin to descend from o, on the scale at any beat of the pendulum; at the end of the second beat, you will hear the rod strike the circular frame, having described twelve inches with a uniformly accelerated motion; and at the fourth beat of the pendulum, it will strike against the square stage at thirty-six, having described twenty-four inches with a uniform motion.

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* By this term I mean only the quantity of sensibly ponderable matter, for that there is other matter is evident from a thousand experiments.

If the same force act on the same mass for different times, one second, two seconds, and three seconds, the velocities generated will then be six inches, 6×2 *inches, and* 6×3 *inches in a second respectively, being in the same proportion with the times wherein the given force acts.*

Set the circular frame to three inches, and the stage to nine inches, let the mass moved be 64 *m*, the moving force, *m*; the weight, A, beginning its descent at any beat of the pendulum, the rod will strike the circular frame at the next beat: here the rod is removed, and A describes the next six inches uniformly in one second, striking the stage at nine at the second beat. By trying the apparatus in other instances, you will constantly find the experiments coinciding with theory.

These experiments show, that if the force, by which bodies are accelerated, be the same, the velocities generated will be in the same proportion as the times wherein the given force acts.

If a body be moved from rest through the same space by different forces, the velocities generated will be in a subduplicate ratio of the forces.

Let the mass be 64 *m*, and the force *m*,—you will find the velocity acquired in describing twelve inches in two seconds, will, when the force is removed, carry it through twelve inches in one second. Now, let the mass be 64 *m*, and the force 4 *m*, and you will find that the body, in describing twelve inches, will acquire a velocity of twenty-four inches, being in the ratio of 1 to 2; whereas the accelerating forces are in the ratio of 1 to 4.

If different quantities of matter be impelled through the same space, and acquire the same velocity, the moving forces must be in the same ratio as the quantities of matter moved.

Let the quantities of matter be 64 *m*, and 48 *m*, the space twelve inches,—the moving forces must be as 4 to 3. The following experiments will illustrate this truth.

Moving forces.	Quantities of matter.	Accelerating forces.	Spaces described in inches.	Velocities acquired in inches per second.
m	64 m	$\frac{1}{64}$	12	12
$\frac{3\ m}{2}$	96 m	$\frac{1}{64}$	12	12
$\frac{3\ m}{4}$	48 m	$\frac{1}{64}$	12	12

You may infer, from the two last sets of experiments, that the moving forces, which impel bodies through the same spaces, are in the joint ratios of the quantities of matter moved, and the squares of the velocities generated.

If a given quantity of matter be impelled from rest through different spaces by the action of the same force, the velocities generated will be in a subduplicate ratio of the spaces described.

Let the quantity of matter be 64 *m*, the force *m*, the spaces 3 inches and 27 inches,—the velocities will be in the ratio of 1 to 3; for it will descend in the first experiment through six inches in a second; in the next you will find it go through 18 inches in a second. Now the spaces are 3 and 27, or as 1 to 9; the velocities acquired as 6 to 18, or as 1 to 3.

Experiments on uniformly retarded motion. The laws observed during the motion of uniformly accelerated bodies, having been made evident to the senses by the preceding experiments, I shall now proceed to illustrate the properties of uniformly retarded motion.

When a body is thrown perpendicularly upwards from the earth's surface, it is continually resisted by a force which is equal to the body's weight; and the weights of all substances being proportional to the quantity of matter they contain, it follows, that the force which retards the perpendicular ascent of any body, being measured by its weight divided by the ascending mass, is the same, being such as destroys a velocity of $32\frac{1}{6}$ feet in each second of the body's motion. But in order to illustrate, by experiment, the general laws according to which bodies are retarded by the action of constant forces, such methods should be made use of as will enable us to apply different resisting forces to the same mass of matter, and the same resisting force to different masses of matter, both of which conditions will be satisfied by the instrument now before you.

A resisting force is to be understood as conveying precisely the same idea as the term moving force; except so far as regards the directions in which those forces act in respect to the body's motion, these directions being contrary to each other.

If equal quantities of matter be projected in free space, with any given velocity, and be resisted by different but invariable forces, the spaces described before the whole motion is destroyed, will be inversely as the resisting forces.

Let the mass projected be 61 *m*, with a velocity of 18 inches in a second, and let it be resisted successively by the forces *m*, 2 *m*, 3 *m*; the spaces described before the motion of the body is destroyed will be $\frac{25.6}{1}$, $\frac{25.6}{2}$ $\frac{25.6}{3}$; these spaces being in the inverse proportion of the resisting forces.

Make A equal 24½ *m*, B equal 25½ *m*; apply to the upper surface of A two rods, each equal to *m*; then will the weight, A, preponderate and descend by the action of a moving force equal *m*, the whole mass moved being equal to 63 *m*. Set the circular frame to 26.44; then the weight A, by describing from rest the space 26.44 inches, will acquire a velocity $= \sqrt{\left(\frac{4 + 193 + 26.44}{63},\right)}$ equal to 18 inches in a second; and at that instant the two rods, each equal to *m*, being removed, the weight will continue to descend with a uniformly retarded motion, which will be precisely the same, as if a mass of 61 *m* were projected with a velocity of 18 inches in a second, in free space, and a force of resistance equal to *m* were opposed to its motion; wherefore A, with the other parts of the system, will lose its motion gradually, and will describe a space equal to $\frac{18^2 \times 61}{4 \times 193} = 25.6$ inches before its motion be entirely destroyed. You will therefore have to descend as low as 52 inches, before it begins to ascend by the superior weight of B.

If any body, moving in a free space uniformly, be resisted by a constant force, for any given time less than that in which the whole motion would be destroyed, the space described will be the difference between the space which measures the initial velocity of motion, multiplied into a number expressing the given time, and that space which the body would describe, if accelerated, during the given time, from quiescence, by a force equal to that of resistance.

Let a mass $63\frac{1}{2}\,m$, be projected with a velocity of 11.877 inches in a second; if it be resisted by a force equal to $\frac{1}{2}\,m$, it will describe 21.95 inches in three seconds. To this end, make A equal 26 *m*, and B equal $26\frac{1}{2}\,m$, and apply a flat rod, $1\frac{1}{2}\,m$, to the upper surface of A; set the circular frame to 11.877 on the scale, then will A descend, the moving force being *m*, and the mass moved equal 65 *m*. When it has described 11.877 inches, it will have acquired a velocity of 11.877 inches in a second; and the rod being at that time intercepted, A will begin to descend with a uniformly retarded motion, and will strike the square stage set at 33.83, at the fifth beat of the pendulum, and thus describe 21.95 inches in three seconds.*

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* For the account of various other useful and scientific experiments, the reader had best consult the excellent Treatise of Mr. *Atwood*, before cited.

This instrument has many other uses, which it is not necessary to describe particularly here; such as, The experimental estimation of the velocities communicated by the impact of bodies, elastic and non-elastic; The quantities of resistance opposed by fluids, as well as for various other purposes; The motion of bodies, resisted by constant forces, are reduced to experiment by means of this instrument, with as great ease and precision as the properties of bodies uniformly accelerated; and, The verifying practically the properties of rotatory motion. The two last are clearly shown in Mr. *Atwood's* Work. This very useful and illustrative instrument requires to be well made, and nicely adjusted; to facilitate which, Mr. *Atwood* has given us the following articles:

1. The pendulum of the clock, which is fixed to the pillar of the instruments, vibrates seconds; it has only one wheel which immediately acts on the pendulum: the small weight which continues the pendulum's motion, after it has been wound up, is half an hour in descending to the ground. The clock will be sufficiently exact if it keep time with a common well-regulated clock for this half hour.

2. The feet of the frame on which the friction-wheels rest, should not be fixed immoveably to the frame underneath, because, in order to adjust the

OF THE DESCENT OF BODIES ON INCLINED PLANES.

Having explained the descent of bodies falling freely by the force of gravity, it will be easy to estimate the force with which they will descend down an inclined plane; in which the direction of the fall is altered, but the absolute force remains the same.

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axis of the wheel, *a b c d*, horizontal, it will be necessary to alter the height of the two anterior or posterior feet, by inserting small plates of wood or brass under them, or by means of screws acting immediately on the feet. A hanging level applied to the wheel's axle, will show when the axis is horizontal.

3. When the axis of the wheel, *a b c d*, has been adjusted horizontal, let two equal weights, A and B, be suspended from the extremities of a silk line of proper length, the thickness of which is no greater than is just sufficient to sustain the weights. When these weights are perfectly quiescent, a small impulse being applied to either, in a vertical direction, will set the whole in motion; which will be continued uniform till one of the boxes arrives at the extremity of the scale. When the box, A, is at the bottom of the scale and quiescent, it must be observed whether the middle line on the scale be every where exactly opposite to the line sustaining A; or, in other words, whether the line in the middle of the scale be in the same vertical plane with the line which sustains A. If it be not, the lower extremity of the scale must be moved along the arm of the base until the adjustment is correct. It is also to be observed, whether the line be every where at equal perpendicular distances from the middle line on the scale: if it be not, the upper extremity of the scale must be removed further from, or nearer to the silk line, until the distances are every where equal. The middle line on the scale will now be vertical, and the circular frame must be so constructed, that the box, A, may pass centrally through it, when the adjustments are correct.

4. The wheel, *a b c d*, previous to this adjustment, should weigh something more than six ounces, provided the figure of the wheels be not very different from those represented in the plate. In the circumference of the wheel, *a b c d*, a small groove should be cut and well polished, to admit the line which goes over the wheel and sustains the weights A and B.

5. The weight of the wheel, *a b c d*, must be so adjusted, that the resistance to motion of its inertia, with that of the friction-wheels shall be equivalent to 2 3-4 ounces. This may be effected by winding a very slender line round the wheel, and applying to the extremity a weight of 30 grains: let the square stage be fixed to the scale of 38 1-2 inches, and let the weight, 30 grains, begin to descend at any beat of the pendulum from 0 on the scale. If it describe the 38 1-2 inches in less than three seconds, the wheel is too light to admit of the experiments being made according to the numbers used in this section. Suppose, then, the weight to be more than three seconds in describing the 38 1-2 inches from the rest: the wheel's weight must be gradually diminished until the time of descent is exactly three seconds. It is however to be noted, that in first making this experiment for the adjustment of the wheel's weight, the line was wound round the cylindrical surface of the wheel's circumference, no groove having been at first found necessary. When a groove is cut, the line must no longer be wound round the wheel, for reasons too obvious to need mentioning: in this case, there-

All bodies endeavour to fall by the shortest course, that is, perpendicular to the earth. When a body is withheld from obeying the impulse of gravity, it is evident, that it is prevented by some obstacle which resists its natural tendency to descend. When bodies, therefore, fall down inclined planes, you must consider them as obeying the usual laws of gravity, as descending with a uniformly accelerated motion, but having the direction thereof changed, and the line of descent lengthened.

If the plane on which the body is placed be either perpendicular or parallel to the horizon, the direction of the line of descent is not changed: when the body is on a plane parallel to the horizon, it finds an invincible obstacle to its descent: when the plane is perpendicular to the horizon, it will fall freely, according to the laws already explained.

But when the plane is inclined to the horizon, it opposes in part the descent of the body towards the earth.

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fore, the adjustment of the wheel's weight is to be obtained by another method. Instead of the weights A and B, let the weights, 48 grains and 24 grains, be suspended from the extremities of the line which goes over the wheel *a b c d*, and set the square stage at 30; then letting the weight, 48 grains, begin to descend from 0 on the scale at any beat of the pendulum, observe the time of its describing the 30 inches as measured on the scale: if the time of descent should be less than three seconds, the wheel is too light: suppose, therefore, the time to be greater than three seconds; the wheel's weight must be gradually diminished until the time of descent is precisely three seconds. When this adjustment is correct, the resistance from the wheel's inertia including that of the friction-wheels, will be exactly the same as if those wheels were removed and the mass 2 3-4 oz. was uniformly accumulated in the circumference of the wheel to which the line is applied.

6. Great attention should be paid to the adjustment of the weights used in these experiments, as a very small error, such as is scarcely perceivable in each, will tend greatly to affect the exactness of the experiments in which many weights are used.

7. In letting the box, A, begin to descend at any beat of the pendulum, the observer must not wait until he hears the beat, at which he intends A's descent shall begin; for, in this case, A's descent will always commence too late: the proper method is to attend to the beats of the pendulum, until an exact idea of their succession is obtained; then the extremity of the rod being withdrawn from the bottom of the box, A, directly downwards at the instant of any beat, the descent will commence at the same instant.

The wheels and their mahogany basis are separated from their stand, and, with the various weights and other apparatus, packed into a small mahogany case. The scales, C, C; the ring, H; the stage, D; the pillar, G; and clock, E, are also made so as to be separated and packed in a portable and convenient manner.

E. Edit.

Those bodies which descend on an inclined plane, have both an absolute and a relative gravity.

Their absolute gravity is the force with which they would descend perpendicularly to the earth's surface, if their fall were impeded by no obstacle; their relative gravity is this same force diminished by the resistance of the plane.

The line A B, *plate* 1, *fig*. 14, perpendicular to the horizon, is called the height of the plane; the oblique line, A D, is the length of the plane; the line B D, parallel to the horizon, is the base of the plane; and the angle, A D B, that the plane makes with the horizon, is called the angle of inclination.

If a body fall freely from a state of rest the length of an inclined plane, A D, 1. *Its motion will be uniformly accelerated.* 2. *The velocity it acquires in its descent, is to that which it would have acquired in the same time, falling freely and perpendicularly, as the height of the plane is to its length.*

Let C be a body placed on an inclined plane, the action of gravity would cause it to descend in the line C G, or line of direction perpendicular to the earth: this line, therefore, represents the absolute force of gravity.

Resolve the line, C G, into two forces; one, C F, perpendicular to the inclined plane; the other, C H, parallel to it. The line, C F, will represent that part of gravity which is destroyed by the inclined plane, or by which the body is prevented from falling; and the line, C H or F G, will represent the remaining force, or the relative gravity which acts on the body to make it descend.

The force, therefore, which acts on the body to make it descend, is to the absolute force, or that which would cause it to fall freely and perpendicularly, as F G to C G; or, the triangles, A B D, C F G, being similar, as A B to A D: and since, in whatever place the body is situate, the triangle, C G F, is the same, it is plain, that at every instant of the descent, the force, F G, which causes the body to descend, is the same, or constant; and the motion produced is therefore uniformly accelerated, and will accord with the laws thereof.

As the motion of a body down an inclined plane is uniformly accelerated, in the same manner as the motion of a body falling freely, it follows, that at the end of any given time, the velocity acquired must be as the accelerating forces: but these are as the height of the plane to the length; and, consequently, the velocities acquired at the end of any given time, must be in the same proportion.

It follows from hence, *that the force which causes the descent of a body on an inclined plane, is only the force of gravity diminished in the ratio of the height of the inclined plane to its length;* that the velocities acquired are as the times, and follow the progression of the ordinary numbers 1, 2, 3, 4, 5; the spaces described from A, are as 1, 4, 9, 16, &c. the spaces described in each interval, are as 1, 3, 5, 7; and, lastly, the space described in descending from A to D, is but half the space the body would have described in the same time, moving uniformly with the velocity the body had acquired at D.

A little reflection, independent of geometry, will render the subject clear to you, and may perhaps be more agreeable. The plane opposes the perpendicular descent of any body, and, consequently, diminishes its absolute gravity only in proportion to its inclination: for, if it were perpendicular to the horizon, it would not at all resist the fall of the body; but the more it is inclined, or the less the perpendicular height in proportion to its length, the more the body is sustained by the plane, and the less is its relative gravity; consequently, the relative gravity of the body is to its absolute gravity, as the length of the plane to its height.

If the angle of inclination become a right angle, the relative gravity becomes the same as the absolute.

If there be no inclination, the relative gravity becomes 0, and the body has no tendency to move on the plane. If the angle of inclination, be infinitely small, the relative gravity will be also infinitely small, increasing with the angle of inclination.

If from the point B, plate 1, fig. 14, *a perpendicular,* B K to A D, *be drawn,* A K *will be the space described by the body* C, *in the time it would have fallen freely from* A *to* B.

For, on account of the right angle at B, it is proved, by the rules of geometry, that as A K is to A B, so is A B to A D; that is, A K: the space described by the body in descending from A to K, is to A B, the space it would have described in falling freely in the same time from A to B, as A B, the velocity acquired by the body in descending from A to D, is to A D, the velocity it would have acquired in falling freely during this time.

But the velocity acquired by the body C, in descending from A to D, is to that which it would have acquired in falling freely during the same time as A B to A D; therefore, while the body, C, falls from A to K, it would have fallen freely from A to B; or, in other words, A B and A K are the spaces that it describes in the same time.

Coroll. If a body be, therefore, at any point, K, of an inclined plane, and the space be required that it would have described in the same time from A, you may find it by letting fall a perpendicular from A D to cut A B, which will give the required point.

Coroll. *A body takes up as much time in describing the length of the chord,* A K, plate 1, fig. 15, *of a circle, as to fall freely from a state of rest, the length of the diameter* A B: for joining K B, the angle, A K B, will be a right angle, and the line, A K, will represent an inclined plane; and therefore, from the preceding article, the spaces A K, A B, ought to be described in the same time.

Therefore the times of descent on all the chords of the same circle are equal; and if A, plate 1, fig. 15, *be the point of contact common to several circles, bodies descending in the same time the length of the chords* A K, A D, A B, A E, *will describe in equal times the parts* k K, d D, b B, e E.

Two bodies setting out at the same time from the same point A, and descending on planes A D, A M, *plate* 1, *fig.* 14, differently inclined;—required the space of the body descending on A M? When the body descending on A D is at K, raise the perpendicular K B, and from the point where it meets the vertical, A B, let fall the perpendicular B L on A M, and L will be the point sought.

The time a body employs in descending an inclined plane, is to the time in which it would fall perpendicularly the same height, as the length of the plane to the height of it. That is, the time a body employs to move from A to D, is as much longer than the time it would fall perpendicularly from A to B, as A D is longer than A B.

We have shown that A K : A B :: A B : A D; from whence it follows, by geometry, that A K is to A D as the square of the first to the square of the second.

You may illustrate this conclusion by numbers: 3 is to 6 as 6 to 12; and 3 is to 12, the third, as 9, the square of the first, is to 36, the square of the second; for as 3 is contained four times in 12, so is 9 in 36. This being premised, we may find what proportion the time of descent, through a part of the plane A K, bears to the time of descent through the whole plane A D.

From the laws of uniformly accelerated motion, we know that the spaces described are as the squares of the times in which they are described; consequently, as A K is to A D, so is the square of the time in which A K is described, to the square of the time in which A D is described: but the square of A K is to the square of A B, as A K to A D; therefore the time of descent through A K, is to that through A D, as A K to A B; but A D is to A B, as A B to A K; consequently, the time of the body's descending through A K, is to the time of the descent through A D, as A B, the height of the plane, to A D, the length of it: but, since a body employs the same time in descending through A K as through A B, we conclude that the time of the perpendicular descent through A B, is to the time of the oblique descent through A D, as the height of the plane is to the length of it.

Hence we conclude, that the time employed to descend the length of any number of inclined planes of the same height, are to one another as the length of these planes respectively, for A K is to A B, as the time employed to descend from

A to D, is to that employed to descend from A to B. By the same reason, A M is to A B, as the time in descending from A to M is to the time spent in going from A to B; therefore the time employed to descend from A to D, is to that employed to descend from A to M, as A D to A M.

The velocity acquired by a body falling perpendicularly the height of an inclined plane, A B, plate 1, fig. 14, *is equal to the velocity acquired at the end of the descent by a body moving down the inclined plane from* A *to* D.

For, the accelerating force of a body falling freely from A to B, is to the accelerating force of a body moving along the plane AD, as AD to AB; but as AB is to AD, so is the time of the descent from A to B, to the time of the descent from A to D; so that the forces which accelerate the motions, are to one another reciprocally as the times they continue to act; consequently, at the end of those times the velocities must be equal.

Or, in other words, the force of gravity, by which a body falls perpendicularly from A to B, is greater than the force by which it descends from A to D; but this last force acts so much longer as it is weaker, and therefore will in the end produce an equal effect or velocity.

Consequently the velocities acquired by bodies falling down inclined planes are equal, when the heights are equal.

If a body descend down several contiguous inclined planes, AB, B C, C D, plate 1, fig. 17, *the velocity which it acquires in its descent from* A *to* D, *is equal to the velocity acquired by the perpendicular fall from* A *to* D, *on a supposition that the body is not disturbed by any resistance it meets with at the angles* B *and* C.

Draw the horizontal lines, H E and D F, through the points A and D, and produce the line C B and D C as far as G and E.

By the corollary of the last proposition, you know that the same velocity is acquired at B, by a body descending from A to B, as in descending from G to B; consequently, the same velocity is acquired in the point C, by a body descending from A through B to C, as in descending from G to C. But the velocity acquired in descending from G to C, is equal to the velocity acquired in descending from E to C; wherefore the velocity in the point D, acquired by the descent along the three planes, AB, B C, and C D, is equal to the velocity acquired by the descent from E to D, which velocity is equal to the velocity acquired by the perpendicular fall from H to D.

It follows from hence, that when a body descends through an arc of a circle, or any other curve, it will have acquired, at the end of its descent, a velocity equal to that which it would have acquired by falling the perpendicular height of the arc.

For curves may be looked upon as composed of an infinite number of right lines.

If a body descend in a curve, and another descend perpendicularly from the same height, their velocities will be equal at all altitudes.

If a body, after its descent in a curve, should be directed upward with the velocity it had acquired, it will ascend to the same height from which it fell: for gravity acts with the same force, whether the body ascend or descend; it will therefore destroy the velocity in the ascent, in the same time it had generated it in the descent.

The velocity of a body descending in any curve, is as the square root of the height fallen from; for, it is the same as in falling perpendicularly and in this case it is as the square root of the height.

LECTURE XXVIII.

OF THE CENTRE OF GRAVITY.

I HAVE now to notice some things relating to heavy bodies, the knowledge of which is highly necessary for rightly understanding mechanics.

In every body there is a certain point, usually termed by the writers on mechanics the *centre of gravity:* the nature and properties of this point are now to be explained.

The *centre of gravity* is that point in any body, about which all its parts, in every position, are in equilibrio, or balance each other.

Now, as the parts of a body are in equilibrio about this point, if this point be supported, the whole body will be at rest. By supporting, therefore, this point, the weight of the whole body is supported; we may, therefore, consider its whole weight as centered in this point. Hence mathematicians, by the *place* of a body, often mean that point where the centre of gravity is situate.

If I take this walking-cane, and attempt to balance it across my finger, I shall at last find some one particular part in it, which being supported, neither of the ends will preponderate. The part which rests upon my finger is the centre of its weight; this being supported, the whole is supported. If I remove my finger from this centre of gravity, towards either of the extremities of the cane, though but the smallest distance, that side in which the centre of gravity is, will sink towards the earth.

Here is a triangular piece of brass, *plate* 1, *fig.* 22, supported by an axis passing through the centre, *c*, of gravity; and you observe that it will rest in any position I place it, whether the side be perpendicular, parallel, or inclined to the horizon.

All bodies by their gravity tend towards the centre of the earth, and that by the shortest way, unless they are hindered by some obstacle opposed to this tendency.

But gravity acts upon bodies as if their whole weight was collected in the centre of gravity. It follows, therefore, that *this centre will continually endeavour to descend towards the centre of the earth;* and where all obstacles are removed, does actually so descend.

But the effort of gravity is in lines perpendicular to the horizon; therefore a body will not rest, except the centre of gravity and point of suspension be in the same vertical line.

A body suspended freely by the centre will not rest, unless the centre of gravity and point of suspension be in the same vertical line.

In every other position, gravity is continually acting to make the body descend; but when the point of suspension is in the same vertical line with, or directly opposed to the centre of gravity, the action of the weight is destroyed, and an equilibrium takes place.

Reciprocally, whenever a body is in equilibrio, we may conclude its centre of gravity is supported in a vertical line.

This will be rendered more evident, by considering that gravity acts in a direction perpendicular to the horizon, and the whole weight of a body being collected into the centre of gravity, that point will always endeavour to descend in a vertical line, and with a force equal to the body's weight: for this reason, a vertical line passing through the body's centre of gravity, is called *the line of direction.*

When the line of direction does not pass through the centre of suspension, the body's weight may be resolved into two forces; one of which impels the centre of gravity directly from the centre of suspension, and is wholly counteracted, because the centre of suspension is fixed; the other force, acting in a direction perpendicular to the former, and having no counterpoise, urges the centre of gravity, in the direction of a tangent to that circle which that point describes about the centre of suspension.

If a body be suspended by a line from a fixed centre, it will be at rest only when the line coincides with a perpendicular to the horizon. An instrument of this kind is called a *plumb-line*, and is of very extensive use, both in philosophy and the mechanic arts. By means of this line, the vertical direction, and the horizontal plane, which is perpendicular to it, are precisely determined. The same may be effected by the spirit-level; but the plumb-line has the advantage of requiring no adjustments; whereas the exactness of the level depends on the position of the points from which the level is suspended, or of the base on which it rests, in respect to the upper surface of the tube. It is, besides, liable to some imperfections, from which the plumb-line is free.

Let us now illustrate the foregoing positions by experiment.

1. I suspend this round board, *plate* 2, *fig.* 2, on the points of the calipers A, in such manner, that the centre of gravity coincides with the centre of motion; and in this instance, the body being both of a regular figure and of a homogeneous substance, the centre of gravity is exactly in the middle thereof. It will remain in any position in which I stop it; and on turning it round, the marks made thereon describe circles round the centre of motion, which is here also the centre of magnitude.

2. I suspend the same board by another centre, *plate* 2, *fig.* 3; and you will now find that the board will not remain at rest, unless the centre of gravity, G, be directly under or directly above the centre of motion. Here the points, K and L, still describe circles about the centre of motion, but the centre of magnitude is not now their common centre.

3. Here is a similar body, *plate* 2, *fig.* 4, which is made denser in one part, by fixing a piece of lead, M, therein. In this case the centre of gravity is no longer at the centre, C, of the board, but at K; about which point only, as a point of suspension, the body will remain in any given position. If it be suspended by any other centre, it will only be at rest when the piece of lead, M, is directly under K, or directly over it.

From what has been mentioned it appears, that *if a body be suspended freely from different centres, its centre of gravity will be in the intersection formed by lines drawn from those centres perpendicular to the horizon.*

Hence we obtain an easy practical method of finding the centre of gravity of any irregular plane figure: suspend it by any point with the plane perpendicular to the horizon, and from the point of suspension hang a plumb-line, and draw a line upon the body where the string passes over; do the same for any other point of suspension, and where the two lines meet must be the centre of gravity; for, the centre of gravity being in each line, it must be at the point where they intersect.

Thus, suspend the body AB, *plate* 2, *fig.* 6, whose centre of gravity is to be found, by any part A, so that it may move freely on that part; let a plumb-line hang from the pin on which it is suspended; mark this line upon the body, and then suspend it by another part, as F, *plate* 2, *fig.* 5, and hang the plumb-line on the point from which it is now suspended, and the line drawn by the plumb-line will intersect the line AB in C, the centre of gravity.

Similar to this is the following mode of finding the centre of gravity mechanically. Place a board of equal thickness on the sharp edge of a triangular prism, so that it may be in equilibrio on the upper edge, and then whatever bodies are laid upon the board in such manner as not to alter its equilibrium, must have one plane of the gravity directly over the edge which supports the board. If the position of the body be changed without destroying the equilibrium, you will obtain another plane of gravity: the intersection of these planes gives a line or axis, in which is the centre of gravity. The bodies may then be placed in a third position, so as to find a third plane of gravity, which shall cut the other planes in a large angle, and the intersection of the three lines will be the centre of gravity of the body.

Thus you may find the centre of gravity of the human body, or of any animal. With respect to the human body, it is observable, that whether a man be fat or lean, or even a mere skeleton, the centre of gravity is always near the same place, in the pelvis, between the hips, the ossa pubis, and the lower parts of the backbone. Raising up the arms and legs will raise up the centre of gravity a little; but still it is always so placed, that the limbs move freely round it; the centre of gravity at the same time moving much less than if it were in any other part of the body.

If a body be placed upon a horizontal plane, and the line of direction pass within the base, it will stand; if it pass without the base, it will fall.

While the line of direction passes through the base, the weight of the body keeps it down with force against the surface on which it rests; but when the line of direction passes without the base, the centre of gravity loses its support, and the body falls until this centre be sustained.

Thus, the inclining body ABCD, *plate* 2, *fig*. 7, whose centre of gravity is E, stands firmly on its base CDIK, because the line of direction, EF, falls within the base.

If a weight, ABGH, be laid upon the top of this body, the centre of gravity of the body together with the weight is raised to I; and as in this state the centre of gravity, I, is not supported, the whole tumbles down together.

Hence you may perceive, how absurd it is for people to rise hastily in a coach or boat, when it is likely to overset, for by this means they raise the centre of gravity so far as to endanger throwing it quite out of the base; if this be done, the vehicle will be effectually overset, whereas by keeping as close to the bottom as possible, they would bring the line of direction farther within the base, and probably prevent the carriage from being overset.

The broader the base is, and the nearer the line of direction is to the centre, the firmer will the body stand; on the contrary, the narrower the base, and the nearer the line of direction to the edge thereof, the more easily may the body be overthrown.

Upon this principle, a body which would fall off the table of itself, will not fall off, although you place a weight upon the part which does not rest upon the table, provided that by this means you throw the centre of gravity of the whole under the table.

I place this piece of wood, *plate* 4, *fig*. 1, upon the table, with the part, B, from the table, and so that the point, c, may be over the edge thereof; it falls off because the centre of gravity is not supported; but by fixing the two wires, DE, FG, with leaden heads into the board at B, it is supported, because the centre of gravity of all the three bodies is now supported by the table.

From this you learn how to balance a body resting upon a point on its under side; this is effected by suspending a weight from each end; for, by this means you bring the centre of gravity below the point of suspension, on which it will then rest: whereas, before the weights were suspended, the centre of gravity was above the point of suspension, and unless it had been exactly over it, which it is very difficult to accomplish, it would descend, and the body must fall.

Various contrivances are executed that take their rise from this principle, that the centre of gravity always tends to the lowest point possible. In France they make use of an odometer, or coach way-wizer, which is fixed between the spokes of one of the wheels; the moving power in this machine is a wheel loaded on one side with a weight, turning freely round with its axis, the centre of gravity being below the axis on which it turns. The centre of gravity in every position keeps the lowest point, so that the arbor and wheel turn once round for every revolution of the wheel; the arbor is properly connected with a train of wheels and pinions, so as to register the revolutions of the coach-wheel.

Upon the same principle are also deduced the modes of suspending the marine barometer, compasses, &c. The toy, called a Chinese tumbler, acts also upon the same principle.

It is also on this principle that a double cone appears to roll up two inclined planes, forming an angle with each other, and lying in the same plane; for, as it rolls, it sinks between them, and by that means the centre of gravity actually keeps descending. To effect this, the height of the planes must be less than the radius of the base of the cone. If the height be equal to the radius, the body will rest in any part of the plane; if the height be greater than the radius it will descend.

The two rules, AB, CD, *plate* 2, *fig.* 8, are joined together by a hinge at one end; the lower sides are straight, on the upper sides one end is wider than the other, so that when opened, and raised by two screws, they form two inclined planes: I place the double cone, EF, near the ends, and you see that as soon as I let it go, it begins to roll towards the upper end of the planes, and thus apparently to ascend; the parts of the cone that rest on the rules growing smaller as they go over a larger opening, and thus letting it down.

A cylinder may also be made to roll up an inclined plane for a small distance; if it be loaded near its circumference with something heavier than itself, and that part be laid towards the top of the plane, then, the centre of gravity being out of the axis towards that part, it will descend whilst the body rolls upwards.

If the line of direction of a body placed on an inclined plane fall within the base, the body will slide down the plane.

Plate 1, *fig.* 20, let A*d* be the inclined plane, R the body, ck the line of direction. Now, the force of gravity represented by *ck*, being greater than the force *ce*, which retains it to the plane, it must therefore either slide or roll down the plane; but it cannot in this instance roll, because to effect this the centre of gravity must rise: which it cannot do; the only method therefore it has to descend, is by sliding down the plane.

If the line of direction of a polyhedron, S, plate 1, fig. 20, *placed on an inclined plane, pass without the base, and the friction of the surfaces be sufficiently great, the body will roll down the plane.*

If CK represent the absolute force of gravity acting on the polyhedron, CE a perpendicular to the inclined plane, represents what is supported by that plane, or what the plane destroys of CE; and CF parallel to the plane, what remains of the force of gravity, or its relative efforts. Now, as motion is only produced by and in the direction of the relative force, the polyhedron tends by the force, CF, to slide down the plane AB; but meeting with a resistance acts as a force in the direction EG, it destroys in part the motion according to CE, and produces at the same time a rotatory motion, which causes the polyhedron to roll. The line of direction, CK, falls without the base, BE.

If the friction be removed to a certain degree, the body will descend partly by rolling and partly by sliding; if it were possible to remove friction entirely, it would descend by sliding without any rotation.

It may seem at the first view rather surprizing, that a sphere should descend along an inclined plane without rolling; but it is certain that there is no force to produce the rotation, except that which arises from the friction of the surfaces, which being entirely removed, the sphere must slide.

As the component parts of any body are subject to the force of gravity, and therefore have weight, any body may be considered as a system of small bodies so connected as to form one; and consequently, if several bodies be united in a machine, or several combinations of bodies to be sustained, no attention is to be paid to to the centre of gravity of the several parts which make the system, but only to the centre of the whole.

If a rod or beam be so constructed, that the line drawn through the centre of gravity and suspension shall be perpendicular to any given line in the rod (for example, that which passes along its upper surface), the rod hanging freely on its axis of suspension, will not be at rest except when that given line is in a horizontal position.

Suspend this triangular piece, *plate* 1, *fig.* 22, by the centre, *a*, and when it has ceased vibrating, you will find the side which is perpendicular to a line draw through the centre of gravity and suspension is in a horizontal position.

That this must happen is evident, because the horizontal position, under the circumstances here described, is the only position in which the line of direction passes through the line of suspension.

Upon this principle the balance, by which the weights of bodies are compared, is constructed.

OF THE SITUATION OF THE CENTRE OF GRAVITY.

The centre of gravity of any line is its middle point.

The centre of gravity in regular, uniform, and homogeneal solids, is at the centre of magnitude.

For, if from all the points on one side of the surface of this solid, lines be drawn through the centre to the opposite side, these lines will be all divided into two equal parts by the centre; the two parts of each line are equal, and of equal weight, and consequently the whole solid will be in equilibrio about that figure.

The centre of gravity of the surface of a sphere, or regular polyhedron, is the centre of the figure; that of a prism or cylinder is in the middle of the axis that passes through the centre of gravity of their opposite bases.

To find the centre of gravity of the surface of a triangle A B C, *plate* 1, *fig.* 19, bisect any two of its sides in D and E, draw lines from the opposite angles to the points, and the centre of gravity will be in the intersection of the lines, F; for E A bisects all the lines drawn parallel to B C, which constitute the surface of the triangle, therefore it passes through the centre of gravity of each line, and for the same reason D C passes through them also.

A similar reason proves, that the centre of gravity of a regular polygon, whose number of sides is an odd number, is the same with the centre of the figure.

The centre of gravity of a triangle may be found by taking two-thirds of the line drawn from any one of its angles to the middle of its opposite side.

TO FIND THE CENTRE OF GRAVITY OF A TRAPEZIUM.

Find the centre of gravity of the two triangles formed by a diagonal, and join them by a straight line; find and join also by a straight line the centre of gravity of two triangles formed by another diagonal; the intersection of these two lines will be the centre required.

By the same means we may easily find the centre of gravity of any polygon.

Divide the polygon into as many triangles less two as it has sides, find the centre of gravity of each triangle, and then considering these centres as so many bodies, find the common centre of the whole system.

TO FIND THE CENTRE OF GRAVITY OF A PYRAMID.

It is evident, 1st. That the elements of a pyramid are polygons similar to the base, and similarly situate, whose dimensions decrease in an arithmetical progression from the base to the summit; therefore, the straight line which goes from the centre of gravity of the base to the summit, will pass through the centre of gravity of all these elements, and consequently through that of the pyramid. 2d. If the pyramid be triangular, each face may be taken for the base, and the opposite angle for the summit; whence it follows, that the centre of gravity of a triangular pyramid S A B C, *plate* 1, *fig*. 21, is at the intersection of the two right lines S F, A F, drawn from the two angles S, A, to the centres of gravity of the opposite faces, or two-thirds of the length of each line S D, A D, drawn on the plane of the opposite face from the angles S, A, to D, the middle point of the side of the base opposite to these angles.

The centre of gravity of a cone or pyramid is three-fourths of the axis, reckoning from the summit or vertex.

For the centre of gravity of the sector of a circle, say, as the arc to its chord, so is two third of radius to the distance of the centre of gravity from that of the circle.

For an arc of a circle, as one-half the arc to the sine of one-half the arc, so is the radius to the distance of the centre of gravity from the centre.

Thus in bodies that are of uniform density, and at the same time admit of geometrical mensuration, the position of the centre of gravity may be ascertained by theory; but if they be wholly irregular, recourse must be had to experiment.

As any body considered in mechanics is only an aggregate of several other bodies or parts, so the centre of gravity of a body is only the common centre of gravity of all its parts; and, consequently if several bodies be united in any machine, or if there be any combination of bodies to be sustained, we are no longer to regard the particular centres of gravity of the compound, but only the common centre of gravity of the whole.

Thus a windmill should be supported under the common centre of gravity of all its parts, and its line of direction should coincide with the axis of the post round which it moves; and a crane on a wharf or a dock, where the whole machine turns round, should have the line of direction in its axis.

Let the line A B, *plate* 1, *fig.* 23*, represent an even rod or wire divided into two equal parts at the point c, its centre of gravity will be at c. If two equal bodies be fixed upon the ends thereof, so as to have their centres of gravity at the same distance from c, they will be in equilibrio about the said point, which will become their common centre of gravity, and continue so, whether the bodies approach to or recede from it, in proportion to their masses.

The same will happen if the bodies be unequal, as A and *b*, *plate* 1, *fig.* 24, whose masses are to each other as two to one, provided the greater body, at A, be twice as near to the common centre of gravity as the lesser, b; and c will be the common centre of gravity of those bodies, though they should move to immense distances from each other, provided their distances from the said point be reciprocally as their masses.

So that when two bodies approach to or recede from each other with velocities reciprocally proportional to their masses, their centre of gravity will remain at rest.

If the bodies be made fast upon the wire, and the centre of gravity be sustained on a pivot, it will remain at rest though the bodies revolve round with the utmost velocity; and the bodies will describe similar circles about it and about each other, the one never overpowering the other.

If they be carried forward in any manner by any external force acting upon them in proportion to their masses, their centre of gravity will go forward uniformly in a straight line, and be moved just as if the two bodies were united into one at that centre; and if they be projected, their centre of gravity will move in the same curve as other projectiles, which is evident by the motion of an arrow, of chain-shot, and of a stick thrown from the hand, the centre of gravity of these bodies moving like a single ball.

So also the earth and moon in their motion round the sun, do neither of them describe the real regular orbit, *but it is described by their common centre of gravity in the same manner as if they were both united in that point*, or in the same manner the earth is supposed to do when these inequalities of motion are overlooked; and if their distances from the common centre of gravity be reciprocally proportionable to their masses, their distances from each other may be greater or less in any proportion.

If to the two bodies A and B, *plate* 2, *fig.* 9, there be added a third, at D, equal to one of the other, let A and B be reduced to their common centre of gravity, and be considered as a body equal to both placed at C, then the common centre of gravity of C and D will be found at K, as much nearer to C, as the mass of the body or bodies, at C, exceeds that of the body at D. If the new body weighed but half as much as the others, it must be removed to d, so as to have the distance, Kd, quadruple the distance K, C. Now, if C D be a wire, and it be supported under K, the three bodies, whether D or d be used, will be thereby sustained. In taking the centre of gravity at K, we consider the wire as a mathematical line without substance or weight.

If these three bodies, united to or acting upon one another proportionably to their masses, be carried round their common centre of gravity, that point will be at rest. Hence also in our system, where the sun and all the planets move round their common centre of gravity, that centre is at rest in the middle of the system. The sun is, on account of its vast bulk, &c. compared with the other planets, considered in general as the centre of the system.

Though the centre of gravity of a body, or of a system of bodies, is often neither within the body itself, nor any of the combined bodies; yet it is to be considered, with respect to its support, descent, or motion, in any direction, as if it were so situate. Thus, let us suppose A and *b*, *plate* 1, *fig*. 24, to be at the distance, A b, from each other, and that A b is no longer a wire but a line representing their distance, we shall then find their centre of gravity at *c*, without the bodies; and if, instead of a wire C D, *plate* 2, *fig*. 9, we suppose D joined to A and B by the wires A D, B D, the centre of gravity, K, will be neither in these bodies nor the wire; so that to support them you must sustain some part of the wire, as G, which being made the centre of motion, the centre of gravity will be under it; or if we support the point H, the bodies will be at rest, because the centre of gravity is over H, the centre of motion.

In the ring A O B E, *plate* 2, *fig*. 10, the centre of gravity is in no part of the ring, but may be supported by any other point, O or E. Thus the centre of gravity of the ring of Saturn is within the body of the planet; and though the common centre of gravity of the sun, moon, and earth, is within the body of the sun, yet the common centre of gravity of the moon and earth is in neither of the bodies, but between them.

When any number of bodies move in right lines with uniform motions, their common centre of gravity moves also in a right line with a uniform motion; and the sum of their motions, estimated in any given direction, is precisely the same as if all the bodies in one mass were carried on with the direction and motion of their common centre of gravity.

For the sum of the motions of the bodies estimated in any given direction is preserved invariably the same in their collisions, without being affected by their actions upon each other, which are equal and mutual, and have contrary directions; and, consequently, their centre of gravity is no wise affected by their collisions on any such actions, but perseveres in a state of rest or uniform motion, as any one body perseveres in its state till influenced by some external circumstance.

The name system does not belong properly to any unconnected assemblage of particles, but can only be applied with propriety to such collections of particles as are connected together by mechanical forces. The varieties in such connecting forces are innumerable, but we only consider here the motions of such systems, whose particles are connected by mutual and equal forces.

Equal and contrary motions communicated to any system of bodies will have no effect upon their centre of gravity, for they would not disturb a body equal to the sum of them all placed in their centre of gravity.

The centre of gravity of a system of bodies will not be disturbed by their mutual attractions, as the motions thus communicated are always equal and opposite; hence the centre of gravity of our system is either at rest, or moves on uniformly in a straight line. The latter is supposed by Dr. *Herschel* to be the case from the change which has been observed in the relative situation of some of the fixed stars.

Hence also the centre of gravity of the earth is not affected by the motions on its surface and bowels: when a cannon-ball, for instance, is thrown upwards, the projecting force, re-acting on the earth, causes it to move in a contrary direction, but as the motions are equal, the centre of gravity remains the same.

The motions and actions of bodies upon each other in a space that is carried uniformly forwards, are the same as if that space were at rest; and any powers or motions that act upon all the bodies so as to produce equal velocities in them in the same or in parallel right lines, have no effect on their mutual actions or relative motions.

Thus the motions of bodies on board a ship that is carried steadily and uniformly forwards, are performed in the same manner as if the ship were at rest. When a fleet of ships is carried away by a uniform current, their relative motions are no wise affected by the current, but approach to or recede from each other, as they would if no such current existed. The motion of the earth and air round its axis has no effect on the action of bodies and agents on its surface, only so far as it is not rectilineal and uniform.

In general, the actions of bodies on each other depend not on their absolute but relative motion, which is the difference of their absolute when they have the same direction, but their sum when they are moved in opposite directions.

GENERAL OBSERVATIONS.

The stability of a body on a horizontal plane depends, as we have already observed, on the position of the line of direction relative to the base of the body; the nearer this line approaches the centre of the base, the more firm the body stands, and the contrary, which we experience daily in a thousand different ways.

When a man is standing, the line of direction passes directly between the feet; when he walks, most of the motion is to preserve this line in the same position.

A man standing with his feet close is not near so firm as when they are at some distance.

A man sitting in a chair cannot rise without bringing his body forwards, and moving his feet backwards, till the centre of gravity be before his feet, or at least upon them, when to prevent falling forwards, he brings one foot forwards.

For when we are sitting on a chair, our centre of gravity is on the seat, and the line of direction falls behind our base; we therefore lean forwards to bring the line of direction towards our feet, and draw our feet backwards at the same time, that we may carry our base towards the line of direction; when the centre of gravity is reduced so as to be exactly over our feet, we are able to raise ourselves upright.

In walking up a steep hill, a man brings his body forwards, and presses only on his toes or fore-part of his feet, so that the centre of gravity may be between his feet, and prevent his falling backwards.

And for the same reason, by an easy and natural motion, we carry the body from right to left, and from left to right at every step.

Rope dancers also use a pole loaded with lead at its two extremities, in order to counterbalance their various movements, and always regulate the motion of the centre of gravity.

In general our motions, and particularly the friction of the feet, serve to modify considerably the effect of our weight, and to preserve constant stability amidst a variety of causes which tend to destroy it.

To consider walking more particularly: in walking, the foot from which we set off is our base at first, till, by turning the ball of it round, we have thrown the centre of gravity forwards beyond it, by which means we should throw ourselves down if we did not take up the other leg from the ground, and set it before us so as to catch ourselves upon it, as upon a prop or support, to prevent us from falling. This is taking one step; in order to take a second, the centre of gravity must be brought directly over the prop; that is, the foot which we put before us must be made our base, from which we may set off in taking a second step. We do this by turning the ball of the foot still farther round, so as to push the ground that we stand on back with our toe, and the ground, by its resistance, pushes our centre of gravity forwards, till the line of direction is got to the place where we want to have it reduced. Hence in walking, the line of direction passes through each foot alternately, and if we set one foot directly before the other in every step that we take, then this line will move evenly forwards; but if we straddle as we walk, then the line of direction does not go on evenly, but is carried out first towards the right side, and then towards the left, at the same time that it goes forwards, which is the reason why those who straddle in their walk, as fat people commonly do, are observed to waddle: indeed every body waddles more or less, but they who walk pretty close do it so little as not to be perceived.

Ducks, geese, and the greatest part of water-fowl, whose legs are set wide asunder for the convenience of swimming and turning quick in the water, have always a waddling motion upon land; but a cock, a stork, an ostridge, and many other birds that are not web-footed, walk directly forwards without waddling, more especially when they walk slow. Quadrupeds seldom or never waddle, because they have commonly three feet upon the ground at a time. The ancients observing, that horses and other quadrupeds, in galloping, lift up their two fore-feet and then their hind-feet, as soon as the two fore-feet are set down, imagined, that in walking, as well as pacing and trotting, a horse has two feet off the ground at one time; and accordingly, in their statues, they have represented their horses with two legs off the ground diagonally opposite: the modern statuaries have fallen into the same error. *Borelli* has shown that this is an error, and inconsistent with the simplicity and wisdom we observe in the works of nature:—to his treatise, *De Motu Animalium*, I must refer you.

In skaiting, the line of direction passes through the foot on which the skaiter moves; and as he raises his foot from the ice, he scrapes it in a contrary direction to that in which he designs to move, and the resistance of the ice upon it pushes him forwards.

In jumping, the centre of gravity rises from the ground, and is carried forwards at the same time. To render this plainer, I will explain it by a very simple instance: if you set one end of a bow stick on the ground, and with your hand bend down the other end, upon letting this end go, the bow flies out at once, and jumps from the ground; for the elasticity of the stick unbends it equally both ways, and as the action of one end is downwards and presses upon the ground, the re-action will be upwards, and the centre of gravity will have a motion communicated to it in that direction, and will therefore continue to rise, and the stick will by this means be carried off from the ground till its weight puts a stop to its farther ascent, and makes it fall back again.

The force of the muscles is that which in jumping supplies the place of elasticity, for by bending the joints of the two heels, the two knees, and the two thighs, there are six bows bent; and if all these be suddenly straightened by the force of the muscles, the centre of gravity of the body will have a motion upwards communicated to it, and the body will by this means be made to rise from the ground. When we are to jump directly upwards, we push the ground on which we stand directly downwards; but when we move obliquely, so as to rise and jump forwards at the same time, we push the ground obliquely backwards with both feet together.

You may now consider what it is that makes the difference between walking and running: if you imagine, that these two modes of motion differ only in the degree of swiftness, and that as walking is a slow motion, so running is only a swifter motion of the same sort, you are much mistaken; for these two motions differ in sort rather than in degree. Indeed they do not differ always in degree, for there are some who will walk faster than others can run; we can walk quick and run slow; in a, word, they are two entire different modes of motion. Walking is moving forwards by steps, so as never to have both feet off the ground at the same time; running is moving forwards by jumps. Indeed, in running, we do not take our jumps with both legs close together, but with one of them before the other; but I call it jumping,. because in throwing ourselves forwards both feet are off the ground at once.

In carrying a load, a man always leans so as to bring the centre of gravity between his feet, and therefore leans the contrary way to the load. He naturally leans forwards when he has a burthen on his back, for if he attempted to retain his usual rectitude of figure, his centre of gravity would be altered, and he must consequently fall backwards; for the same reason, when the burden is on his breast, he counterbalances the weight by altering his figure in the opposite position. In almost every instance of our motions we are obliged to make use of these balancing arts to keep ourselves upright; and it is usually the study of a fine painter, to know how far the human figure may be bent without losing its centre of gravity. *De Vinci*, one of the first painters after the revival of the art, has laid down rules upon this subject, which have been improved by succeeding painters.

LECTURE XXIX.

ON THE MOTION OF PROJECTILES, PENDULUMS, &c.

*On the Motion of Projectiles.**

IF a heavy body be projected directly upwards or downwards, it will be moved in a right line, but its motion will be, as we have shown you, a uniformly accelerated or retarded motion, according as it is projected upwards or downwards; but if it be projected in a horizontal direction, or in any other direction that is oblique to the horizon, it will be carried in a curve line.

Galileo first investigated the effects of gravity on falling bodies, and upon that foundation demonstrated, that all projectiles would move in a *parabola* in a *non-resisting medium;* and not taking the resistance of the air into the account, he proved, that a ball shot horizontally would, in its flight, describe half a parabola; and when the piece had an elevation above the horizon, the ball would describe a whole parabola, supposing it to fall on the plane of the battery; and by the same method of reasoning he showed, that whatever the ranges of the projected body or the elevations of the piece were, the ball would still trace that curve line, of a greater or a lesser amplitude, by the time it descended to the level of the place from whence it came.

Galileo confined his projections to the horizontal plane of the battery; but *Toricelli*, his disciple, soon after carried the theory farther, by tracing the shot to its fall, whether that place were above or below the plane; and still found by geometrical deductions, that it flew in a parabola of a larger or smaller amplitude, according to the angle of elevation of the piece, and the strength of the powder.

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* Sir *John Pringle's* Discourses.

Galileo suspected, that the resistance of the air would divert the projectile from its track, and proposed some means for ascertaining the inequalities arising from thence; yet those who came after him, *Newton* and *Huygens* excepted, all asserted, that the resistance of the air was so small, that bodies moving through it would not sensibly deviate from the path they would describe in vacuo, the velocity and direction of projection being the same.

Dr. *Halley*, and other eminent mathematicians, acquiesced in the justness and sufficiency of the principles of gunnery invented by *Galileo*, and enlarged by *Toricelli*; nay, so far were these theorists from suspecting any defect or fallacy in these principles, that they seemed rather to reproach the practical artillerists, for not profiting more by the instructions they had so liberally imparted to them.

Yet, that either the theory of projectiles was defective, or the resistance of the air was such as rendered it inapplicable to practice, was evident from numerous experiments. It has been found by experiment, that a musket-ball, three-fourths of an inch in diameter, fired with half its weight of powder from a piece forty-five inches long, moves with a velocity of near 1700 feet in a second; now if, according to theory, this ball flew in the curve of a parabola, its horizontal range at 45° would be found to be about 17 miles; but all practical writers assure us, this range is really short of half a mile. One more instance: an iron bullet of 24 pounds weight, fired from a piece of the common dimensions, with its greatest allotment of powder, hath a velocity of 1650 feet in a second; now the horizontal range of this shot at 45°, computed according to the parabolic hypothesis, will come out to be about sixteen miles, which is five or six times its real quantity; for practical writers all agree in making it less than three miles.

It is not only when projectiles move with these very great velocities, than their flight sensibly varies from the curve of a parabola; the same aberration often takes place in those that move slow. In short, says M. *de Ressons*, a French officer of artillery, distinguished by the number of sieges at which he had served, by his high military rank, and by his abilities in his profession, "Although it be agreed, that theory joined to practice does constitute the perfection of every art, experience had taught him, that theory was of every little service in the use of mortars; that practice had convinced him, that there was no theory in the effects of gunpowder, for that having endeavoured, with the utmost care he was master of, to point a mortar agreeable to theory, yet he had never been able to establish any solid foundation thereon."

Sir *Isaac Newton*, in his Principia, has indeed particularly considered the resistance of the air to projectiles moving with small velocities; but as he had no opportunity of making experiments on those which move with prodigious swiftness, he did not imagine, that a difference in velocity would make such a resistance as it is now known to do. Sir *Isaac* found, that in small velocities the resistance was increased in the duplicate proportion of the swiftness with which the body moved; that is, a body moving with twice the velocity of another of equal magnitude, would meet with four times as much resistance as the first; with thrice the velocity, it would meet with nine times the resistance, &c. But even this theory is found to be erroneous, with regard to military projectiles.

After the theory of projectiles, with respect to gunnery, had exercised the powers of the ablest mathematicians for near two hundred years, and for almost fourscore years of that time had rested on fundamentals which had never been contested; yet you find M. *de Ressons*, one of the most competent judges, pronouncing it to be entirely useless.

Nothing further was done till this subject was taken up by Mr. *Robins*, a man deeply versed in geometry and the doctrine of numbers: but he knew also the limits as well as the powers of both, and how insufficient they were for establishing any theory where matter was concerned, without preparing the way by finding, by varied experiments and attentive observation, the physical properties of that matter. Those who had heretofore treated of the foundation of gunnery, by being too forward in the application of their mathematics, had in some measure hurt the credit of that admirable science.

Mr. *Robins* perceived the error of his predecessors in this inquiry, and corrected it. Persuaded of the great resistance of the air to bodies moving in it, and also of the uncertainty of the force of gunpowder, and of the variations in the flight of shot, occasioned by unavoidable varieties in the make of the shot, and in the make of the pieces of artillery which discharged it; apprized of so many causes of aberration, he justly concluded, that the affair here was at least as much an affair of physics as of geometry; and that though the art of throwing bombs had not been advanced by theory, it was not because the art admitted of none, but because the theory, which had hitherto been devised, was both defective and erroneous.

To free this subject from its difficulties, and to account for its irregularities, Mr. *Robins* was obliged to institute a set of experiments, to discover the initial velocity of the shot in given circumstances. To this end he endeavoured to ascertain the force of gunpowder, that he might thence estimate the velocity of the shot expelled by its explosion. This being done, he proceeded to measure the quickness of a musket-bullet, shot out of a given barrel with a given quantity of powder; and as the great velocities communicated to bullets by gunpowder rendered it impossible to make any direct observation on them, he was obliged to use an indirect method.

For this purpose he contrived a machine, by which the velocity of the bullet might be diminished in any given ratio, by being made to strike on a large body of a weight justly proportioned to it; whereby the swiftest motions, which otherwise would escape our examination, were to be exactly determined by these slower motions, that had a given relation to them.

The machine was simple, and the idea new. It consisted of a large wooden pendulum, which swung freely; but in so slow a manner, that its vibrations could be easily counted, with whatever celerity the bullet acted.

Assisted by this, he inquired into the resistance made by the air to projectiles of rapid motion, and thereby discovered, that the curve described by any shot was very different from a parabola; being neither a parabola, nor approaching to it, except when the bullets are projected with very small velocities; and, consequently, that all application of the properties of that conic section to gunnery was so erroneous, as to be totally useless; thus giving an instructive instance of the fallacy of the most specious theories, that do not proceed hand in hand with experiment.

Since Mr. *Robins's* time, many important and accurate experiments have been made on this subject by Dr. *Hutton*, Mr. *Thompson*, and *Lovel Edgworth*, Esq. but I should exceed the bounds of these lectures, were I to enter more minutely into their experiments, and must therefore refer you to those authors, who have treated particularly on these subjects. After what you have heard of the imperfection of this part of science, it would be pedantry to dwell on the mathematical theory of projectiles, and thus endeavour to make you appear wise in your own eyes, or that of others, by a parade of demonstration, and a useless display of diagrams.

I shall therefore content myself with laying before you one or two of the principal propositions that are furnished by theory.

1. *If the force of gravity were constant, and acted in parallel lines, and* there were no resistance from the air, *bodies projected near the earth's surface would describe a parabola.*

But the resistance of the air is so great, that it will cause the body projected to describe a path altogether different from a parabola; so that no practical conclusions can be drawn from this theory. This you may easily imagine, when you consider that the resistance of the air to a cannon-ball of twenty-four pounds weight, moving with its initial velocity from a full charge of powder, is between five and six hundred pounds; which resistance is above twenty times the ball's weight.

2. *If the velocity of projection be given, the horizontal range is the greatest, when the angle of elevation is* 45°.

This position also is only true on a supposition that the resistance of the air has no effect; whereas it so influences military projectiles, that the greatest range is at an angle much less than 45°.

In the art of gunnery, aberrations will take place from a variety of causes, which can by no means be foreseen or prevented. A difference in the density of the atmosphere, in the dampness of the powder, or in the figure of the shot, will occasion variations in the range of a bullet that cannot be reduced to any rules, and render the event of each shot very precarious. The resistance of the atmosphere, simply considered, is a problem that, notwithstanding the labour of Mr. *Robins*, &c. has not been completely solved.

It is an objection that has been made to the mathematical philosophy, and to which in many cases it is most certainly liable, that it considers the resistance of matter, more than its capacity of giving motion to other matter. Hence, if in any case matter acts both as a resisting and a moving power, and the mathematician overlooks its effort towards motion, founding his demonstration only upon its property of resisting, these demonstrations will be certainly false, though they should be supported by all the powers of geometry.

FURTHER OBSERVATIONS ON PROJECTILES.

The air presses as much upon the back-part of the ball, as it resists on the fore-part; and, of consequence, a ball moving through the air with any degree of velocity, ought to be as much accelerated by the action of the air behind, as it is retarded by the action of that before. Here, then, it is natural to ask, If the air accelerates a moving body as much as it retards it, how comes it to make any resistance at all? yet certain it is that this fluid doth resist, and that very considerably. To this it may be answered, that the air is always kept in some certain state or constitution by another power which rules all its motions, and it is this power, undoubtedly, which gives the resistance. It is not to our purpose, at present, to inquire what that power is; but we see that the air is often in very different states: one day, for instance, its parts are violently agitated by a storm, and another, perhaps, they are comparatively at rest in a calm. In the first case, nobody hesitates to own, that the storm is occasioned by some cause or other, which violently resists any other power that would prevent the agitation of the air. In a calm, the case is the same; for it would require the same exertion of power to excite a tempest in a calm day, as to allay a tempest in a stormy one. Now, it is evident, that all projectiles, by their motion, agitate the atmosphere in an unnatural manner; and consequently are resisted by that power, whatever it is, which tends to restore the equilibrium, or bring back the atmosphere to its former state.

If no other body, besides that above-mentioned, acts upon projectiles, it is probable, that all resistance to their motion would be in the duplicate proportion of their velocities; and accordingly, as long as the velocity is small, we find it generally is so; but when the velocity comes to be exceedingly great, other sources of resistance arise. One of these is a subtraction of part of the moving power; which, though not properly a resistance, or opposing another power to it, is an equivalent thereto. This subtraction arises from the following cause: the air, as we have already observed, presses upon the hinder part of the moving body by its gravity, as much as it resists the fore-part of it by the same property; nevertheless, the velocity with which the air presses upon any body by means of its gravity is limited; and it is possible that a body may change its place with so great velocity, that the air hath not time to rush in upon the back-part of it, in order to assist its progressive motion. When this happens to be the case, there is, in the first place, a deficiency of the moving power equivalent to fifteen pounds on every square inch of surface, at the same time that there is a positive resistance of as much more on the fore-part, owing to the gravity of the atmosphere, which must be overcome before the body can move forwards.

This deficiency of moving power, and increase of resistance, do not only take place when the body moves with a very great degree of velocity, but in all motions whatever. It is not in these cases perceptible, because the velocity with which the body moves frequently bears but a very small proportion to the velocity with which the air presses in behind it. Thus, supposing the velocity with which the air rushes into a vacuum to be 1200 feet in a second, if a body move with a velocity of 30, 40, or 50 feet in a second, the force with which the air presses on the back-part is but $\frac{1}{25}$ at the utmost less than that which resists on the fore-part of it, which will not be perceptible; but if, as in the case of bullets, the velocity of the projectile have a considerable proportion to the velocity wherewith the air rushes in behind it, then a very perceptible, and otherwise unaccountable resistance is observed, as we have seen in the experiments already related by Mr. *Robins*. Thus, if the air press in with a velocity of 1200 feet in a second, and the body change its place with a velocity of 600 feet in the same time, there is a resistance of 15 pounds on the fore-part, and a pressure of only $7\frac{1}{2}$ on the back-part. The resistance, therefore, not only overcomes the moving power of the air by $7\frac{1}{2}$ pounds, but there is a deficiency of other $7\frac{1}{2}$ pounds, owing to the want of half the pressure of the atmosphere on the back-part, and thus the whole loss of the moving power, is equivalent to 15 pounds; and hence the exceeding great increase of resistance observed by Mr. *Robins*, beyond what it ought to be, according to the common computations. The velocity with which the air rushes into a vacuum is, therefore, a desideratum in gunnery. Mr. *Robins* supposes that it is the same with the velocity of sound; and that when a bullet moves with a velocity greater than that of 1200 feet in a second, it leaves a perfect vacuum behind it. Hence he accounts for the great increase of resistance to bullets coming with such velocities; but as he doth not take notice of the loss of the air's moving power, the anomalies of all lesser velocities are inexplicable on his principles. Nay, he even tells us, that Sir *Isaac Newton's* rule for computing distances may be applied in all velocities less than 1100 or 1200 feet in a second, though this is expressly contradicted by his own experiments.

Though, for these reasons, it is evident how great difficulties must occur in attempting to calculate the resistance of the air to military projectiles, we have not even yet discovered all the sources of resistance to these bodies, when moving with immense velocities. Another power by which they are opposed, and which at last becomes greater than any of those hitherto mentioned, is the air's elasticity. This, however, will not begin to show itself in the way of resistance, till the velocity of the moving body becomes considerably greater than that by which the air presses into a vacuum. Having, therefore, first ascertained this velocity, which we shall suppose to be 1200 feet in a second, it is plain, that if a body move with a velocity of 1800 feet in a second, it must compress the air before it; because the fluid hath neither time to expand itself, in order to fill the vacuum left behind the moving body, nor to rush in by its gravity. This compression it will resist by its elastic power, which thus becomes a new source of resistance, increasing, without any limit, in proportion to the velocity of the moving body. If, now, we suppose the moving body to set out with a velocity of 2400 feet in a second, it is plain that there is not only a vacuum left behind the body, but the air before it is compressed into half its natural space. The loss of motion, therefore, in the projectile is now very considerable. It first loses fifteen pounds on every square inch of surface, on account of the deficiency of the moving power of the air behind it; then it loses fifteen pounds more, on account of the resistance of the air before it; again, it loses fifteen pounds, on account of the elasticity of the compressed air; and, lastly, another fifteen pounds on account of the vacuum behind, which takes off the weight of the atmosphere, that would have been equivalent to one-half of the elasticity of the air before it. The whole resistance, therefore, upon every square inch of surface moving with this velocity is sixty pounds, besides that which arises from the power tending to preserve the general state of the atmosphere, and which increases in the duplicate proportion of the velocity, as already mentioned. If the body be supposed to move with a velocity of 4800 feet in a second, the resistance from the air's elasticity will then be quadrupled, or amount to sixty pounds on the square inch of surface; which, added to the other causes, produces a resistance of 105 pounds upon the square inch; and thus would the resistance from the elasticity of the air go on continually increasing, till at last the motion of the projectile would be as effectually stopped, as if it were fired against a wall. This obstacle, therefore, we are to consider as really insuperable by any art whatever, and therefore it is not adviseable to use larger charges of powder than what will project the shot with a velocity of 1200 feet in a second. In this velocity the elasticity of the air will not make great resistance, if indeed it do make any at all; for, though Mr. *Robins* hath conjectured, that air rushes into a vacuum with the velocity of sound, or between 1100 and 1200 feet in a second, yet we have no decisive proof of the truth of this supposition. At this velocity, indeed, according to Mr. *Robins*, a very sudden increase of resistance takes place; but this is denied by Mr. *Glenie*, who supposes that the resistance proceeds gradually: and indeed it seems to be pretty obvious, that the resistance cannot very suddenly increase, if the velocity be only increased in a small degree. Yet it is certain, that the swiftest motions with which cannon-balls can be projected are very soon reduced to this standard; for Mr. *Robins* acquaints us, that "a twenty-four pound shot, when discharged with a velocity of 2000 feet in a second, will be reduced to that of 1200 feet in a second, in a flight of little more than 500 yards."*

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* It is proper to notice here another particular circumstance attending the motion of bodies projected with considerable force. It has been observed, that bullets, in their flight, are not only depressed beneath their original direction by the action of gravity, but are also frequently driven to the right or left of that direction, by the action of some other force. If the action of gravity were the true cause of the variable direction of bullets, their

CONCERNING PENDULUMS.

A *pendulum* is a heavy body hanging to a small cord or wire, which is moveable upon a centre. It is that well-known instrument so useful in measuring time, and ascertaining with accuracy its nicer divisions.

A body thus suspended being put in motion, describes an arc; in one half of which it descends, and ascends in the other.

For instance, here is a pendulum, that is, a heavy body, P, *plate* 2, *fig.* 11, hanging by a small thread, and moveable therewith about this point, B, to which the thread is fixed. If, when the thread is stretched, the weight be raised as high as C, and thence let fall, it will by it own gravity descend through the arc C P, as low as possible.

If it were entirely free, and not retained by the thread, it would fall in a vertical line, P L, from the point where I held it; but being retained by the string, it can only partially obey the action of gravity, and is constrained to describe an arc, C P.

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horizontal direction from the mark aimed at should increase in the proportion of the distance of the mark from the piece simply; but experiments prove the contrary: the piece which will carry its bullet within an inch of the intended mark at 10 yards distance, cannot be depended upon to 10 inches in 100 yards, and much less to 30 in 300 yards. The experiments by Mr. *Robins* and Dr. *Hutton* have undoubtedly proved this.

Besides the resistance of the air, as one cause given for this irregular motion, there is another more probable one given by Dr. *Hutton*, which is a whirling motion acquired by the bullet about an axis, by its friction against the side of the piece; for by this rotatory motion, combined with the progressive one, each part of the ball's surface will strike the air in a very different direction from what it would do without such whirling, and the obliquity of the action of the air arising from this cause will be greater, according as the rotatory motion of the bullet is greater in proportion to its progressive motion. M. *Euler*, on the contrary, attributes this deflection of the ball to its figure, and very little to its rotation; for, if the ball were perfectly round, though its centre of gravity did not coincide with that of its magnitude, the deflexion of the axis from the cylinder, or line of direction sidewise, would be very inconsiderable. But when it is not round, it will generally go to the right or left of that direction; and so much the more as its range is greater. And he infers, that iron cannon-shot, which are rounder and less susceptible of change of figure on passing along the cylinder than those of lead, are more certain than musket-shot. For further particulars, as well as a mathematical investigation of the laws of projectile motion, see Dr. *Hutton's* Mathematical Dictionary, vol. ii. p. 288.

E. EDIT.

The body in falling from C to P through the arc C P, acquires as much velocity (motion) as it would have acquired in falling perpendicularly from E to P, consequently enough to carry P to the same height, through the same curve, in equal time, supposing the direction to be changed without affecting the velocity.

The thread is the cause which changes the direction impressed on the body, without affecting the velocity; not considering, for the present, the effect of friction at the point of suspension.

The body, when arrived at P, the lowest point of the curve, cannot descend to the earth, because it is withheld by the string, but it retains all the velocity it has acquired in falling from C to P; so that if at this instant gravity ceased to act on this body, and that it was no longer withheld by the string, it would go on, by the first law of motion, in a line P D a tangent to the circle C P, in which the body moves: but the thread opposing an invincible obstacle to its gravity, and the force which it has acquired, to go on in the tangent P D, it only endeavours to fly off in this tangent; the thread drawing it back, and the impressed force pushing it on, it attempts to move in the direction of another tangent, which it is again withheld from pursuing by the thread; and by thus causing it continually to change its direction, it describes the arc P R, equal to C P.

When the body has arrived at R, all the force it had acquired is expended, and it falls again by its gravity from R to P; from whence it again ascends to P, and then descends again. And thus it will continue its motion backwards and forwards along the curve, which motion is called an *oscillatory* or *vibratory* motion; and each swing from C to R, as also from R to C, is called a *vibration* or *oscillation*.

The body being retained by the thread in the circumference of the circle C P M, of which the thread is the radius, the arc, C P R, that it describes, will be the arc of a circle.

You see clearly, that the thread, by which the body is suspended, is an obstacle that opposes in part its descent towards the earth, and changes the direction in which it moves; but, at the same time, you see that it is gravity which is the cause of its motion or vibrations.

A horizontal line, perpendicular to the plane of the oscillations, and passing through the point B, about which the pendulum oscillates, is called the *axis* of oscillation; the point, B, to which the thread is fixed, is called the *point of suspension*.

In considering pendulums, it is customary to suppose the weight of the body to be concentrated in one point.

Pendulums may be *simple* or *compound*.

Simple pendulums are those to which only one weight is suspended: *compound pendulums* are those to which several weights are fixed; these are placed at different distances from the point of suspension.

If the pendulum suffered no retardation in its motion from the resistance of the air, nor from the friction of the thread against the centre about which it moves, the arcs described in each vibration would be exactly equal, and the motion of the pendulum would continue forever.

But as the motion of the pendulum is continually retarded by these causes, the arcs described in each vibration must grow less and less continually, and at last vanish together with the motion of the pendulum.

The vibrations of one and the same pendulum, vibrating in unequal circular arcs, are performed very nearly in equal times, provided the arcs are but small.

Thus in the pendulum A B, *plate* 2, *fig.* 20, the vibration through the arc, C A D, is performed very nearly in the same time wherein the pendulum vibrates through the arc E A F, on the supposition that the arcs, C A and E A are very small.

Now, as the arcs are very small, they will not differ much, either in length or declivity, from their respective chords; consequently, the times of describing the arcs, by a heavy body running along them, will be nearly equal to the times of describing the chords: but I have already shown you, that the times of describing the chords are equal; wherefore, the times of describing the arcs, C A, E A, must be nearly equal; and so likewise must be the double of those times, or the times wherein the pendulum vibrates through the unequal arc, C A D and E A F.

To render this clearer, let us recur to a diagram, to show that the velocities of the bodies which oscillate in different arcs, C B, D B, *plate* 2, *fig.* 12, are, when they are arrived at the point, B, as the chords of the arc they have described. Draw the horizontal lines, C F, D E. Now I have shown you, that the velocity bodies acquired in falling through the arcs C B, D B, are the same as they would have acquired in falling perpendicularly from F to B, and from E to B. Now, the velocity acquired in falling from F to B, is to the velocity acquired in falling from G to B, in a subduplicate ratio of F B to G B, or as the line C B to G B. In the same manner, the velocity acquired by a body in falling from E to B, is to that it would have acquired in falling from G to B, in a subduplicate ratio of E B to G B, or as D B to G B; and, consequently, the velocity acquired in falling from F to B, is to that acquired in falling from E to B, as the chord C B is to the chord D B; that is, the velocities acquired in falling through the arcs, are as their respective chords.

From hence it is clear, that if in the circle, G B, you take the arcs, B 1, B 2, B 3, of which the chords are respectively as 1, 2, 3, &c. the velocities of a pendulum descending successively through the arcs B 1, B 2, B 3, &c. will be as 1, 2, 3, &c. that is, as the respective chords.

This you may confirm by experiment. Here are two pendulums of an equal length; I will let them go at the same instant of time, but in such manner that they may vibrate through small but unequal arcs. They will for a long time keep pace together, and continue to begin and end their swings, without any sensible difference as to point of time, during a great number of vibrations.

It is the characteristic of real genius to distinguish facts one from another, and to draw important consequences from what would be of no value to others. Thus *Galileo*, considering the oscillations of a lamp suspended from a roof; a circumstance that had often been attended to before, but to little purpose; perceived that its vibrations were apparently equal, though their extent continually diminished, till the motion of the lamp entirely ceased: this phenomenon, which is called the *isochronism of pendulums*, appeared to him important and useful. He perceived also, that every thing else remaining the same, the vibrations were slower, in proportion as the lamp was farther from the point of suspension; and saw that, by these observations, he was furnished with an instrument proper to measure what we call time; an instrument which always gave equal intervals; and that he could increase or diminish these intervals at pleasure, by making the pendulum longer or shorter; and thus obtain an accurate chronometer or instrument for measuring time.

You have seen it proved, by the experiments with Mr. *Atwood's* machine, that a falling body describes spaces which are as the squares of the times employed in falling, or the velocity acquired at the end of each of these times.

Now, in the vibrations of a pendulum, the spaces described are arcs of a circle, whose radius is the length of the pendulum. Let there be two pendulums, A B, C D, *plate* 2, *fig*. 13, vibrating in similar arcs, E B F, G D H. The time of a vibration of the pendulum, A B, will be to the time of a vibration of the pendulum, C D, in a subduplicate ratio of the length, A B, to the length, C D. Now, as the arcs are to each other as the radii; and, from the foregoing principle, the time of a vibration in the arc E B, is to the time of vibration in the arc G D, in a subduplicate ratio of A B to C D, it follows, that these, or in other words, *the times of the vibrations of pendulums that describe similar arcs of circles, or which have equal angles of vibration, are as the square-roots of the lengths of the pendulums;* so that if one pendulum be four times as long as another, the shorter will vibrate in half the time, or perform two vibrations in the same time that the longer performs one. Again, if one pendulum be nine feet long, and another four, the square roots of which lengths are three and two, the short one will make three vibrations while the other is making two.

A disproportion in the lengths of two pendulums occasions a great difference, you see, in the times of their vibrations.

Of several pendulums vibrating in similar arcs, the vibrations of the longest are slower than those of the shorter ones; and, consequently, if a pendulum be required that shall vibrate seconds, it must have a determinate length, as the length of the pendulum fixes the time of its oscillation. This length has been ascertained to be 39.13 inches.

From this principle, you may find how long a branch is which hangs down from the roof of a church; and, consequently, by measuring from the ball of the branch to the floor, and adding this to the length of the branch, you may find how high the church is. Let us suppose the branch to vibrate once in three seconds; then, since the times of vibration are as the square-roots of the pendulums lengths, it follows, that the lengths of pendulums are as the squares of the times of vibration; and that the length of the branch is to the length of a pendulum which performs a vibration in one second, as the square of 3 to the square of 1, or as 9 to 1. Now, the length of a pendulum that vibrates seconds being 39.13 inches, multiplying this by 9, we obtain 352.17 inches for the length of the branch.

Pendulums which are of the same length vibrate in the same time, whatever be the proportion of their weights; or, in other words, the time of a pendulum's vibration is no-wise altered by varying the weight thereof. This follows from the property of gravity, which is always proportional to the quantity of matter; consequently, all bodies in the same circumstances are moved by the force of gravity with the same velocity. To confirm this by experiment, here are two unequal weights, so suspended by two threads, as to constitute two pendulums equal in length. Let them at the same instant of time fall from equal heights, they will keep pace together so as to perform their vibrations in equal times.

From the motion of a pendulum, it is clear, that in any one place *the quantity of gravitating matter in any body is proportional to its weight.* For, we find by experiment, that pendulums of equal lengths, whatever quantities of matter they contain, vibrate in the same time. They have equal velocities in the same time: the velocity and time being given, the quantity of matter is as the force of gravity.

What I have hitherto said on this subject, extends only to simple pendulums; that is, those to which only one weight is suspended, and where the thread by which it is suspended, is considered as without gravity or weight: for, when the rod by which the weight is suspended is of any considerable weight, the pendulum must be considered as compounded, since the weight of the rod has the same effect as a second weight fastened to the same thread. And I defined a compound pendulum, as one to which several weights were fixed, at invariable distances one from the other, as well as from the point of suspension.

Compound pendulums follow the same laws as those that are simple, only with some modifications.

OF THE CENTRE OF OSCILLATION.

To determine the times of the vibrations of a compound pendulum, we must consider a thing of which I have not yet spoken to you; that is, *the centre of oscillation.*

The centre of oscillation of a compound pendulum, is that point in which the efforts, or actions, of the weights which compose it may be considered as united, to cause the pendulum to vibrate in a certain time; or, in more technical terms, the centre of oscillation is that point of a pendulum, on each side of which the quantities of motion are equal, or in which all the gravity of the pendulum might be collected, without altering the times of its vibrations. Though the centre of oscillation is different from the centre of gravity, yet it is evident, from these definitions, that they have a necessary relation to each other.

The centre of oscillation of a simple pendulum, whose thread is considered without weight, is not in the centre of gravity, but a little below it; and is nearer or farther from the centre of gravity, according to a certain proportion between the radius of the ball forming the pendulum, and the length of the thread by which it is suspended. Mr. *Huygens* has shown how to find the centre of oscillation from the proportion between the radius of the weight or bob and the length of the pendulum.

The real length of a simple pendulum is not, therefore, the same as the length of the thread from the point of suspension to the ball attached to it, nor even to the centre of gravity thereof; but its length is to be estimated from the point of suspension to the centre of oscillation, which differs from the centre of gravity, except when the length of the thread exceeds to a certain degree the radius of the ball, when the difference becomes insensible.

When the line of the pendulum is possessed of weight, which becomes sensible with respect to that which is attached to it, the centre of oscillation is no longer where it was in the suspended ball, but in a point somewhat above it; and this point is farther removed in proportion as the weight of the rod is heavier when compared to the weight of the suspended ball.

In this case, the true length of the pendulum is the distance between the point of suspension and the centre of oscillation, and the vibrations of this pendulum will be quicker than if the rod were without weight, because the pendulum is shorter.

You have seen, that the longer the pendulum, or the farther the weight from the point of suspension, the slower are its vibrations. Thus, if to an inflexible line, A C, *plate* 2, *fig.* 14, four feet long, from whose extremity a weight, P, is suspended, you add, at Q, a second weight, B, somewhat higher than the other, as three feet from the point of suspension; then the weight P, which is four feet from the point of suspension, ought to make its vibrations slower than the body B, which is only three feet therefrom: but as they are both attached to the same inflexible line, they are forced to perform their vibrations in the same time, which will be between the slowness with which it would have vibrated if there had been only the weight at A, and the greater velocity of its vibrations if there had been only the weight at Q. Thus the second weight quickens the vibrations of the first, and the first retards those of the second; and the centre of oscillation of this pendulum will be in that point in which if these two weights were united, the pendulum would perform its vibrations in the same time as that of the pendulum with the two weights.

To find therefore the centre of oscillation of a compound pendulum, is to find the length of a simple pendulum that would perform its vibrations in the same time as the compound; and the real length of the compound pendulum is to that of a simple isochronous pendulum as the pendulum, C R, is to the pendulum C Q A. Now, as the lengths of pendulums are as the squares of the times of their vibrations, it is easy to see that the simple pendulum C R, whose vibrations are isochronous to those of the compound pendulum C Q A, should be more than three feet and less than four feet long; and, consequently, a simple pendulum is always shorter than a compound pendulum, whose vibrations are performed in the same time, and the centre of the oscillation of the compound pendulum, C Q A, will be between the two weights P and Q, that is, somewhere about the point O.

It is easy to perceive, from what I have already explained to you, that in a compound pendulum, C Q A, consisting of two weights, the nearer one of these weights is to the point of suspension, or the farther the two weights are from each other, the more the centre of oscillation approaches the point of suspension; and, on the contrary, if the two weights be equally distant from the point of suspension, their centres of oscillation would become the same, and the compound pendulum might be considered as a simple one.

What has been said of a pendulum consisting of two weights, equally applies to one consisting of three, or more weights.

Before I proceed any further, it will be proper to observe to you, that that part of the preceding account which relates to the ascent of pendulous bodies, though generally received, is not deemed satisfactory by all. There are some able philosophers who consider it as imperfect;* they allow, that the body descends to the lowest

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[*] Jones's Physiological Disquisitions, p. 34.

point of the arc by the force of gravity, but do not agree so well with what happens when it gets beyond that point.

Does it, they inquire, fall by the cause of gravity, and rise again by no cause at all. If you say, it rises by the motion it has acquired in falling, they ask, what is this motion? It is nothing but an effect, and to say, that it *moves by motion*, is saying nothing at all; for motion is not a thing by itself, as philosophers seem very falsely to have considered it; it is not a quality which a body can get possession of and run away with it; it is a mere effect, and as such must be referred to some cause, or be given up as unintelligible.

It is said, that a pendulum cannot rise by the power of gravity, because this is inconsistent with the direction of gravity; but notwithstanding this apparent difficulty, it is not improbable, that the same power, which naturally carries a body downwards, will carry it any way according to the circumstances of the case. Is not the whole arc of a pendulum described out of the line of gravity, except the first and last points?

In falling to the lowest point of the arc, the body moves for a little time very nearly in the tangent of its curve, at right angles to the line of gravity; or, in other words, its gravity gives it motion in a horizontal direction. But if gravity can give thus much, why not all the rest? Why must it leave the body at one point of the tangent line, and not continue to act upon in the same line? If the cause and the effect can be kept together ever so little beyond the perpendicular line, the difficulty is over, and the body performs the whole course of its vibrations by one and the same cause; and the reciprocations we observe in the moving body, are occasioned by a medium, whose vibrations are continued with infinite freedom.

This is surely more satisfactory than that species of reasoning, which assigns a cause for one half of the motion, and a law, which cannot execute itself, to account for the other half; and that says, there is a certain mathematical point at which they are miraculously changed one for the other.

If the difficulty of making a body rise by gravity shock you, word the matter differently, and say it *rises by the cause of gravity.*

Motion is either a cause or an effect: it cannot be a cause for this reason, that nothing can be the cause of itself; and if you allow it to be an effect then all the consequences follow.

Sometimes our corporeal senses are able to discover the relation between the effect and its proper cause, and sometimes causes are to be inferred by rational induction grounding itself on past experience.

Thus when the mercurial fluid rises an inch higher to-day in the tube of a barometer than it stood yesterday, you neither see nor feel the cause of this alteration, but you discover the cause by very sure deduction; for we know that the Torricellian vacuum is filled by the column of mercury, till the mercury becomes a counterbalance to the pressure of the external air; and that the air is the impelling cause, which drives the mercury up into the tube, and keeps it suspended there. Hence we conclude, that an alteration in the pressure of the air has occasioned an alteration in the tube, and that the pressure of the atmosphere being about $\frac{1}{30}$th part greater to-day than it was yesterday, forces the mercury so much higher till it is in equilibrio therewith.

In the same manner most of the other motions that are in the world may be referred to their proper causes, either by immediate experiment or rational deduction.

It does not follow, that no material cause is concerned in any particular effect, because we neither see nor feel its operation.

There are many obvious cases, in which the cause of motion may be assigned with certainty, though it is not perceived by any of the outward senses; and partly for this reason, because that which is manifest in some instances is occult in others, and distinguishable only by rational deduction.

Thus, when a thermometer is held before the fire, we discover, by its rising, that the fire enters the pores both of the glass and of the inclosed fluid, and this being thereby expanded and increased in its dimensions, necessarily rises higher in the tube. Here, though we feel the element of fire by its heat, and see it by its light, yet its effects are only to be discovered by rational deduction.

Motion is never to be considered as a thing by itself, but as an effect, which, like all other effects, must be referred to its proper causes: and there is no occasion to understand motion as the cause of its own continuation, when there is an active medium adequate to all the effects of gravity, which, moving with infinite freedom, can so far deceive us by the subtilety of its vibrations, as to make us believe there is no cause where the most powerful of all secondary causes is present. If motion be an effect, this reasoning is necessary and natural.

Before I proceed to the application of pendulums to clocks, it may not be amiss to recapitulate their properties, as it is the only part of a clock which has a natural tendency to measure time.

A pendulum once put in motion would continue to vibrate in equal arcs, if no external cause tended to destroy its motion.

All those equal vibrations would be performed in equal times.

Pendulums of equal lengths, though of unequal weights, will, *cæteris paribus*, perform their vibrations in equal times.

A pendulum cannot rest but when its centre of gravity is directly below its point of suspension.

The times wherein pendulums of any length perform their oscillations, are as the square-roots of their lengths directly, and the square-roots of the gravitating forces reciprocally.

If an iron bar be suspended at one end and put into motion, the oscillations will be performed in less time than by a single pendulum of the same length: for the upper parts of the iron will endeavour to oscillate according to their respective distances from the centre of suspension, and so accelerate the motion.

A single pendulum, whose length is two-thirds of the bar's length, will be found to perform its oscillations in equal times with the bar. Hence a point taken one-third of the length of the bar from the lower end, is called the centre of oscillation.

Hence it is evident,

1. That the bar oscillates in the same manner as it would do if the gravity of all its parts were contracted into the centre of oscillation, and all its other parts remained without gravity.

2. Therefore, if any obstacle stop the centre of oscillation, the whole motion of the bar is destroyed at once, and consequently the whole force of its motion is spent upon, and received by the obstacle.

3. Therefore, the quantities of motion on each side the centre of oscillation are equal.

4. Therefore, if any other part of the bar strike against an obstacle, the motion will not be destroyed immediately, but the centre of oscillation will endeavour to go on, and, by that endeavour, either break or bend the bar.

From these particulars it is easy to explain the *centre of percussion*, or what part of any instrument or weapon will strike with the greatest force. For if the stroke be not given with that part which is the common centre of oscillation of the weapon, and so much of the arm as moves with it, whether it be from the wrist, elbow, or shoulder, the whole force of the motion will not be spent upon that which receives the blow.

All oscillating bodies, of what shape soever, have their proper centres of oscillation; which, if the rod of the pendulum be small, and the bob large, will fall within the bob.

By the length of a pendulum is always meant the distance between the centre of suspension and the centre of oscillation.

If a pendulum be heated, it grows longer, and consequently oscillates slower.

Pendulums of the same length oscillate slower the nearer they are brought to the equator, and that for two reasons; because the semidiameter of the earth's equator is about seventeen miles longer than the semi-axis of the earth; and because of the centrifugal force, arising from the diurnal rotation, which at the equator is the $\frac{1}{289}$th part of gravity, from whence it decreases continually, till at the poles it quite vanishes.

Much has been said of the advantages of making a pendulum vibrate in the arc of a cycloid, in which it would describe the greatest and least entire vibrations in the same time; but this property is only demonstrated on a supposition that the whole mass of the pendulum is concentrated in a point, but this cannot take place in any really vibrating body. And when the pendulum is of finite magnitude, there is no point given which determines the length of the pendulum; but, on the contrary, the centre of oscillation will not occupy the same place in the given body, when describing different parts of the track it moves through, but will continually be moved in respect of the pendulum itself during its vibration. This prevents the determination of the time of the vibration in a cycloid, except in the above-mentioned imaginary cases.

There are many other obstacles which concur in rendering the application of this curve to the vibration of pendulums, a source of errors far greater than those which its peculiar property is intended to obviate; and it is therefore now wholly disused in practice.

To find the length of a pendulum, that shall make any number of vibrations in a given time.

Reduce the given time into seconds, then say, as the square of the number of vibrations given is to the square of this number of seconds, so is 39.13 to the length of the pendulum sought in inches.

Example. Suppose it makes 50 vibrations in a minute; here a minute is = 60 seconds; then,

As 2500 (the square of 50 : 3600 the square of 60) : : 39.13 : the length $= \frac{3600 \times 39.13}{2500} = \frac{140868}{2500} = 56.34$ inches, the length required.

If it be required to find a pendulum, that shall vibrate such a number of times in a minute; you need only divide 140868, by the square of the number of vibrations given, and the quotient will be the length of the pendulum.

If the pendulum be a thread with a small ball at one end, then the distance between the point of suspension and the centre of the ball is esteemed the length of the pendulum. But if the ball be large, say, as the distance between the point of suspension, and the centre of the ball, is to the radius of the ball, so is the radius of the ball to a third proportional. Two-fifths of this set from the centre of the ball downward, will give the centre of oscillation. Then the whole distance from the point of suspension to this centre of oscillation, is the true length of the pendulum.

If the bob of the pendulum be not a whole sphere, but a thin segment of a sphere, as in most clocks; then to find the centre of oscillation, say, as the distance between the point of suspension and the middle of the bob, is to half the breadth of the bob, so is half the breadth of the bob to a third proportional. Then one third of this length set from the middle of the bob downwards, will give the centre of oscillation. And as before observed the distance between the centres of suspension and oscillation, is the exact length of the pendulum.

Having the length of a pendulum given, to find how many vibrations it will make in any given time.

Reduce the time given into seconds, and the pendulum's length into inches; then say, as the given length of the pendulum is to 39.13, so is the square of the time given to the square of the number of vibrations, whose square-root is the number sought.

Example. Suppose the length of the pendulum is 56.34 inches, to find how often it will vibrate in a minute.

1 Minute = 60 seconds. Then 56.34 (the length of the pendulum) : 39.13 : : 3600 (the square of 60) to the square of the number of vibrations $= \frac{3600 + 39.13}{56.34} = \frac{140868}{56.34} = 2500$, and $\sqrt{2500} = 50 =$ the number of vibrations sought.

If the time given be a minute, you need only divide 140868 by the length, and extract the square root of the quotient for the number of vibrations.

OF TIME.

As time in itself does not fall under the notice of our senses, and as the parts thereof go on in a continued succession one after the other, no two existing together, it is impossible to discover the equality or inequality of any two portions of time, by an immediate comparison of one with the other.

It is therefore necessary, in order to distinguish the parts of time, to have recourse to something sensible, and of a different nature from time, as a measure thereof.

In the first ages of the world, men observing the frequent risings and settings of the sun, took the one or the other for their first measure of time; calling that portion of time which passed between the two risings or settings, which immediately succeeded each other, by the name of a day. In like manner, it is rational to suppose, that observing the frequent returns of the new and full moons, they made the one or the other their second measure of time, calling that space which passed between two successive new or full moons, by the name of a moon or month. For some time they contented themselves with those measures, without knowing or considering whether they were accurate or inaccurate.

In proportion as science dawned upon the human mind, men became better acquainted with the motion of the heavenly bodies; they then discovered irregularities in the apparent motion of the sun, and, of consequence, an inequality in the natural days, which depend on that motion. By considering the causes of this inequality, they were led to make such alterations in the natural days, by adding to some and taking from others, as reduced them all to a mean equal length, each day being made to consist of 24 equal hours, each of which is subdivided into 60 equal parts, called minutes, and these into 60 other equal parts, called seconds, and so on in a sexagesimal progression; and these parts of time, thus reduced to an equality, constitute the mean or equal time, as it stands distinguished by astronomers from the unequal or apparent time, as measured by the apparent motion of the sun.

In order to have a constant measure of equal time, *Huygens*, a man who added the acutest penetration to the most indefatigable industry, contrived a method of adapting pendulums to clocks, whereby their motions are so exactly regulated, that in one, whose movements are rightly adjusted, the seconds, minutes, and hours, are for some time pointed out with the greatest exactness.

The principle of motion in a clock is derived from the power either of a weight or spring; either of these forces is sufficient to actuate or put in motion the system of wheels and pinions which compose the intermediate parts of the clock; the indexes, fixed on the axis, point out the proper sub-divisions of time on appropriate circles upon the face of the clock; and a pendulum or balance is added, to regulate or render uniform the motion communicated to the machine.

The force of the weight is derived from the power of gravity, and this being always the same in a given quantity of matter, the force of the weight may be considered as a constant quantity, or always remaining the same in the same medium, and is therefore a uniform power or principle of motion.

If the pendulum be put in motion by a push of the hand, it will continue to move backwards and forwards till the resistance of the air, and friction at the point of suspension, destroy the original impressed force. But, at every vibration of the pendulum, the teeth of the crown-wheel act upon the pallets, so that after one tooth has communicated motion to one pallet, it escapes, and the opposite tooth acts upon the opposite pallet, and escapes in the same manner; and thus each tooth escapes after having communicated its motion to the pallets in such manner, that the pendulum, instead of being stopped, continues to move.

Let F G E, *plate* 2, *fig.* 15, represent the swing-wheel of a clock, that is to revolve in the direction E G F; let C and D represent the pallets moveable on an axis at A, and so connected with the pendulum AB, as to be made to vibrate along with it. Suppose the ball to vibrate from R to B, one of the teeth of the wheel resting against the pallet C, whose figure is seen enlarged at I K L; when the pendulum returns towards Q, the pallet, C, is drawn out, the wheel pressing first along the plane I K, and afterwards on the inclined plane K L; the pressure of the wheel on K L pushes the pallet, or assists its motion, till at length the tooth slips off the point L. During this time the pallet D, whose figure is seen at P N O, is carried in between the teeth on the other side of the wheel, so that when the wheel escapes from the pallet C, another tooth drops on the plane, N P, of the pallet D; the returning vibration again draws out the pallet, the tooth of the wheel assisting its motion by pressing along the inclined plane, N O, till it escapes at O. At this instant the pallet, C, has acquired its original situation, and therefore receives the adjacent tooth, and the whole proceeds as before.*

The piece of mechanism I have just described, is termed the *dead-beat escapement*, because the second hand, in clocks of this construction, falls with a dead stroke without recoil on the division line of the dial, and remains motionless during that part of the vibration in which the tooth rests on the plane, I K or P N, of the pallet.

From hence it is easy for you to see, that it is, 1st. The weight that turns all the wheels, and at the same time continues the motion of the pendulum. 2d. That the quickness of the motion of the wheels is determined by that of the pendulum. 3d. That the wheels point out the parts of time divided by the uniform motion of the pendulum.

Considerable irregularities in clock-work arise from the tenacity of the oil applied to the moving pivots, for the oil is less fluid in cold than in hot weather; and any diminution in its fluidity tends to destroy the momentum which the pendulum had acquired, as well as to diminish the maintaining power by its endeavour to overcome the rigidity of the cooled oil; the action on the pallets will therefore be less forcible, and the vibration will be less. To avoid this defect, as far as relates to the suspension of the pendulum, it is usual to suspend it by a straight flexible spring, instead of an axis.

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* *Nicholson's* Introduction to Natural Philosophy, vol. i. p. 88.

In all that I have hitherto said, the length of the pendulum of the clock has been considered as invariably the same; but the contrary happens in practice, for heat dilates, and cold contracts all metals, by which means the same pendulum is longer in summer than in winter, and the clock consequently goes slower.*

Various expedients have been thought of for correcting this imperfection; all of these, except one, depend on opposing expansion to expansion, in such manner that one shall shorten the pendulum as much as the other lengthens it.

The only method that does not depend on the above principle, is by making the pendulum-rod of the straightest grained wood, because the longitudinal expansion is so small, that it answers with sufficient accuracy for ordinary purposes. Deal wood makes very good pendulum-rods; the wood called sapadillo† is said to be still better; baking varnishing, gilding, or soaking these woods in any melted matter, is said to render them less accurate, and that they should be simply rubbed on the outside with wax and a cloth.

Of the various contrivances to remedy this evil by contrary expansion, I shall mention only one, namely, the *gridiron-pendulum.*‡ The expansion of brass is allowed to exceed that of steel in the proportion of five to three; on which supposition, if any piece of steel by the application of a certain degree of heat, expands three-tenths of an inch, a piece of brass of the same length will expand five-tenths of an inch.

Now, as the whole length of the brass expands five-tenths of an inch, and the expansion through the whole is supposed uniform, if it be divided into five equal parts, each part will have expanded one-tenth of an inch; if, therefore, we take away two-fifths of its whole length, the expansion of the remaining three-fifths will be three-tenths of an inch; consequently, the expansion of a bar of steel is equal to that of a bar of brass three-fifths in length.

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* If the rate of the going of a clock be too fast, it is easily regulated by letting down a little the bob of the pendulum, by unscrewing a small screw usually placed under it for that purpose; and if the rate be too slow, by screwing it upwards. E. Edit.

† *Cumming's* Elements of Clock Works, p. 92.

‡ *Nicholson's* Introduction to Philosophy, vol. i. p. 91.

Let AB, *plate* 2, *fig.* 16, represent the length of any given pendulum, to which a gridiron is to be applied. Bisect AB in C, and make BD, DE, EF, each equal to CB, the half of AB; then AF will be to BF, or HG, as 5 to 3.

If, therefore, AF be steel, and HG be brass, their contractions and expansions, with any degree of heat and cold, will be equal; and if the brass bar, HG, be firmly attached to the steel bar, at their extremities F G, and the steel bar be suspended by the point A, the brass bar will expand upwards, as much as the steel bar downwards, and the points, A and H, will always keep the same distance from each other. Consequently,

If a pendulum be constructed, whose point of suspension is A, *and its centre of oscillation* H, *it can neither be lengthened nor shortened by any change of heat and cold.*

It appears, that whatever be the length of a pendulum, three half lengths must be of steel, and one half length of brass, in order to have as much expansion upwards as downwards; and when the whole length of the bars is thus ascertained, they may be cut into several lengths as may best suit the purpose.

Always observing, that the expansion of the steel tends downwards, and that of the brass upwards; in which case they will balance each other, without regard to the number of pieces into which they are cut; for the effect of the whole consists of the effects of all its parts.

If the steel bar, AF, be cut into three unequal lengths, and the brass bar, HG, into two, and those five pieces be applied and connected with each other, as in *plate* 2, *fig.* 17, the brass bars will expand upwards, as much as the steel downwards; and the centre of oscillation, B, always keep the same distance from A, the point of suspension; nor will the expansion of the little cross pieces that connect the bars be of any effect, as they only act laterally.

Though in theory five bars only are necessary for constructing a gridiron, nine are requisite in practice;* and in order that the bar, to which the pendulum is immediately suspended, may be equally supported on each side. From *plate* 2, *fig.* 19, you will see clearly, that the corresponding bars on each side the centre co-operate in such manner, as to move both ends of the cross-bars equally, and by that means prevent such bending of the upright ones, as must otherwise happen by the weight of the ball B. These additional, bars properly applied, do not increase the expansion.

Plate 2, *fig.* 18, represents the manner of connecting the bars in a gridiron-pendulum. The bars are represented as broken in the middle: the letters on the round bars denote the metal of which each is made; and that part of the middle rod which is above the cross-piece E, serves only to keep the three bars on each side thereof in the same plane with the two outside ones; it also answers a like purpose at the lower end of the gridiron, and is pinned only at the cross-piece E; moving freely, but without shake, in the pieces C, D, F, and G. The upright bars are generally connected towards the middle by one or more cross-pieces: to prevent any trembling that might arise from their elastic flexibility, these pieces are pinned to the two extreme bars, and allow the other seven to move freely.

In all this it is supposed, that the expansion of brass is to that of steel exactly as five to three, and that all the dimensions are accurately laid down. If the contrary appear, the gridiron may be adjusted by shifting the cross-piece, F, upwards or downwards, as occasion may require.

Before we proceed to a fresh subject, I shall beg leave to say a few words on the nature of time; as, unless you are able to form abstract ideas of time and space, you will never truly apprehend spiritual subjects; whereas just notions thereof will throw very important light on religious and metaphysical speculations. "If nobody ask me what time is," says St. *Augustine*, "I think I know; but if any body ask me, I cannot tell:" and indeed it is not easy to give a clear idea thereof. Are days, and months, and years, descriptive of time? Days, and months, and years, are measured by the laws of motion; but the measure is different from, and only relative to time. Time would be, were days, years, and months, to be confounded. Lord *Monboddo* defines time as the measure of the duration of things that existed in succession, by the motion of the celestial bodies.

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* On the supposition that steel and brass are the metals used.

All the subjects to which time is applicable, are said to be in time; and are all, some way or other, affected by time. But it is evident, that beings which suffer no change in substance, qualities, or energies, cannot, without some restriction, be said to be in time. Of this kind we conceive the Divine Being to be.

For time cannot be applied to any being of staple and permanent duration, without change or variation of any kind; but only to beings that are in a constant flux, and always changing, either as to their substance, or their qualities and energies. The common distinction, therefore, between eternity and time appears to be well founded; the former only applying to a being without change or variation of any kind; the latter, to beings which exist only by succession. Of the first kind we believe the Deity to be, in whom is eternity, and who is without change, or shadow of change. Eternity infers immutability, nor was ever separated from it in the minds of men; for all who believe a God, believe not only that he always was, but likely that he continues without variation the same, yesterday, to-day, and for ever.

Without body moving by a motion revolving into itself, we cannot conceive any standard or measure of time. Hence, if the visible world had a beginning, it is easy to conceive a period when there was no time; and again, if this visible world should be totally destroyed, a period will be when there will be no time: for in both these cases, there is neither a subject to which the duration of time can be applied, nor is there any standard or measure for it. It is only, therefore, in the material world that there can be such a thing as time. Hence St. *John* speaks of this annihilation of time: "And the angel which I saw stand upon the sea and the earth, lifted up his hand toward heaven, and swore by Him that liveth for ever and ever, &c. &c. that there should be time no longer."

The efflux and succession of time is owing to the power and will of God, and may, therefore, only take place among certain orders of his creatures. Mr. *Locke* says, we obtain our notion of time by observing a succession of ideas. We may readily allow him, that this way we come by the measures of time which we use in computation: but succession implies a previous idea of first and last, before it can be attained; for, a variety of ideas affords us no notion of succession, unless we perceive that one came before the other. Nor will their degrees of vividness or faintness answer the purpose: for, suppose a man to stand with a candle in his hand between two glasses, he will see a number of flames in the glass before him, each fainter than the others; yet the whole scene will appear quiescent, nor exhibit any idea of succession: and the ideas of things in our remembrance, though fainter, as more remote, would do the like, unless we had another idea of precedence annexed to them: so that our idea of precedence seems to be original; not derived from any other, but attained by our manner of existence, extending to the length of time wherein there is a first and a last.

Mr. *Locke* equally erred, when he attributed the length of duration to the quick succession of ideas, and its shortness to a slow succession; for, the length or shortness depends on the state of the mind, not on the quick or slow succession of ideas.

Time, it is true, appears long when a man is impatient under any pain or distress, or when he is eager in expectation of happiness. On the other hand, when he is pleased and happy in agreeable conversation, or delighted with a variety of agreeable objects, time flies away and appears short.

Now, according to Mr. *Locke's* notions, in the first of these cases the succession of ideas is very quick, and in the last, very slow; the contrary of which appears to be true: for, when a man is racked with pain or with expectation, he can hardly think of any thing but his distress; and the more his mind is occupied by that sole object, the longer time appears. On the other hand, when he is entertained with cheerful music, with lively conversation, and brisk sallies of wit, where the succession of ideas is quickest, the time appears shortest.

Dr. *Clarke*, by confounding the ideas of successive and unsuccessive duration, advanced a strange paradox, which he thus expresses: "How any thing can have existed eternally, that is, how an eternal duration can be actually past, is utterly unconceivable; and yet, to deny that an eternal duration is now actually past, is an express contradiction." Now, he should have said quite the contrary; namely, that to maintain that an eternal duration is actually past, is altogether repugnant and contradictory. It is indeed not only unconceivable, but absolutely impossible. Wherever there is a last, there must be a first: now, in eternity there can be no first, and therefore no last. Whatever is successive cannot be eternal: no additions of finite successions can make absolute infinite.

This subject cannot be considered too closely, if you wish to avoid the errors of modern materialists; for, you will find one of them declaring, that he can easier admit the non-existence of God himself, than the non-existence of space or duration: thus conceiving, as Mr. *Hindmarsh* has justly inferred, time and space, which in themselves are unsubstantial, inanimate, and destitute of intelligence, to be more necessary and independent in their existence, than him whom we call God.

I will conclude this part of the present lecture with a short extract from that part of Mr. *Tucker's* Light of Nature, which he terms, The Vision. "And there stood before him an angel; his countenance was mild and lively, and his raiment white and shining. He had spangled wings growing from his shoulders, his sides, and his legs. And he said, 'O man, come along with me: I will teach thee what to think of glories thou canst not comprehend, and make thee more sensible of the infinite distance between the creature and the Creator.' So saying, he took me up, and carried me to the utmost bounds of the universe. And he said, 'Try now if thou canst create a new world beyond this.' And I said, 'Far be it from me to attempt incroaching upon the divine prerogative. Thou knowest I am poor and weak, unable to act without corporeal instruments; and the little power I have, is given me.' And he said, 'I know thy weakness; that the power is not thine; nor do I expect that thou shouldest operate. I am commissioned for thine instruction. Know, then, that creative power awaits thy direction, for a trial of what thou canst perform by it.' 'Alas!' said I, I know not where to begin, nor how to proceed!'

And he said, 'Stretch forth thy right-arm, and thrust it straight from thy side.' And I essayed, but could not thrust it out. Not that I found any thing to resist me; but when my arm came to the utmost verge of the universe, it seemed as if I had lost the use of it, so that I could not move it further. And I asked the angel, 'Wherefore cannot I move my arm this way? I can trust it above or below, before or behind me; but I cannot stretch it out from my side.' And he said, 'Because there is no space to receive it.' And I said, 'Since it hath so pleased the creative power, and this for mine instruction, may there be space.' And he said, 'Put forth thine arm again.' And I put it forth with ease straight from my side, as I could have done any other way.

And he said, 'Wave now thy fingers to and fro; moving them in order, one after another.' Accordingly, I endeavoured to do as I was bidden, but could not perceive whether my fingers moved or not; for I had the same feel as at first, when I stretched out my arm. Wherefore I asked, 'Why cannot I perceive whether my fingers move, or not?' And he answered, 'Because there is no time; neither without time can there be a succession of ideas or motions.' Then I said, 'May time begin her course:' and presently I felt my fingers move to and fro, in the manner I had intended to move them.

And the angel said, 'Now will space continue, and time run on her course, for ever; until the same power which gave them birth shall interpose to destroy them.

LECTURE XXX.

ON THE COMMUNICATION OF MOTION BY COLLISION.

NATURAL Philosophy is little else than the measuring of such motions as are obvious, or accounting for such as proceed from a hidden cause. I have already explained many of the one kind, and described the measures of the other. I have now to consider the laws of communication of motion by collision. Little more can be expected than the statement of these laws; for philosophy is still unable to explain how one body becomes possessed of a power of communicating its motion to another.

The laws of motion I am here going to explain, arise from a principle laid down in the beginning of these Mechanical Lectures; namely, that action and re-action are equal, and in contrary directions.

If a body impinge against another body, moving in the direction of a line which joins their centres of gravity, the two bodies mutually act on each other, in any given instant of their collision, with equal forces; which equal forces will always have the effect of retarding the striking body, and accelerating the body struck, if it move in the same direction with the striking body; and of retarding it, if it move in the contrary direction

This mutual action and re-action of bodies may be illustrated by supposing a spring in the form of a helix, the axis of which coincides with the direction of the motion, to be interposed between the two bodies; then in whatever degree the spring is compressed, it will exert the same elastic force on each of the bodies, the inertia of the spring not being considered. When the spring first begins to be compressed, the force by which it retards the striking body, and accelerates the body struck, is the least of all; it afterwards will continually increase during each successive instant, till its compression be the greatest possible. The intensity of this force may vary according to any imaginable law; but whatever may be the variation, it will act equally on both sides.

When the two bodies have acquired a common velocity, the spring can acquire no further compression, and may then begin to restore itself with various degrees of force, from 0 to the force with which it was compressed. When the restitutive force is equal to that of compression, the action of the spring will be similar to that of perfectly elastic bodies.

Motion cannot be communicated to any body instantaneously, but must be produced by gradual acceleration; it not being conceivable that any really existing body should pass from quiescence into finite motion, or from one degree of finite motion to another, without having passed through all the intermediate degrees of velocity.

Suppose a spherical body to impinge on another body of the same form, moving in the same direction with the line which joins their centres, and let both of them be perfectly non-elastic.

When the surfaces of these spheres are just in contact, the distance of their centres will just be half the sum of their diameters; but as the figures are gradually changed by the impact, their centres will become nearer than before. During the time of their approach, the centre of the first ball will move with a greater velocity than that of the ball struck, and the resistance which the spheres oppose to the change of their figures, will act equally in both spheres, but in contrary directions; that is, the force by which the striking body is resisted, will urge forwards the body struck; which, therefore, will be gradually accelerated, and the striking ball retarded, until the centres are at their greatest distance, the change of the bodies' figures being then the greatest; at which instant all acceleration of the ball struck, and retardation of the striking ball, will cease, and the two centres begin to go on with a common velocity.

This reasoning, concerning collision of bodies, is equally applicable to the case in which both bodies are non-elastic, or when one is perfectly non-elastic, and the other perfectly hard. It is likewise equally applicable to the collision of bodies which are elastic in any degree, so far as regards the velocity communicated to the body struck, during that small but finite portion of time in which the figures of the spheres receive their greatest change, their centres then beginning to go on with a common velocity in bodies of every kind of texture.*

OF ELASTICITY.

The action whereby bodies, whose figures are changed, restore themselves to their former figure, is termed *elasticity*. An elastic body is, therefore, one, whose figure being changed, it recovers, or has a tendency to recover, its former figure.

There are in nature, as you have already seen, varieties of activities; in some of which the causes are rendered manifest by experimental inquiries; in others, and among these we may reckon elasticity, no cause at all is discoverable by the senses.

A variety of experiments prove the existence of an elastic force. The separation of two bodies after impact, is a proof of elasticity. Metals, semimetals, stones, gems, fossils, cartilages, most fluids, as air, and even water, exert an influence opposite to the direction of the force compressing them, and discover a tendency to e-turn to their natural state: which tendency is in all of them perhaps imperfect, and less than the force impressed; but most perfect in air, glass, ivory, hardened steel, and cartilages.

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* See *Atwood* on Rectilinear Motion.

Elasticity is increased by augmenting the density of a body: thus metals are rendered more elastic by being beaten by a hammer; and their elasticity, which was scarcely sensible before, by this process becomes very sensible. Steel is more elastic when tempered, and its density is increased in the ratio of 7809 to 7738.

Elasticity is sometimes increased by cold; thus the range of a cannon-ball is said to be greater when the cannon is cold, than when heated; and the string of a violin, or a steel lamina, is inflected, and also recovers its situation, with less force in hot than in cold weather.

Metal fibres, and thin steel laminæ, exhibit no elasticity, unless stretched to a certain degree, and inflected by a certain force; as appears from lax cords, which, if a little stretched and removed from their natural state, discover no tendency to return to it; and when the inflection of fibres is very great, the influence of elasticity seems to be in some cases annihilated; as appears by the fibres of wood, which, inflected to a certain degree, remain quiescent, and have no tendency to recover their former situation. The limits where the elastic power begins, or where it terminates, are unknown.

Motion is supposed to be communicated or diffused, in elastic bodies, from the point of impact to the remote parts; and this supposition is grounded on the following experiments.

Here are two ivory balls suspended, so that their centres are in one line. The surface of one of them, B, is fresh painted. By letting them touch each other gently, A has received a small point of paint upon its surface. Now I shall raise the ball A, so that it may impinge with some violence on B, and it is evident that the surface of each ball has been flattened by the blow; for there is on each a circular mark, shown on one by the paint struck off, and on the other by the paint received; and, as the balls retain their spherical figure after the impact, it is clear that the parts of the surface not only lost, but again recovered their figure.

Two glass balls may, with a proper degree of velocity, so impinge on each other, that the interior parts of the ball will be broken, though the exterior, contiguous to the point of impact, be unbroken.

Suspend two ivory balls from the same point, by strings of the same length, and let the smaller ball A, impinge upon B, at rest, with a given velocity. A will be reflected always to the same height, and B will be impelled to the same height. But if either A or B be hollowed, and lead inserted in the centre, or near to the posterior surface, neither ball, though the weight be the same, will ascend as high as before the insertion of the lead. The progressive motion of the parts from the point of impact is stopped by the insertion of the lead; and consequently, the force of restitution and the change of figure, is less than before it was inserted.

The motion diffused from the point of impact to the remote parts of an elastic body, it continued for some time, and diminishes gradually till it vanishes, and there seem to be two kinds of vibrations in the parts of an elastic body; one of which is quick, and called a tremor of its minute parts; and the other slower and longer, by which its figure is changed, and an impinging body repelled.

A stroke or friction upon the edge of a glass, containing water, communicates a tremulous motion to the parts of the glass, which is visibly communicated to the water. A reed or stick placed across the bottom of a large glass bell, will fall when the glass is struck, the stroke producing a change of figure. If you hold a piece of metal near the brim or lip of a bell, without touching it, and the bell be stricken by a hard body, you will see it touch the piece of metal, and will hear a succession of sounds gradually decaying. If the edge of the bell be pinched, and the fingers be suddenly withdrawn, the same sound is heard, without producing any sensible motion towards the piece of metal, or displacing the reed across it.

Those laws according to which alterations are produced in the rest and motion of bodies upon their collision, are called the *laws of communication of motion.*

When one body strikes against another, if the line of direction of the impulse pass through the centre of gravity of both the bodies, the impulse is called *full* or *direct*, otherwise it is called an *oblique impulse.*

The relative velocity of two bodies, is that velocity with which they approach to, or recede from each other; and is equal either to the difference of the velocities of bodies moving the same way, or to the sum of their velocities moving contrary ways: for, if one of the bodies were at rest, and the other moved towards it or from it, with the fore-mentioned difference or sum of velocities, the body in motion would approach to, or recede from, the body at rest, just as fast as the two moving bodies approached to, or receded from, each other.

If two bodies move in the same direction with equal velocities, they are relatively at rest.

When the impulse is direct, and the bodies are void of elasticity, the laws of the communication of motion are these that follow:

1. In all cases, the velocities after the stroke are equal; for, the impulse ceases when the impinging bodies are relatively at rest, and not before.

2. If two bodies move the same way, and in the same right line, that which moves the faster will overtake the other, if before it, and the sum of their motions will be the same after the stroke as before: for, by the third law of motion, so much motion as the slower body gains, the swifter body loses.

3. If two bodies move contrary ways, the sum of their motions after the stroke will be equal to the difference of their motions before the stroke: for, whilst the stronger motion destroys the weaker, it loses also an equal part of itself, by the third law of motion.

4. If the sum of two conspiring motions, or the difference of two contrary motions, be divided by the sum of the quantities of matter in both the moving

bodies, the quotient will give their common velocity after the stroke.

5. If the velocity after the stroke be multiplied into the quantity of matter in each body, the products will express the quantities of motion in each body after the stroke.

6. The difference between the quantities of motion in either of the moving bodies before and after the stroke, is equal to the quantity of the stroke.

Example 1. Let a ball of three ounces, moving with nine degrees of velocity, overtake another ball, of two ounces, moving with four degrees of velocity; then will the quantities of motion before the stroke be 27 and 8, the common velocity after the stroke will be 7, the quantities of motion after the stroke will be 21 and 14, and the quantity of the stroke will be 6.

Example 2. Let the same balls move with the same velocities contrary ways; then will the quantities of motion before the stroke be as before; the common velocity after the stroke will be $3\frac{4}{5}$, the quantities of motion after the stroke will be $11\frac{2}{5}$, and $7\frac{3}{5}$, and the quantity of the stroke $15\frac{3}{5}$.

Wherefore, if two equal bodies move in contrary directions, with equal velocities, as soon as they strike, both the motions will be destroyed. If a body in motion strike against an equal body at rest, it will communicate half its velocity or half its motion. And if one moving body overtake another moving body equal to the first, the common velocity after the stroke will be equal to half the difference of their velocities before the stroke. But if they move in contrary directions, the velocity after the stroke will be equal to half the sum of the velocities before the stroke.

7. If a moving body strike against an immoveable obstacle, after the stroke the whole motion will be destroyed, and the quantity of the stroke will be equal to the whole quantity of the motion.

8. If the moving body gravitate towards the immoveable obstacle, as when a stone falls upon the earth, the quantity of the stroke is equal to the sum of the quantity of motion added to the weight of the moving body;

for the weight remains upon the obstacle when the impulse is destroyed.

The sum of the motions of two bodies void of elasticity, may be less after the stroke than it was before, but it cannot be more.

9. If a body move after the stroke the same way that it moved before, the difference of the velocities before and after the stroke will be equal to the velocity lost or gained. Thus, in the first example, the velocity lost s 2, and the velocity gained 3.

10. If the direction of the motion after the stroke be contrary to the direction of the motion before the stroke, the sum of the velocities before and after the stroke will express the quantity of the velocity gained. Thus, in the second example, the velocity lost is $5\frac{1}{5}$ the velocity gained $7\frac{4}{5}$.

When the impulse is direct, and the bodies perfectly elastic, the laws of the communication of motion are different from the foregoing. For,

1. Upon the collision of two elastic bodies, the force of elasticity is equal to the force of compression; and the force of compression in each body is equal to the quantity of the stroke.

2. The whole force of elasticity, exerted at the restitution of both the springs, is double the quantity of the stroke; for it is the result of two forces in contrary directions, each of which is equal to the quantity of the stroke. Or it may be conceived thus: the sum of the elasticities is equal to the sum of the quantities of the stroke in both bodies together; that is, to double the quantity of the stroke in each single body.

3. The effect of the elasticity in each body will be equal to the effect of the stroke, and in the same direction: for the two equal and contrary elasticities in the former case, are equivalent to action and re-action in the latter.

4. Wherefore, to find the velocity of either body after the collision, first, find the common velocity with which the bodies would move after the stroke, if they had been void of elasticity, and find also the velocity lost or gained; then subtract the velocity lost from the

common velocity, or to the common velocity add the velocity gained; so shall the difference or sum be the velocity sought. But if the velocity lost be greater than the common velocity, then subtract the common velocity from the velocity lost, and the remainder will give the velocity sought in a contrary direction.

Thus in the first example before stated, if we suppose the bodies to be elastic, the velocities after collision will be 5 and 10, and, consequently, the quantities of motion will be 15 and 20.

In the second example, the velocities after collision will be $1\frac{2}{5}$ and $11\frac{3}{5}$, and the quantities of motion $4\frac{1}{5}$ and $23\frac{1}{5}$.

For a third example, let us suppose two bodies in proportion as three to two; the first at rest, and the second moving towards the first with 40 degrees of velocity: then will the quantities of motion before the the stroke be 0 and 80, the common velocity after the stroke 16, the quantities of motion after the stroke 48 and 32, the quantity of the stroke 48, the velocity gained 16, the velocity lost 24, the velocities after the restitution of the springs 32, and 8 in a contrary direction; the quantities of motion after collision 96 and 16.

In the first example, the sum of the motions before collision is equal to the sum of the motions after collision; in the second, it is greater; in the third, less.

Two elastic bodies always recede from each other after collision, and that with the same relative velocity with which they tended towards each other before collision. Thus, the relative velocity before and after collision is 5 in the first example, 13 in the second, and 40 in the third.

And in all cases, whether the impinging bodies be elastic or not, the sum of the motions in the same direction, and the difference of the motions in contrary directions, are the same both before and after collision.

If two equal elastic bodies move towards each other with equal velocities, they will recede from each other after collision each with the same velocity. If one of them in motion strike against the other at rest, it will communicate the whole velocity, and remain at rest itself. If one overtake the other, they will interchange velocities, and continue to go on the same way as before. If they meet each other with different velocities, they will interchange velocities, and fly off from each other in contrary directions.

If an elastic body strike upon an immoveable elastic obstacle, it will rebound with the same velocity that it came.

If one of the impinging bodies be hard, and the other elastic, the laws of the communication of motion are the same as if both the bodies were elastic; for the spring will give way, till the force of elasticity becomes equal to the force of compression, that is, double to the quantity of the stroke in each body; and the re-action of a hard body against double the quantity of elasticity, will produce the same effect as a single quantity of elasticity in each body, acting in contrary directions.

If the impulse be oblique, it must be resolved into two impulses, one direct, and the other parallel to the tangent at the point of collision. The effect of the direct impulse must be computed according to the laws before mentioned: the parallel impulse will continue after the collision the same in all respects that it was before; nor will it produce any other effect, except by means of the friction to make the bodies revolve each about its own centre of gravity.

It often happens, that the communication of motion is the indispensable means of obtaining some other effect; thus it is used to nail and flatten bodies; in these cases it is not sufficient, that the striking body has a certain quantity of motion, but the mass or quantity of matter must be so proportioned to the velocity, as to produce the required effect, without splitting or destroying the body struck.

In architecture there are many occasions, where it is necessary to drive piles; if the mass which strikes the pile be small, it will not have force sufficient to drive it; and if it be moved with considerable velocity, it will split the head of the pile; a large mass must therefore be used, moving with less velocity. If small hammers and a great velocity be used to drive pivots of iron into large pieces of wood, the head of the pivot will yield, and yet not be driven into the wood; whereas, the effect will be answered by using hammers of considerable weight, moving with less velocity.

Gold-beaters, and other workmen who flatten metals, make use of heavy hammers, and move them slowly; if they used small hammers, and moved them swiftly, the parts of the metal would be broke and divided by the strokes.

To prevent the propagation of motion, and to deaden the blow, those that work in their chambers, and are obliged to use anvils, place the blocks that hold the anvil upon a roll of matting, or upon springs; without this precaution, great part of the force impressed by the hammer would be transmitted to the floor, and would create a shaking to the prejudice of the building.

The principles above laid down will assist you in explaining the recoil of cannon, fusees, and other firearms; for you may consider the gun-powder as a spring which unbends or expands itself in every direction, but which can only act efficaciously against the breech of the cannon and the ball.

The action of the powder is that of two equal forces against two unequal powers of resistance, the ball and the cannon; it may therefore be supposed, *cæteris paribus*, to impress them both with an equal quantity of motion, but with different degrees of velocity; that of the ball being as much greater as its mass is less than that of the cannon, &c.

The cannon, the musket, &c. especially if you take into the account the obstacles which retain them, are much more difficult to move than the ball with which they are charged, which, of course, receives from the inflamed powder a much greater degree of velocity.

There are other circumstances which contribute to augment the velocity of the ball, as the length of the piece, &c. which do not come properly before us. As to the recoil in general, supposing the quantity and quality of the powder the same, a gun recoils so much the more as the bullet makes more resistance either by its weight or the wadding.

A rocket flies upwards, because its lower part, which is fired, performs the office of a spring, which acts one way against the body of the rocket, and the other way against a volume of air; and as this spring is continually renewed by the successive inflammation of all the parts of the rocket, its motion is accelerated, first, because it is contained in the body itself, and is therefore continually adding to its velocity; and, secondly, because the weight or resistance of the rocket is diminishing every instant by the dissipation of the parts as they burn away.

GENERAL OBSERVATIONS ON MOTION.*

If you take only a cursory view of the operations in nature, you must be convinced, that there is somewhere an *inexhaustible source of impulse*, though undiscernible by our senses: consider the dissolving power of menstruums, the violence of fire, the strong contraction of our heart and arteries, the stability of heavy masses held down by the force of gravitation to the earth, the firm cohesion of bodies, and you must own all these must have some prodigious fund, though you know not whence to derive the force they exert.

The heavens, the earth, and all the elements are in constant motion; the matter of the earth, &c. is constantly passing into the substance of trees and plants, and as constantly returning from them again in their dissolution; the sea is moved with tides and currents; the air is agitated with winds and storms; and light is continually flowing from the sun, stars, &c. to the utmost boundaries of the universe.

Wherever the sources of this fund may be, or however copious the supplies, they cannot answer all the demands made upon them by the constant collision and other sources of decay. Every time you clap your hands together, you take something from the stock of motion which is to carry on the operations of nature; but the collision occasioned by human actions are very trifles in comparison with that great quantity of force which is spent in the workings of nature. The bearing of rivers against their winding banks; the dashing of seas against the shore; the opposition of winds one against another, or from mountains; the systole, or contraction of circulating vessels in plants and animals; but above all, the gravitation of bodies to the earth and sun; the cohesion of compounds; the opposition of centripetal and centrifugal forces; the various repulsions, &c. which must all occasion every moment an immense consumption of force, and require things to be so constructed, that there may be continual recruits to supply the perpetual diminution.

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* See *Jones*'s Essay on the First Principles of Philosophy.
———————— Physiological Disquisitions, p. 27 and 64.
Tucker's Light of Nature.

We have no knowledge of substances, or of any being, or of any thing, abstracted from the action of that thing or being.

All our knowledge of things consists in the perception of the power, or force, or manner of acting of that thing; that is, of the action of that thing on our senses, or of the effects of that thing on some other thing, whose action affects or is the object of our senses, and in the perception of the relations or ratios of these actions to each other: for if they produce no alteration on the senses, if no change be perceived by the mind through the medium of the senses, we can have no evidence of their existence; every effect must be produced by some cause or action.

Every thing that you know is an agent, or has a power of action, for you know nothing of any being but its action, and the effects of that action; the moment it ceases to act it is annihilated as to you, you can form no notion of its existence.

Whatsoever produces any effect or alteration on another thing, must possess a power either inherent or communicated.

The property or quality of any thing is nothing else but the action of that thing; and the different quality or properties of any thing or substance are no other than the different actions or action of that thing.

For every property must be occasioned by some cause, and you can have no conception of a cause without action. The power of doing something seems to be the first idea that rises when you contemplate real beings; it is that which distinguishes them from beings merely possible or ideal.

The more you consider the subject, the more you will find it necessary to admit of an active agency; for the effects perceived cannot possibly be the mere effects of composition; for composition, however exalted or refined, is only the union or co-arrangement of parts which before were separated, from which arise a certain order and relation more or less perfect, according to the quality and adjustment of such constituent and integrant elements.

But if such elements be themselves void of all energy, what can mere arrangement generate but various passive relations and analogies? If individually they possess neither the power of action or re-action, of attraction or repulsion, will the most artificial transposition or co-ordination, produce in the composite either energy, or motion, or resistance of any kind? To suppose it, is a manifest absurdity; it is to suppose that real energy can be generated out of nothing, or out of no antecedent subject; it is to suppose a realizing of non-entity, an evocation of something out of nothing.

When out of a block of marble the artist fashions a statue of Venus, he adds nothing internally to the marble; or when, from the fused mixture of salts and sand, the most beauteous forms are called forth, is any thing more effected than a new disposition of parts which before existed in another disposition? But, in educing active composite from inactive elements, not only a new position takes place, but a real active quality is super-added, which did not pre-exist in the separate elements.*

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* *Berrington's* Immaterialism Delineated.

Here, however, it is necessary for me to enforce upon you one consideration, that matter acts upon matter not by an essential but by a mechanical power, that is, by its motion; for in the natural as in the moral world there is no power but of God.

Motion is never to be considered as a thing by itself, but as an effect, which, like all other effects, must be referred to their proper causes; for the parts of the world have motion not of themselves, but as the limbs have by means of their connexion with the body to which they belong.

The parts of every system have some general reference or connexion with one single point or part, by which they become a kind of unity or one system. In the animal, such connexion is held by means of the nerves, or nervous fluid, or both, so that all parts of the system have communication with the centre. Whatever new part is so added as to be united to the common reference, becomes part of the same system. Whenever any part is so removed as to have no further reference to, or communication with the common centre, it is no longer a part of that system.

It is absurd therefore, in accounting for the phenomena of nature, to consider the motion of any body abstractedly as a thing by itself, for no such motion exists in nature.

There is nothing insulated in nature: it is a system of parts connected and related, and every particular part must be considered under this relation, without which neither the nature nor design of it can be understood.

To account, therefore, for the motion in the world, it must be taken as a connected system, and the effects considered as they stand related to their proper causes; and as motion is not a cause but an effect, there can be no motion without a cause of motion. If the effect be permanent, so must be the cause.

It is by no means necessary, that there should be but one cause of motion acting on a body at the same time. In many instances there is a concurrence of causes contributing to the same effect. A ship may be at once moved by the wind and tide, and the cause that acts on projected bodies.

Descartes, in order to make the universe a mere independent machine, feigned, that natural bodies are indifferent to motion or rest; that if at rest, they will continue so; if in motion, they will continue to move till they are stopped by some new force; thus endeavouring to frame a machine of a most extraordinary kind, of which there is no example in art or nature; a machine that continues to move after the moving power has ceased to act, and upon which it has no subsequent influence. It is a principle that naturally leads to atheism, for neither the power nor providence of God are necessary to that body, which moves to-day only because it moved yesterday.*

Rest and motion are not only different in their natures, but so opposite, that it implies a contradiction to suppose an equal disposition to either to be inherent in any body. Motion is positive, implying power or force wherever it acts; rest is a mere negative, in which body exerts no powers; it weighs, acts, or presses neither backwards nor forwards, up nor down; it produces no effect, and is in the exercise of no power. Notwithstanding this contrariety between rest and motion, yet they are confounded together in this law of *Descartes*.

Natural power may be properly defined to be matter actually moving, or exerting a nisus to motion like a suspended weight; but there can be no power where there is no tendency to action; the same body cannot therefore, at the same time, be indifferent to the exerting of actual power, and to the exerting of none at all.

All natural life, whether animal, mineral, or vegetable, or whatever characterizes matter to our senses, depends upon motion, activity, and power in matter, not on the inherent properties thereof; motion is therefore to be considered not as a natural condition of matter, but something superadded thereto, and constantly supported, to constitute life, variety, and mutability.

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* *Jones*'s Physiological Disquisitions.

Indeed so clear is this opinion, that we find even Dr. *Clarke* assert, " that all the great motions in the world are caused by some immaterial power perpetually and actually exerting itself every moment, in every part of the world ; and further, that the very laws of motion cannot continue but by something superior to matter continually exerting on it a certain force or power according to such determinate law."

ON PERMANENT MOTIONS IN NATURE. EXTRACTED FROM THE REV. WM. JONES'S PHYSIOLOGICAL DISQUISITIONS.

" The absurdity of the foregoing principle of *Descartes* will appear more fully by transferring it to a parallel case. Thus life is an effect as truly as motion; and as no body can continue to live without the constant operation of those causes which are acting for the support of life, so no inanimate body can continue to move without the proper causes of motion. When you are told, that bodies are indifferent to rest and motion, you learn no more than if you were told, in other words, that they are indifferent to life and death; and if you went on from this principle to assert, that a body once moved would move always, unless there were something to stop it, you would be as much mistaken as if you should affirm, that the body which lives once will therefore necessarily live on till something interposes to kill it.

" In accounting for the nature of human life, he would be thought to assign a very mean reason, that should urge, a man lived to-day only because he lived yesterday ; for there are certain physical principles on which the animal life is preserved, and without which it cannot possibly subsist."

In the same manner, a body continues to move only so long as the natural causes of motion continue to act upon it, and rest, which is mechanical death, must inevitably follow. Absolute rest is absolute death, the privation of all power, all faculty, all instrumentality ;

it can suffer agency, but exercise none; it can fill up no place in a chain of being, because incapable of any agency even in the lowest degree subordinate.

Matter is active under certain circumstances: where no cause or origin is apparent, among the different activities of which we have no experimental cause, or cannot by the senses discover their origin, you may reckon what we denominate life. Those also whereby bodies approach towards or recede from others, as attraction, repulsion, gravity, elasticity, &c.

" By considering the different kinds of motion observable in nature, you will find a uniform motion in a right line to be very rare indeed; the far greater part is in curve lines, or with velocities not uniform; the wind blows with a serpentine motion, as may be discerned by tracing its impressions over a plane covered with snow, or a field of standing corn. The water of the sea is driven into waves, which rise and fall alternately; the fire of lightning descends in crooked lines; the clouds fly in curve lines; the planets move with unequal velocity in their orbits. Light indeed moves in right lines, but whether its velocity be uniform at very great distances from its source, has not yet been ascertained.

" When you consider motion as an effect arising from impulse, you will always find that a body receives its motion in the same direction with the acting cause.

" If the causes of motion be various, and in different directions, you have seen, that the body acted upon must take an oblique or compound direction; from hence it is clear, that a curvilinear motion cannot possibly be simple. Such a direction must arise from the joint effect of different causes concurring at the same instant to act upon the body.

" As all motion is in the direction of its cause, it will follow, that when you see a body moving in any direction, you may affirm, that there is a cause acting upon it in the same direction; thus, if light bodies move outward from an electrified body, we conclude, that they are driven by a current of electrified ether, proceeding in right lines from the body. When you see the like bodies moving inwards towards the sphere, you must either conclude, as you did before, that there is another flux of matter in a contrary direction to the former, or give up one of the plainest principles in philosophy, that motion is in the direction of its cause. Bodies moving in contrary directions must therefore infer causes acting in contrary directions.

" As all motion is in the direction of its cause, there can be no such thing as a power of attraction ; all appearances from whence such a power is inferred must necessarily be resolved into impulse. If the earth attracts a stone by a power in the earth, the power exerts itself in a direction contrary to that of the effect, and can therefore never happen consistently with the laws of mechanism.

" In the motion of different bodies, it is observable, that some retain the motion they have acquired without any diminution, while others are soon reduced to a state of rest.

" When a body maintains its motion without diminution, it is moved by such a cause or causes as would renew its motion, if by any means it came to be stopped. If you stop the motion of the lungs by an effort of the muscles, the natural causes which act upon the body tend to renew their motion, and cannot be easily hindered from effecting their purpose.

" Every lasting motion is of such a nature, that it will be renewed upon its own principles. If it were possible to stop the planet Jupiter in his orbit, or any of the satellites, the established causes that act upon them would renew their motion without any artificial impulse ; for impulsion, in the sense it is commonly received, has no more share in any undecaying motion, than in the circulation of the blood in an animal or the sap in a vegetable. The heavens are therefore moved by such causes as would renew their progress, if it were possible for them to be stopped, otherwise their motion is not maintained upon physical principles. This is not meant as an objection to the schemes of motion deduced from the properties of curve lines, because geometrical without physical evidence will prove nothing on this subject. Motion is to be considered as a fact wherever you find it, and must be derived from principles agreeable to the general order of nature.

" Nothing can act where it is not, and therefore nothing can communicate its action to any other thing which is at a distance from it, but by the intervention or communication of some middle thing or medium, which extends from one thing to the other. We perceive, that the sun communicates some kind of action to all the planets, by which they circulate round it, and by which they are retained in their orbits, or gravitate towards it; so likewise we observe, that the earth communicates action to bodies at a distance from it, by which they gravitate towards the earth: there must therefore be some medium, some middle thing, by which the sun and the earth communicate action to other things at a distance from them.

" That two distant bodies, both supposed at rest, should be able to act upon one another without the assistance of any third substance, is to invest matter with a power of beginning motion itself, which is making motion essential to matter; and on this the materialist builds his system; it is a notion which the French *soi-disant* philosophers have turned to the purposes of atheism. Nor need you have recourse to such a principle, for every thing proves to you, that there is active* matter present where it is neither seen nor felt, as fire, air, elasticity, &c. and all the phenomena that surround you evince the continuity of material agency.

" All the experiments made upon the elements, which are the most useful experiments for advancing physical knowledge, tend to show how matter interposes to produce such changes and motions as we observe in bodies distant from each other. How does the sun act upon the fruits of the earth, but by the mediation of its light? How do the clouds water the earth, but by the mediation of air? How does the chemist produce so many changes, but by the mediation of fire? In short, wherever distant bodies affect each other, there is always something to mediate, whether it be or be not visible; and when this mediation can be traced no further, natural philosophy terminates. Amuse not, therefore, yourselves with names and qualities which contradict the known laws of mechanism, and supersede the operation of the elements.

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* Not active by its own nature, but according to the mechanism of the world.

"In order to account for the lasting motion of the heavenly bodies, it is most agreeable to analogy, reason, and observation, to admit a general principle of circulation in all fluid matter, which circulation certainly prevails in such parts of nature as are more immediately subjected to examination, and may with propriety be extended to the sun itself, and the elementary matter in the celestial spaces.

"It has been a question in philosophy, whether bodies can move in a space which is filled with matter, commonly called a *plenum?* But this question cannot be answered without first considering the condition of the matter, and stating the circumstances of the moving body.

"If the matter, so filling any space, be in a fluid condition, so that the parts can slide freely over one another, they will be able to move in different or even contrary directions at the same time; and while the place of the whole mass continues the same, the place of the parts which compose it may change every moment.

"The fulness of the space is therefore no objection to a free motion of the parts of fluids among themselves, neither is it an objection to the motion of any solid body in such a fluid medium.

"If a vessel be filled with water and closely stopped, any solid body that floats therein will move freely from one side to the other, or from the top to the bottom, because the parts of a fluid which are displaced before fall into the space behind, as the body leaves it; so fast as the body proceeds, just so fast do the parts of the fluid recede, without finding impediment or vacuity. Again, there may be fulness of matter and yet there may be motion, provided there is a circulation among the parts of the medium.

" It is easy to foresee what will happen to any solid body moving in a fluid medium, by attending to the circumstances. When the body is moved by any artificial force or effort of violence, contrary to the nature of the medium in which it moves, the parts of the medium in recovering their natural state will resist the motion of the body till the equilibrium be restored, and the body be at rest.

" But if the motion of the body arise from the motion of the medium, then the resisting nature of the medium is no longer any objection to the motion of the body; for it is absurd to imagine, that the cause of motion can resist the motion which it causes.

" No inferences from the resistance of mediums can lead us to the necessity of a *vacuum*. A vacuum can only be wanted when a motion is proposed, which is independent of the action of every medium; but there is no such motion in nature; on the contrary, we every where find examples of an unresisted motion, from impelling fluids in resisting mediums."

" At the extremities of this steel rod are two small lamps; over the lamps are two vanes of brass placed with contrary aspects, and inclined at an angle of about 45°; the rod is suspended on a point, so that the whole will turn freely. As soon as the lamps are lighted, the machine, you see, begins to turn freely upon its centre; it now makes several revolutions in a minute, and will continue thus to move so long as the lights continue burning; and supposing the lights to have a continual supply, the consequence would be a perpetual motion in the machine.

" Air is a resisting medium; yet, instead of retarding the motion of the machine by its resistance, it preserves that motion by its impulse; and though the motion discontinues at last, it is not owing to any defect or irregularity in the cause, but by the imperfection of the materials; if these would but continue in the same state, the motion would be unretarded so long as air and fire, which are the causes of it subsist in the world.

The causes of motion are not artificial, but such as are supported by nature itself in its regular mode of acting; these begin and continue the motion.

" Thus, you see, if a fluid act as an impelling cause to a moving body, its resistance with respect to the same body becomes of no account; it is in a manner annihilated, without removing a single particle of matter. The like is true of all floating bodies; when the motion of the fluid conspires with the motion of the body, the body moves with the velocity of the fluid, and loses nothing by communication. In fluids which have a motion of their own, bodies under proper circumstances will not lose motion, but acquire it. If the matter of the heavens has a motion of its own, the planets will thence derive their revolutions, instead of being retarded thereby.

" In all these experiments, where there can be no reasonable doubt about the explanation, matter is found to act upon other matter for producing the effect; and we are able to trace this in such a variety of instances, that unless the world be governed by opposite and contradictory principles, the same rule must obtain throughout the whole.

" The body of man, which is the highest piece of machinery in nature, is made to see, hear, and speak, upon mechanical principles; and it dies without the constant impression of a material force upon it from the element of air. By the pressure of the atmosphere, the mercury is made to rise in the tube of a barometer; hail, snow, and vapours are formed therein by its different temperatures; clouds are sustained by and driven about in it; plants grow and are nourished by it; and for those effects, where the cause is not so obvious, we have a more subtile element, that of fire or light; light as it illuminates, fire as it burns, warms, &c. These are the instruments which God has manifestly ordained as secondary and subservient to his own power in the economy of the material world, and they are so universally extended and incorporated with other things, as to be serviceable in the motion of all its particular parts.

"These powers are present to all those effects which have fallen under the observation of philosophers, and there are no other causes to be found, unless you ascribe unintelligible and innate powers to inert matter, thereby making the effect to be the cause of itself; or that you can suppose that God, who hath created these means, hath made it a rule to act without them. What fruit can then be expected from the labour of an inquirer, who, neglecting these elements, would banish them for such causes as can only operate in a vacuum? He must deface the beauty of nature, which is no where so conspicuous as in the dependence that is established between effects and their causes; destroying the connexion between the parts of matter, he can only present you with such a picture of nature as resembles the bones of a skeleton, which is incapable of stirring one step upon natural principles; whereas the work of God is worthy of its author, and the frame of nature is a perfect and well connected body, furnished with all its proper muscles and ligaments.

If you inquire into the primary and original cause of all motion and energy, your question may easily be resolved: that cause can be no other than the power of God; for, as matter did not make itself, so neither can it move itself, the motion thereof must commence, and be continued by the power of God. He is strictly the source of motion, as the source of life; neither life nor motion can remain for one moment, but so long as they depend either on his own immediate power, or such means as that power hath established in subordination to itself. Where we can observe and understand these means or intermediate causes, it is not necessary to recur at every step to the primary cause; and as it is the constant and ordinary means of Divine Providence to work with natural means, reason will require us to understand them, whether we can or cannot observe them. That matter is instrumental in the hands of God, for bringing about his purposes in nature, is the avowed doctrine of the Sacred Scripture; some unthinking and ignorant readers may despise its philosophy, but those who examine the works of God without prepossession, will soon discover by experience, that it contains the seeds of much knowledge. Let us fix in the present case upon one instance, namely, that of vegetation; the earth is said to bring forth grass, this is the effect. The first cause of it is the power of God, to whom all things owe their being, and in whom all things consist; for he causeth the grass to grow for the cattle, and herb for the service of man. But then it is also said, that the tender grass springeth out of the earth by the clear shining after rain. Therefore God is the primary cause, the sun is the instrumental cause; while the earth, softened and dissolved by the drops of rain, supplies the materials. This account, plain and simple as it is, gives us all that can be said upon the subject, and guards us against every fundamental error; it condemns the impiety of the heathen idolator, by referring the glory of the whole work to a power superior to matter; corrects the well-meaning ignorance of those modern philosophers, who would confute atheism by leaving out second causes; and at the same time leaves us in the possession of that ancient and valuable doctrine, that some of the elements are appointed to rule over and govern the rest."

LECTURE XXXI.

OF THE MECHANICAL POWERS.

THE knowledge of mechanics is one of those things that contribute to distinguish civilized nations from barbarians. From it the works of art derive much of their beauty and value; without it, we can make very little progress in the knowledge of the works of nature. By this science we are enabled to improve every power and force in nature, and render the motions of the elements, water, air, and fire, subservient to the purposes of life.

A weight, greater than the natural strength of man could manage, without some mechanical contrivance, could never be lifted from the earth: he is obliged, therefore, to seek assistance, and call in other forces to make such alterations as his pleasures or necessities may require. By means of levers he lifts weights much greater than his strength could overcome; with the wheel and axle, pullies, &c. he lifts them to considerable heights, and produces such effects as would to an unexperienced savage appear the work of enchantment.

Nor were the ancients without a great knowledge in this art of increasing strength by machinery. The stones which are laid upon the tops of the pyramids of Egypt, each of which is as big as a small house, create the wonder even of a modern mechanic, and teach him to reverence the superior arts of antiquity.

Practical mechanics will show you how to employ a given force, so as to produce a proposed effect by the aid of some machine or engine. It also shows how to modify by machines the action of a given power, or the quantity of a known effect.

Three things are therefore always considered in treating of the mechanical engines; a *weight* to be raised, the *power* by which it is to be raised, and the *instrument* or *engine* by which it is to be effected.

Mechanics may be consequently reduced to two problems: 1. To determine the proportion which the power and weight ought to have to each other, that they may just sustain each other in equilibrio. 2. To determine what ought to be the proportion of the power and weight to each other in a given engine, that it may produce the greatest effect in a given time.

Machines or engines may be considered as different means employed to facilitate the action of a power against a resisting obstacle.

The number and nature of mechanical engines vary according to the object for which they are designed; but, however varied or multiplied, you will find them to be only a combination of a small number of simple machines, commonly called *mechanical powers.*

I shall consider six simple machines; the *lever*, the *pulley*, the *wheel and axis*, the *inclined plane*, the *wedge*, and the *screw.*

These are all calculated to communicate motion to bodies, and sustain a pressure, for which the power unassisted by them is incompetent; and the artifice in all consists in distributing the weight among such a number of agents, that the part sustained by the power may bear a small ratio to the whole.

Thus, a power incapable of communicating motion to, or supporting the pressure of a body, without mechanical assistance, may effect its purpose by transferring a part of the weight upon the support or fulcrum, distributing it among a number of pullies, or placing it upon an inclined plane, or screw.

Postulata. The following postulata are necessary for the mathematical consideration of the mechanical powers.

1. That a small portion of the earth's surface may be considered as a plane.

2. That heavy bodies descend in lines that are parallel to each other; for, though all bodies tend to the centre of the earth, yet the distance from which they fall is so small, when compared to their distance from the centre of the earth, that their inclination is inconsiderable.

3. That the effort of any given power or weight is the same in all points of its direction; or, if a body be acted on by any power in a given direction, the action will be the same, in whatever part of that direction it be applied.

4. That though all matter is rough, all machines imperfect, &c. yet, in order to make calculations more easy and elegant, and to render theory more perfect, we suppose all planes perfectly even, all bodies quite smooth, all lines straight and inflexible, without weight

and without thickness, all cords to be extremely pliable, and all machines without friction.

These suppositions go no further than to show that the reasonings are concerning perfect instruments; but as there are none really so, the difference between theory and practice is to be afterwards investigated, and, when discovered, allowed for.

5. The effort of the power and of the weight is equal in all points of their directions; *i. e.* if you push or draw any body with a stick, supposed inflexible and without gravity, this body is pushed or drawn with the same force, whether the stick be long or short. Or if a weight be suspended by a long or short string, it will neither weigh more nor less, the weight of the string excepted; for, though gravity varies according to the different distances of a body from the earth, this variation is totally insensible in the length of a cord by which a weight is suspended from a machine.

Here it may be proper to recall to your mind what I have before shown you, that the momentum of any power or weight, is that force wherewith it either moves or endeavours to move; and that it is always proportional to the product arising by multiplying the power or weight into the velocity wherewith it moves, or would move, if it were not hindered by some opposite power or weight. If the product arising from the multiplication of one weight or power into its velocity, be equal to the product arising from the like multiplication of any other weight or power into its velocity, the momenta of those two powers or weights must be equal. And this will always happen when the weights or powers are to each other reciprocally as their velocities.

Hence, two weights or powers will balance, when one exceeds the other in weight, as much as the latter exceeds the former in velocity; and herein consists the force and efficacy of mechanical engines; for, they are so contrived, as to diminish the velocity of one weight or power, and to increase that of the other; by which means a very small weight or power may balance a very large one.

In general, therefore, there is always an equilibrium, when the sum of any number of acting powers is equal, and directly opposed to any one power, or to the sum of the forces of any number of powers; and reciprocally, if there be an equilibrium among the acting powers the sum of the acting powers on one side, will be equal to the sum of the powers acting on the opposite side.

Effects are always in proportion to their adequate cause; or, the change of motion in any body is always in proportion to the force which produced it, and in the direction wherein that force acts; for instance, if a certain force affects a certain motion, double that force will occasion double that motion.

It follows from this axiom, that if an effect depend on several heterogeneous causes, or, if several different circumstances concur in producing any effect, this effect will be as the quotient resulting from dividing the product of the circumstances that contribute to augment the effect, by the product of those that concur to diminish it.*

Or, in other words, an effect produced by several dissimilar causes, is in a ratio compounded of the direct ratio of the quantities which must be increased in order to augment the effect, and the inverse ratio of those that must be diminished to increase the effect.

To illustrate this by an example, suppose a waggon to be sent to any particular place; it is clear that the facility of conveying it, depends on the weight, the number of horses employed, the strength of the horses, length of the way, goodness of the road, and time to be employed.

Now, it is easy to see, that the facility of moving the waggon will increase, in proportion as the number of horses, their strength, the goodness of the road, and the time are increased, and the weight of the load, and length of the way diminished.

Six things are to be considerd in mechanical powers: 1. The power. 2. The resistance. 3. The fulcrum or support. 4. The velocity of the power and resistance. 5. The centre of gravity. 6. The line of direction.

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* *De la Caille's* Elements de Mechanique, p. 5.

OF THE LEVER.

The lever is an engine, which, when treated mathematically, is considered as an inflexible line, void of all gravity; though in practice it is a solid bar, that is flexible and weighty.

In the lever, you will more particularly direct your attention to three circumstances: 1. The fulcrum or prop by which it is supported, or axis about which it turns. 2. The power to raise and support the weight. 3. The resistance or weight to be raised or sustained.

The axis of a lever is the point about which it turns.

The points of suspension are those points where the weights really are, or from which they hang freely.

The weights are always supposed to act at right angles to the lever, except it be otherwise expressed.

As the lever is generally applied to lift or sustain weights, by means of a power, and fulcrum or prop, it has taken different names, from a difference in their relative situations.

It is called a lever of the *first kind*, when the fulcrum is between the power and the weight.

When you stir the fire with a poker, you make use of this lever. The poker is the lever; it rests upon one of the bars of the grate as a fulcrum; the incumbent fire is the weight to be overcome; and the hand at the other end is the power.

In the *second kind* of lever, the weight is placed between the fulcrum and the power.

Thus, in raising a water-plug, &c. the workman puts his iron lever through the hole in the plug, till he reaches the ground on the other side; and making that his fulcrum, lifts the plug by his strength applied to the other end of the lever.

In the *third kind* of lever, the power is situate between the weight and the fulcrum. The two above-mentioned levers assist the strength, but with the third kind the workman acts at a disadvantage; as in raising a piece of timber or ladder up on one end. In this the man's strength or power is in the middle, the part of the ladder already raised is the weight, and the end upon the ground is the fulcrum.

There will be an equilibrium on a straight lever of any kind, when the power is to the weight as the distance of the weight from the fulcrum is to the distance of the power from the fulcrum.

For it is evident, that there is always an equilibrium between equal and opposite forces; but forces are equal when the velocities of the weights are inversely as the quantities of matter, but the distances of the weight and power from the fulcrum will express their velocities, because these distances may be considered as the radii of arcs that they would describe in the same time, and these arcs are to each other as their radii; and, consequently, the velocity of each point of a lever will be as the distance of it from the centre of motion. Therefore, as on the present hypothesis the velocities are inversely as the masses, and the forces act in opposite directions, they are consequently in equilibrio.

Or the proposition may be proved thus: if there be not an equilibrium, let one of the weights preponderate. The preponderating weight has therefore more momentum than the other; but since, by the hypothesis, the weights are inversely as the arms of the lever, it follows, that the velocities of the weights are inversely as their quantities of matter, whence the moments of both are equal; but they were proved unequal. This contradictory conclusion results from denying the proposition affirmed, which is therefore true.

To estimate, then, the advantage of a power over a resistance, it is only necessary to determine what force will maintain an equilibrium between them; for, if their power be increased, it will overcome and raise the weight.

From hence, then, we may deduce,

1. That a power acts more efficaciously against a weight, in proportion as the power is more distant from the fulcrum.

2. That two equal masses, acting in opposite directions on the same lever, cannot be in equilibrio but when they are equally distant from the fulcrum.

3. That two unequal weights exert equal forces, when the distance of the small weight from the fulcrum exceeds the distance of the large weight from the same, in the same proportion as the large weight exceeds the small one; or, in other words, when the distances from this fulcrum are reciprocally as their masses.

These propositions are also demonstrated by the following experiments.

In making experiments on the mechanic powers, some difficulties arise from the weight of the materials; but as it is impossible to find any that are without weight, we take care that the levers, &c. are perfectly balanced themselves, before the weights and powers are applied, as they would otherwise alter the proportion between the power and weight; but, being balanced, the effect is not influenced by the weight of the materials.

The bar, therefore, used in making the experiments on levers, has the short end so much thicker than the longer arm, as will be sufficient to balance it on the prop; the hooks are all at equal distances from each other.

Of the lever of the first kind. Before you is a lever, A B C, *plate* 3, *fig.* 1, of the first kind, turning on an axis, D, which is the fulcrum that supports it. The parts, A B, B C, on different sides of the prop, are called the arms of the lever. The shorter arm is made so thick and heavy, as just to balance the longer arm: thus the lever is reduced to a horizontal position; and, being in equilibrio with itself, may be considered as without weight. The longer arm is divided into six equal parts: the distance of the hook from the fulcrum of the shorter arm, is exactly equal to one of these parts; so that the whole lever may be considered as divided into seven equal parts.

Now, from the proposition just explained to you, it is clear, that the power gained by this lever, or added to the natural strength or force of the agent, is in proportion as the one arm, B C, is longer than the other, A B. The velocities of the different parts of the bar, are as the distances of those parts from the fulcrum.

To illustrate this by experiment, suspend on the longer arm a weight of one ounce E, at the sixth hook from the fulcrum, and hang a weight of six ounces, F, on the hook of the shorter arm, and then the weight of one ounce will just balance or support the weight of six ounces. The weight of one ounce may be called the power, and that of six ounces the resistance or weight. The resistance is as much nearer to the fulcrum, as the power is lighter than the resistance. A man, therefore, who can without the help of any machine support one hundred weight, will be enabled by this lever to support six hundred.

As in this lever, the fulcrum may be placed exactly at an equal distance from the power and weight, or nearer to the one than the other, it is clear that the power and weight may counterbalance each other when they are equal, or when the one exceeds or is exceeded by the other, according to the different situations of the fulcrum.

Of the hammer-lever. This lever differs in nothing but its form from a lever of the first kind. Its name is derived from its use, that of drawing a nail out of wood by a hammer.

Suppose the shaft of a hammer to be five times as long as the iron part which draws the nail, the lower part resting on the board as a fulcrum; then, by pulling backwards the end of the shaft, a man will draw a nail with one-fifth part of the power that he must use to pull it out with pincers; in which case the nail would move as fast as his hand, but, with the hammer, the hand moves five times as much as the nail, by the time that the nail is drawn out.

To confirm this, in our apparatus, *plate* 4, *fig.* 2, and *plate* 5, *fig.* 2, we have a lever whose arms, AB, BC, are at right angles to each other. The heel turns on a pin, B, as the fixed axis. Hang a weight F, of five ounces on the shorter arm, then hang a weight E, on the fifth division of the longer arm, by means of a string going over the pulley, D, in the frame opposite to it, and these weights will balance each other.

It may be proper to observe, that though weights will be at rest when they are in equilibrio, yet it is not absurd to reason concerning their velocity, and to say, that they are in equilibrio, because they are to each other as their respective velocities: for, here we do not mean the velocities with which the power and weight move; that, indeed, might be absurd, as neither of them move at all; but we mean the velocity with which they must begin to move, if they were put in motion. When, therefore, in giving a reason for an equilibrium, we say, that the power and weight are to each other as their respective velocities reciprocally, we do not mean the velocities with which the power and weight move, but the velocities with which they must begin to move, if put in motion. Thus, if the centre of motion divide a lever of the first kind in the proportion of one to six, a power of a hundred pounds, applied at the end of the longer arm of the lever, will be in equilibrio with a weight of six hundred pounds suspended at the end of the shorter. And when we say, that the power and weight are to each other as their respective velocities reciprocally, we mean, that the weight cannot begin to move without making the power move with six times its own velocity; so that the weight cannot move in one direction, unless it can make the power move in a contrary direction, with a momentum equal to its own; and as equal forces, acting in contrary directions, destroy each other, they will remain in equilibrio.

Of the second kind of lever. In this kind the weight is between the fulcrum and the power.

Thus the bar, ABC, *plate* 3, *fig.* 2, is supported on the table, D, at the end A. The weight, E, is hung any where upon the bar, as at B; and the power, or hand, is applied at the other end, C, of the bar, to lift up that end, and raise the weight.

The power or advantage gained by this lever is as great in proportion as the distance of the hand from the table or prop, D, exceeds the distance of the weight from it. Thus, if the power or hand, C, be seven times as far from D, as the point B, on which the weight, E, is hung, a power equal to one-seventh part of the weight will support it; so that here, as in the preceding case, the power is increased in proportion as its distance from the fulcrum exceeds the distance of the weight from the fulcrum.

To confirm this by experiment, fasten a hook to the fifth hook of the longer arm of the lever, ABC, fixed at HI, *plate* 4, *fig*. 2, and carry it over the pulley placed directly above it; suspend a weight of one ounce from the end of the string, and this will balance a weight of five ounces suspended from the first hook from the fulcrum; so that a man taking hold of the bar at C, and pulling upwards, could support five times as much weight hanging from the point D, as he could support without the lever; but the end of the bar must be raised five inches, in order to raise the weight one inch.

The string is put over the pulley, in order that the power may act in a contrary direction to the weight. There is a slit in the bar over the lever, that the pulley may be placed over any part thereof. When used, it should be so placed, that the string going over it may be perpendicular to the lever, the lever being in a horizontal situation.

Of the third kind of lever. In this machine, the power is between the fulcrum and the weight: it may be considered as the second kind reversed, for the power in this must exceed the weight as much as the distance of the latter from the fulcrum exceeds that of the power from the fulcrum.

If you place the end, A, of the lever, ABC, *plate* 3, *fig*. 3, under the table, and divide the bar into seven equal parts from A to C, and suspend a weight of one ounce at the end C; then a hand placed at B, one division from the fulcrum, must pull up with a force equal to seven ounces, to support the weight at C, of one ounce; and if the hand rise one inch, the weight will be raised seven inches.

To illustrate this by experiment, fix one end of a string to the first hook of the lever, ABC, *plate* 5, *fig*. 2, and bring it over the pulley D, which for this purpose you must now place over this hook, and to the other end of the string hang a weight of five ounces, which acts as a power to pull the lever upwards. This will be counterpoised by a weight of one ounce at the fifth hook, C. No mechanical advantage is gained by this lever, because the power here must exceed the resistance, as much as its distance is nearer the fulcrum than that of the resistance; it is therefore seldom used.

If you examine the instruments in general use, you will find, that most of them are levers of one or other denomination. Thus, this pair of pincers is made up of two levers of the first kind, whose centre of motion is the rivet. The power is applied at the handles to press them together, and thereby pinch the body, as a weight, at the opposite ends. In this case, if the distance of the power, in both levers, be triple that of the weight from the rivet, the power acts six times stronger on the weight, than if it were applied directly to it.

A *pair of scissars* is also made up of two such levers. In these it is clear, from what has been said, that the nearer the resistance is to the points, the greater will be the difficulty of cutting with them: the difficulty is lessened by bringing it nearer to the rivet. The force and advantage of the lever, in this way, is very evident in the tinman and brazier's *shears*, whereby one man, pressing upon the handle, and raising the lower shear, is able to cut a plate of brass or copper a quarter of an inch thick; the upper shear is rivited to a couple of strong standards, fixed into the block. Nippers, snuffers, &c. are also levers of this kind.

The *cutting knife*, used by druggists and pattenmakers, to cut drugs, or the woods they use, is a lever of the second kind. One end is moveable on a joint, or centre of motion; by this it is also held down to the board on which it is used. The power is applied at the handle; the resistance is the wood or drug to be cut.

A *door* is also a lever of the second kind. The hinges are the centre of motion, the body of the door is the weight, the hand applied to that part near the lock is the power.

A *pair of bellows* are two levers of the second kind, whose common centre of motion is the ends of the boards where the nose begins. The power is applied to the handles, while the air to be pressed out acts against the boards as a resistance or weight.

Nut-crackers, with a joint at the end, are levers of the second kind.

The *oars* and *rudder* of a boat are also levers of the second kind. The boat is the weight or resistance to be moved, the water is the fulcrum, and a man at the handle the power. The *masts of ships* are levers of the second kind; the bottom of the ship is the fulcrum, the ship the weight or resistance, the wind acting or gathering in the sail is the moving power.

Sheep-shears are two levers of the third kind. The centre of motion is at the springing bow, the power, or hand, is applied at, or near the middle; the wool to be cut is the weight or resistance.

A *ladder*, or a *pole*, to be reared against a wall, are also levers of the third kind.

The *use of the third kind of levers* can only be fully contemplated in the animal body. Here the Sovereign Mechanic has displayed a wisdom and power that puts at nought every effort of human art. Animals move their limbs with great velocity, by applying the power of muscles very near the centre of motion; giving the muscles, at the same time, such very great force, as to perform their offices suddenly, raising the limbs even when great weights hang at their extremities, as when we lift weights with our hands or feet. There is scarce a bone in the animal body, but what is a lever of the third kind. In what manner, or by what means, power is communicated in the human frame, we are totally ignorant, nor is it to be accounted for upon the usual principle of mechanism, for their is no loss of time to compensate for the increase of power. The mind can impart no more of her might to the limbs, than the fibres are capable of conveying. What could *Goliah* or *Samson* do, if you allowed them only a single cobweb to work with? They would not have power to stir a silver thimble: for, if they went to push, the string would bend; if to pull, it would break. Yet when one tossed his weaver's beam, and the other carried the gates of Gaza, they performed their feats by tender filaments, lighter than a cobweb, undiscernable by a microscope.

When two powers, applied to the ends of a lever, support a weight resting upon the lever, they are to each other reciprocally as their distances from the weight. In other words, when two powers support a weight by the help of a lever, the sum of the powers must be equal to the weight; and the weight being placed between them, their respective distances therefrom must be reciprocally as the powers.

To illustrate this, I shall suspend this divided beam from the points 4 and 3, by weights going over the pullies in the bar above the beam; these weights being to each other reciprocally as the distances from the point O, where the resistance is to be suspended, will be in equilibrio with seven ounces, suspended from the point, O, of the beam.

Upon this principle, horses of unequal strength may be made to draw equally in a coach; for, by dividing the spring-bar unequally, that horse must exert more strength that is applied to the shorter end of the bar. Two men also, who carry a barrel hanging from a staff, are unequally pressed upon their shoulders, if the barrel do not hang in the middle; that man carrying most who is nearest.

If at one end of a lever be fixed a weight, which, moving with the end of the lever, does not hang freely, while the power acting at the other end is a heavy body hanging freely, or an inanimate power pressing perpendicularly towards the earth, such a fixed weight would vary in force, according to the position of the beam. The force will also vary, according as the centre of gravity of the weight is above or below the beam.

When the centre of gravity of the fixed weight is *below*, the beam, as in *plate 5, fig.* 7, will become heavier, or act more strongly, when raised above the horizontal line, by inclining the lever; and lighter when below the horizontal line, or when the lever is inclined contrary to its former situation.

But, on the contrary, if the centre of gravity of the fixed weight be *above* the beam, as in *plate 5, fig.* 8, it will be heavier when depressed below the horizontal line, and become lighter when raised above the horizontal line.

But the fixed weight will act in the same manner as if it hung freely, when the lever is in a horizontal situation.

When the lever with the weight, C, fastened above it, is in a horizontal position, the line of direction is c p, and p is the point on which the centre of gravity acts. When the lever is in the position SOM, the point e, through which the line of direction passes, is nearer the lower end of the lever than p. When the lever is in the position TOR, the point, a, through which the line of direction passes, is nearer the centre of the lever. It is therefore evident, from a bare inspection of the figure, 1. That when a fixed weight, whose centre of gravity is above the lever, is below the horizontal line, the point of action gets farther from the centre of motion, and its momentum, or effect, is therefore increased. 2. That when it is above the horizontal line, the point of action approaches nearer the centre of motion, and the momentum, or effect of the weight, is thus diminished.

When the centre of gravity of the fixed weights is below the lever, *fig.* 7, p is the point of action when the lever is horizontal; but by inclining it in the position KR, i c becomes the line of direction, and i the acting point, which will be nearer to O than the point p. By inclining it in the contrary direction, c e will be the line of direction, and e the acting point, farther from O than the point p. Consequently, when a fixed weight, which has its centre of gravity below the lever, is beneath the horizontal level, the point of action gets nearer the centre of motion, and the effect of its weight is diminished; but when the weight is above the horizontal level, the point of action is removed farther from the centre of motion, and the effect of the weight is increased.

Nothing of this kind will happen when the weight is freely suspended by a rope; because the point of suspension, or point of action, is not altered. What has been here proved, concerning levers of the first kind, is equally applicable to the other two kinds.

When two draymen carry a barrel on a coulstaff, from which it is suspended by a chain, the point on which the weight acts is not altered by inclining the staff in going up or down hill; there will be no inconvenience suffered, or advantage gained, by the first or last man: but if they carry the barrel upon two dogs, then the weight does not swing, and the centre of gravity is below the lever; therefore the point on which the weight acts, will, by inclining the lever, be made to approach the higher end; and the first man, in going down hill, by having this point removed from him, will be eased in part of his burden, and the last man will have his equally increased. When two chairmen are going down hill, if the centre of gravity of the weight they carry be above the plane of the poles, the man that walks first will have the disadvantage: if it be below the plane of the poles, the man that walks last will have the disadvantage; but if the centre of gravity be in the plane of the poles, the action will be equally divided in going down hill.

To show that a power is most effectual when it acts at right angles. Unhook the scales from the brass beam G H I, *plate* 4, *fig.* 2, and raise the transverse arm, M N, so as to be only a few inches distant from the beam; and place two pullies, so that their outer edge may be exactly three inches distant from the point where the string falls, by which a weight of four ounces hangs freely from the beam. This will be balanced by a weight of four ounces, at an equal distance, on the opposite arm of the beam. But if you put the string over either of the pullies, by which means the weight affixed thereto acts obliquely on the beam, it is over-balanced by the equal weight on the opposite arm; and so much so, that, in the present instance, it requires one ounce additional weight to make it counterpoise the other.

The quantity of motion produced by a power which acts upon a lever is always proportional to the moving force.

We say, indeed, that the weight a man is obliged to lift, by the help of a lever, is greater than his strength;

but by this we mean only that the weight is greater than he could raise by his strength directly applied to it, in such a manner as to make his power, and the weight that is to be lifted, move with the same velocity.

He is enabled to raise it by the help of a lever, because, by suspending the weight at the shorter end of the lever, and applying his strength at the longer arm, the weight is made to move slower than the power. And though the momentum of the weight, when it moves with the same velocity as the power, may be greater than his strength, yet the proportion which the velocity of the weight bears to that of the power, may, by means of the lever, be diminished, till the moving force which he exerts shall be equal to the momentum of the weight.

No motion is therefore produced by the lever itself: the only use of it is to make the weight move slower than the power; so that, by means of this mechanical instrument we can indeed lift a greater weight than we could without it, but at the same time we only produce the same quantity of motion, for as much as the momentum we can raise with a lever exceeds its weight, so much it falls short in velocity of the momentum we could raise without one. This reasoning is applicable to all the other mechanical powers.

MECHANICAL ARITHMETIC.

Several weights suspended at different distances from the axis on one side of a balance, will counterpoise several others suspended at different distances on the other side; provided the product arising from the multiplication of the weights on one side, by their respective distances from the axis, be equal to the sum of the products arising from the like multiplication of the weights on the other side into their respective distances.

Thus, suspend a weight of one ounce at two inches distance from the axis of the beam, G I, *plate* 4, *fig.* 2; another of two ounces, at three inches; and a third of three ounces, at four inches: and if on the other side you suspend one weight of five ounces at one inch from the axis, and another of three ounces at five inches distance, the two latter will balance the three former ; for the product of five inches into one, added to the product of three by five, gives twenty, equal the product of one into two, added to that of two into three, and three into four.

From this principle you may easily deduce a method of finding the centre of motion, about which, when equal or unequal weights are suspended at the ends of a balance of known length and weight, they will be in equilibrio.

From hence you will perceive how easy it is to perform what is called *mechanical arithmetic*, by means of a divided beam and a proper set of weights.

Addition. Let the numbers to be added be two, three, and seven. I apply an ounce weight at the second division, and another on the same arm at the third ; I now take an ounce weight, and move it along the other arm till the beam be in equilibrio, which I find to be the case when the weight is at the twelfth division.

Subtraction. To subtract five from twelve. Hang an ounce weight at one end of the arm at twelve inches, and another at the other end at five ; then move a third ounce weight along the arm till the equilibrium be restored, and you will find it to be at the difference seven.

Multiplication. To multiply four by three. Suspend a four ounce weight at the third division on one arm, and move an ounce weight on the other till the beam be in equilibrio, and you will then find it mark out the product twelve.

Division. To divide twelve by four, suspend an ounce at the twelfth division, and move a four ounce weight on the other arm till there be an equilibrium, and you will find it at the quotient three.

OF THE BALANCE.

The balance, as you all know, is an instrument of very extensive use, being applied to estimate the weights of bodies, that is, to compare the weight of any given substance with certain standard weights previously determined.

The balance is a lever of the first kind, whose arms are of an equal length; in other words, the points from which the weights are suspended are equally distant from the centre of motion. In treating of a balance, four parts are principally to be considered: 1. The beam itself. 2. The centre of motion. 3. The arms of the beam. 4. The points of suspension.

To conceive more clearly the principles of the balance, it is considered theoretically as an inflexible mathematical line; a point in the middle of the line represents the fulcrum, and the nearer the balance in practice can be brought to this abstracted view of it, the more perfect is the construction thereof. From this view, you must clearly see, that points equally distant from the fulcrum or centre of motion, will describe equal arcs; if, therefore, equal weights be freely suspended at equal distances therefrom, their moments will be equal, and the beam will be in equilibrio. For since all bodies tend towards the earth, with forces proportional to their weights, if two weights be equal, their efforts will be equal; and if they be suspended from the extremities of a lever, whose arms are equal, they act with equal power on each arm, and consequently one does not tend more to the centre of the earth than the other; therefore, if the fulcrum be immoveable, the two extremities will be so likewise, and remain equally distant from the centre of the earth, or parallel to the horizon. It follows also, that the further the points of suspension are from the centre, or the longer the beam, the more the momentum of the weights is increased, and consequently the balance is rendered more sensible.

In practice, the beam of a balance is a solid bar, and is made of the most homogeneous and inflexible materials, and is proportioned in strength to the weight it is intended to support; for if the beam be too weak, or loaded too much, the arms will bend, which will render the beam less sensible; and if the arms bend unequally, will make it untrue.

The arms should be accurately of the same length, and as long as possible, relative to their thickness and the weight they are intended to support.

If the bearing edge of the axis be very obtuse, the distance of the extremities of the balance relative to the fulcrum will be altered.

The two pivots, which form the axis or fulcrum, should be in a right line, and at right angles to the beam.

The axis should be well polished and very hard, and its supporting part should be thin, and yet not so sharp as to cut, but a little rounded to avoid friction; these edges are first made sharp, and then rounded with a fine hone or piece of buff-leather, which causes a sufficient bluntness or rolling edge, on the regular form of which depends chiefly the excellence of the instrument.

The edge of the axis must be more or less blunt, in proportion to the weight it is to support.

The axis should be at right angles to a plane supposed to pass through the middle of the beam.

The rings, or the pieces on which the axis bears, should be hard and well polished, parallel to each other in order to prevent friction, and of an oval figure, that the axis may keep its proper bearing, or always remain at the lowest point.

The index is a slender bar perpendicular to the length of the beam.

Particular care should be taken, that the points of suspension, on which the scales hang, be in the same line with the centre of motion, for if the points of suspension be higher than the axis, the balance will vibrate too much.

If they be below the axis, the vibration will be too slow; and, in either case, if the beam incline, it will divide a horizontal line passing through the fulcrum into unequal parts, and therefore destroy the equilibrium, though equal weights may be in the scales.

The index placed perpendicularly to the beam, in order to ascertain when it is horizontal, would add to the weight of the descending arm, and so destroy the equilibrium, if it were not corrected by a weight placed on the opposite side of the beam.

As the horizontal position of the beam is the only one by which we can well judge of the weight of bodies, particular care is to be taken of the position of the fulcrum or centre of motion, for you will find, that the situation of this part alters in a degree the necessary perfections of the balance. There are three situations to be considered, whether, 1st. The centre of motion should coincide with the centre of gravity of the beam. 2d. Whether it should be above; or 3dly, Below it.

1. When the axis of motion or fulcrum passes through the centre of gravity, and the three edges lie all in one right line, the beam will rest in any position whatsoever, whether the scales be on or off, with or without weights.

A beam, where the centres of motion and gravity are coincident, can have no other tendency to a horizontal situation but what it may derive from its own weight, and will therefore turn with the same additional weight, whatever may be the weight applied, except so far as the greater load produces a greater friction.

If two weights were suspended from such a beam, at an equal distance from the axis, the quiescence of the rod would denote an inequality between the weights acting upon it; but it is more convenient, that some fixed position of the beam should denote the equilibrium of the weights, and there is no position so eligible as that which is commonly used where the beam is horizontal, when the weights suspended from the extremities of the arms are equal.

2. If the centre of motion or fulcrum be a little above the centre of gravity, the beam will not rest in any position but the horizontal one; for when the beam is quiescent, the centre of gravity will be immediately under the centre of motion; consequently, if it be removed from this situation, it will vibrate backwards and forwards, and at last rest in a horizontal position.

The nearer the centre of gravity of the beam is to the centre of motion, the nicer will be the balance, and the slower its vibrations; thus, if *a c b c*, *plate* 4, *fig.* 7, be the beam, and C the centre of motion, the difference between the effect of having the centre of gravity at K, or *c*, will be the same as if we compared the velocities of two pendulums of the lengths C K and C *c*, which I have shown you to be in a subduplicate ratio of their lengths. The tendency to a horizontal position is, therefore, increased by lowering the centre of gravity, in which ease it will also require a greater additional weight to cause it to turn or incline to any given angle, and is therefore less sensible with a greater load.

3. When the fulcrum is below the centre of gravity, if the centre of gravity be moved out of the perpendicular line, the beam will be overset, that is, turned out of its horizontal position, and will not return thereto, so that equal weights may appear unequal.

Fixing the centre of motion in a balance is, you perceive, of the utmost importance, for upon this depends the ease with which it will be affected by a smaller weight, and the readiness with which it will return to its horizontal position; and it is clear, from what I have already told you, that the best position is that in which the centre of motion is a little above the centre of gravity; and even in this, it should be proportioned to the distance of the weights from the fulcrum, and the quantity of matter to be weighed, which in different beams can only be attained by the practice and experience of the maker.

Hence, great care is taken in all balances, that the axis be placed a little higher than the centre of gravity; to determine the best distance of these two points, an adjustment is added to some beams to raise or lower the centre of gravity.

The scales should be suspended in such manner, that in all positions the strings of the scales may be parallel one to the other; for if this be not observed, the weights, although equal, will not be in equilibrio.

To impress more strongly on your mind the alterations occasioned by the different positions of the centres of motion and gravity, I shall exhibit to you again the triangular piece of brass, *plate* 1, *fig*. 22, with three holes, one in the centre of gravity, one above, and the other below that centre.

Suspend it on a pin passing through the centre of gravity, and you will see it rest in any situation; suspend it on the hole above the centre of gravity, and set the beam in motion by raising the centre of motion, and you will observe it descend with an accelerated velocity, and continue oscillating like a pendulum for a considerable time. When suspended with the centre of gravity above the centre of motion, it will only be level, when the centre of gravity is directly over the point of suspension; if it preponderate ever so little, it will descend till the centre of gravity is below.

When the arms of a balance are unequal, the balance is said to be false, as not giving the true weight of the body weighed, whether it be suspended from the shorter arm or from the longer one. There are, however, several properties of the false balance, which are extremely useful in the estimation of weights, as well as in correcting errors which may have arisen in the adjustment of the true balance.* A false balance is instantly detected by changing places between the weight and the material weighed.

A balance with unequal arms will weigh as accurately as another, provided the standard weight itself be counterpoised, then taken out of the scale, and the thing to be weighed be put into the scale, and adjusted against the counterpoise.

Or when proportional quantities only are considered, the bodies may be weigned against the weights, taking care always to put the weights in the same scale; for then, though the bodies may not be really equal to the weights, yet their proportions amongst each other will be the same as if they had been accurately so.

A weight which counterpoises an ounce, when suspended from the longer arm of a false balance, being added to the weight which counterpoises an ounce, suspended from the shorter arm, will always be greater than two ounces. The excess is that part of an ounce, which is expressed by a fraction, of which the numerator is the square of the difference of the arms, and the denominator the product of the arms.

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* *Nicholson's* Principles of Chemistry.

Atwood's Analysis of a Course of Lectures.

If any substance be successively weighed from the longer and shorter arms of a false balance, the true weight will be a geometrical mean between the false weights.

Further properties of the balance may be known by referring to Mr. *Nicholson's* work, as before cited; by whom is recommended the following set of weights to accompany the balance, for chemical and other operations, *viz.* 1000 grains, 900 g. 800 g. 700 g. 600 g. 500 g. 400 g. 300 g. 200 g. 100 g. 90 g. 80 g. 70 g. 60 g. 50 g. 40 g. 30 g. 20 g. 10 g. 9 g. 8 g. 7 g. 6 g. 5 g. 4 g. 3 g. 2 g. 1 g. $\frac{9}{10}$ g. $\frac{8}{10}$ g. $\frac{7}{10}$ g. $\frac{6}{10}$ g. $\frac{5}{10}$ g. $\frac{4}{10}$ g. $\frac{3}{10}$ g. $\frac{2}{10}$ g. $\frac{1}{10}$ g. $\frac{9}{100}$ g. $\frac{8}{100}$ g. $\frac{7}{100}$ g. $\frac{6}{100}$ g. $\frac{5}{100}$ g. $\frac{4}{100}$ g. $\frac{3}{100}$ g. $\frac{2}{100}$ g. $\frac{1}{100}$ g. and the peculiar property of these is, that the philosopher will always have the same number of weights in his scales, as there are figures in the number expressing the weights in grains: thus 742.5 grains will be weighed by the weights 700, 40, 2, and $\frac{5}{10}$ E. EDIT.

OF THE STEEL-YARD.

On the equilibrium that arises from the suspensions of weights, when the distances are reciprocally proportioned thereto, is founded the *steel-yard*, which consists of two arms of very unequal length, but equally poised by means of a weight annexed to the shorter, from which likewise a scale is suspended to receive such things as are to be weighed; the longer arm is divided into a number of equal weights, beginning from the axis, and sustains a weight which slides from one end to the other, which weight being applied to the second division, will counterpoise double the weight in the scale of the shorter arm, of what it will balance when at the first division, and triple when applied to the third division, and so on, it being so graduated, that the weight and distance are proportional.*

HELSHAM'S PROPERTY OF THE BALANCE.

If a man standing in one scale, and counterpoised by a weight in the other, lays his hand to any part of the beam and presses it upwards, he will thereby destroy the balance, and make the scale wherein he stands to preponderate.

To illustrate this property, fix the beam, A B, to the upper part of the pillar of the apparatus, represented in *plate* 4, *fig*. 4; A X is a perpendicular piece suspended from the end, A, of the beam, and is prevented from being pushed out of its perpendicular position by the plate D G.

H I is a pushing pipe, the lower point is put into a hole at X, the upper point is put into a small hole on the under side of the beam; the pin I, by means of an internal spiral spring, is pressed up against the beam, as a man might do with his arm.

The perpendicular piece A X, and the pushing pipe being suspended, the spring, at *b*, stopping by *a*, is prevented from acting; the whole is then counterpoised by weights on the opposite arm of the beam, the spring is then to be freed, and the beam immediately preponderates, and an additional weight, *a*, must be hung to the opposite end to restore the equilibrium.

OF THE WHEEL AND AXIS.

The wheel and axis is a machine much used, and which is applied to a variety of purposes, and in a variety of forms. The power acts on the circumference of the wheel; the weight is fastened to one end of a rope, whose other end winds round an axis that turns with the wheel.

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* *Plate* 3, *fig*. 1, is a representation of the principle of the steel-yard. The weight, at F may represent the scale, or matter to be weighed suspended to the short arm A B, and E the general moving weight that moves from one end to the other of the longer arm B C, and by the proper figure thereon denoting the weight. E. EDIT.

This machine is generally used with a handle: thus, to wind up a jack, I turn a handle, which motion coils a rope round the axis in the middle of the engine. By turning a handle we raise up a bucket from a well, &c. &c. but in all these cases, the greater the distance of the handle from the centre of the wheel, the greater is the power we thereby obtain.

The axis A, and wheel C D, *plate* 3, *fig*. 6, are fastened together, so that one cannot move without the other; when a weight, W, is to be raised by this engine, it is fixed to the cord which goes round the axis, but the power, P, is applied to one of the several circumferences of the wheel.*

To turn the wheel once round, pulling at the rope P, as much rope must be drawn off as winds once about the wheel, or through a space equal to the circumference of the wheel; but whilst the wheel is turning once round, the axis is also turned once round, and of course, the rope by which the weight is suspended will wind once round the axis, and the weight will be raised through a space equal to as much rope as will go round the axis, *i. e.* equal to its circumference.

Since, therefore, in the time the machine is turned once round, the space which the power describes is equal to the circumference of the wheel; and the space which the weight describes is equal to the circumference of the axis; and as the velocities are as the spaces described in the same time; it follows, that *the velocity of the power is to the velocity of the weight, as the circumference of the wheel is to the circumference of the axis.*

Though this machine will present itself to you in a variety of forms, you will always find the abovementioned proportion between the power and weight, when the power is applied to the circumference of the wheel, and the weight to the axis.

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* At *plate* 5, *fig*. 3, the apparatus, A B C D E, is a compound engine, in which the wheel, F, may occasionally be detached to show the nature of the wheel and axis.

Hence on the wheel and axis the weight and power will be in equilibrio, when the power is to the weight as the circumference of the wheel is to the circumference of the axis; and the momentum of the power, or its weight multiplied by its velocity will be equal to the momentum of the resistance, or the weight multiplied by its velocity.

Now, geometricians prove that the circumferences of circles are as their diameters; consequently, *if the power bear the same proportion to the weight, as the diameter of the axis bears to the diameter of the wheel*, the power and weight will balance each other. Thus, suppose the diameter of the wheel to be eight inches, and the diameter of the axis to be one inch, then one ounce acting as a power, P, will balance eight ounces as a weight, W; and a small additional weight will cause it to descend, and turn the wheel and its axis, and so raise the weight W, and for every inch the weight rises, the power, P, will fall eight inches; so that

The wheel and axis may be considered as a lever, whose fulcrum is a line passing through the centre of the wheel and the middle of the axis, and whose long and short arms are those radii of the wheel and axis that are parallel to the horizon, and from whose extremities the cords hang perpendicularly.

Suppose that the power does not act by a rope winding round a wheel, but that it is moved by a man's strength applied immediately to the spokes or handles, K, E, F, G, H, I; if the man first lay hold of the handle H, and push it down to G, his hand passes through the space HG, and the handle will be brought down to G; he then lays hold of I, and pushes it to G, and so on, till he has turned the wheel once round; and his hand, which is now the power, will describe the whole circumference of a circle, which is to be considered as the circumference of the wheel.

Hence a wheel and axis may be considered as a kind of perpetual lever, on whose arms the power and weight always act perpendicularly, though the lever turns round its fulcrum; and in like manner, when wheels and axes move each other by means of teeth on their circumference, such a machine may be considered as a perpetual compound lever.

In using this machine, the weight is not always suspended to the axis by a rope, but is sometimes fixed to the axis itself. This is the case of a bell that is moved in ringing by a wheel and an axis; here, in once turning the wheel round, the velocity of the bell is as the circumference described by its centre of gravity.

On the other hand, in a crane-wheel the power is not applied to the wheel by means of a rope, nor does it act upon any handle or spokes, but by a man walking within the wheel; as the man steps forward, the part upon which he treads becomes the heaviest part of the wheel, and descends till it be the lowest; thus he keeps going on, and by treading upon every part of the wheel's circumference in its turn, he walks over the whole thereof, and brings the wheel once round.

A capstan is a cylindrical axis, that is turned by four, or more, bars that cross it; these are the spokes of an imperfect wheel, and the velocity of the power applied to them is as the circumference which it describes in one revolution of the capstan.

When the parts of the axis differ in thickness, and weights are suspended at the several parts, they may be sustained by one and the same power applied to the circumference of the wheel, provided the product arising from the multiplication of the power into the diameter of the wheel, be equal to the sum of the products arising from the multiplication of the several weights into the diameters of those parts of the axis from which they are suspended.

Thus a weight of five ounces hung from a part of the axis whose diameter is one inch, and another of ten ounces from a part whose diameter is half an inch, will be both balanced by a weight of two ounces hanging from the circumference of a wheel whose diameter is five inches; for, five by one, added to ten by one-half, is equal to ten, which expresses the momentum of the weights; and two by five, or ten, is also the momentum of the power.

In reasoning upon the wheel and axis, I have supposed the rope that sustains the weight to be of no sensible thickness; but if it be a thick rope, or if there be several folds thereof round the axis, you must measure to the middle of the outside rope to obtain the radius of the axis; for the distance of the weight is increased just so much by the coiling of the rope round the axis.

You have seen that the velocity of a weight, or any other force applied to the axis of a wheel, will increase as the circumference of the axis increases. Thus a weight, or any other force, acting upon the axis, B, will have a greater velocity than one which acts upon the axis A; if the force were applied to T, its velocity would be still further increased. Now, the fusee of a watch is an axis in the shape of a cone, and the spring, by means of a chain, acts upon the smallest part of this axis when the watch is wound up; as the watch goes down, the chain is applied to different parts of this cone, and to the thickest part of all when the watch is just down. Therefore, as the elasticity or absolute force of the spring decreases, the velocity with which it acts to move the watch increases, by being applied to a larger part of the axis in proportion; and consequently, if the fusee be properly regulated, the momentum produced in the wheels and hands of the watch will be always the same. In other words, the motion of a watch is rendered uniform by the fusee, though the force of the spring is not always the same, but is greatest when the watch is wound up, and keeps decreasing as the watch goes down.

OF THE PULLEY.

The pulley is a small wheel turning about its axis, with a drawing rope passing over it; the small wheel is commonly called a *sheeve*, and is so fixed in a box or block, as to be moveable round a centre-pin passing through it.

Pullies are of two kinds: 1. *Fixed*, which do not move out of their place. 2. *Moveable*, which rise and fall with the weight.

A rope going round one or more pullies to raise a weight, is called the running rope of the fixed pulley; this kind only turns on its axis, but does not move out of its place, see B, *plate* 3, *fig*. 4, or V, *plate* 5, *fig*. 3. It changes the direction of the power, but gives it no mechanical advantage; so that you can raise no greater weight by means of this pulley, than you can raise without it by your natural strength.

Two equal weights suspended to the ends of a string that goes over a fixed pulley, will balance each other, for they are equally stretched by the weights; and if either of them be pulled down through any given space, the other will rise through an equal space in the same time; and, consequently, as the velocity and the weights are equal, they must balance.

Though this pulley gives no mechanical advantage, it is a source of great convenience, as it takes off the necessity a man would otherwise be under of ascending along with the weight, and thus lessens his labour; besides having this further convenience, that by means thereof, the joint strength of several persons may be made use of to raise the same weight.

OF THE MOVEABLE PULLEY.

Of this kind are C and D, *plate* 3, *fig*. 4, or T, *plate* 5, *fig*. 3, the lower of which rises or falls along with the weight, and adds to the momentum of the power.

Some writers have thought that the nature and effects of the pulley might be best understood, by considering a fixed pulley as a lever of the first kind, and a moveable pulley as one of the second. But this mode seems only calculated to introduce the parade of demonstration, as the pulley cannot with propriety be called a lever; for, when any power sustains a weight by means of a system of pullies, that power will sustain the same weight if the pullies be removed, and the ropes be brought over the axis on which the pullies turned. Now, in this case, no one I believe would say, that these axes should be considered as levers; if it were required to raise a weight, there would be a very great resistance arising from the friction of the ropes on the axes, and it is merely to avoid this resistance that pullies are used, because they turn upon their axes with very little friction.*

The best and most natural method of computing the proportion of any power to the weight it sustains by means of a system of pullies, or of explaining their effects, is by considering that every moveable pulley hangs by two ropes equally stretched, and which must, consequently, bear equal parts of the weight; and, therefore, when one and the same rope goes round several fixed and moveable pullies, as all the parts on each side are equally stretched, the whole weight must be equally divided among all the ropes by which the moveable pullies are suspended.

Hence, if the power which acts on one rope be equal to the weight divided by the number of ropes, that power must sustain the weight; in other words, the power and weight balance each other, when the power is in proportion to the weight, as one is to the whole number of cords. Upon this principle, the proportion of the power to the weight it sustains by means of a system of pullies, may be computed in a manner so easy and natural as to be obvious to every common capacity.

It is necessary to observe here, that in making your computation, the weight of the moveable pulley must be balanced before the power and weight are applied, or else it must be included in the weight supported.

Let us now consider the moveable pulley C, *plate* 3, *fig.* 4, in our apparatus; here a weight, W, hangs at the lower end of the moveable pulley, D, and one end of the cord goes round the pulley, and is fixed to the hook at the top of the frame, while the other end goes over the fixed pulley, and is sustained by the power P.

Now, it is evident that the two ropes, F, G, support the whole weight, W, the cord, F, supporting one half, and G the other half; consequently, whatever holds the upper end of either rope, sustains half the weight.

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* *Hamilton's* Four Introductory Lectures.

If I take hold of the cord, F, and pull upwards, I only feel half the weight, the cord G, supporting the other half; if I put the rope, E, over the fixed pulley, this only changes the direction, and therefore in pulling the rope downwards I only feel half the weight; one ounce suspended from E will balance two ounces at H, and you will be enabled to lift twice as much weight by the assistance of a single moveable pulley as you could raise by your own actual strength.

The power, P, moves twice as fast as the pulley, D, with the weight W; for, to raise the weight through a space equal to HI, each of the ropes, F and G, must be shortened by a space equal to HI, that is, twice HI; and this quantity must be drawn over by the rope to which the power is affixed: therefore, the space described by the power will be equal to twice the space described by the weight, or the velocity of the weight is to that of the power as one to two. And you will readily observe, that when the weight has been raised two inches, or two of the silken lines which are fixed at the distance of one inch from each other, the power has descended four inches or four of the lines.

When the upper and fixed block contains two pullies, which only turn upon their axis; and the lower moveable block contains also two, which not only turn upon their axis, but rise with the block and weight, D, *plate* 3, *fig*. 4, or S, *plate* 5, *fig*. 3, the advantage gained is as four to one.

At E, *plate* 3, *fig*. 4, and R, *plate* 5, *fig*. 3, there are three pullies in the fixed block, and three in the moveable one, the number of cords six, and the power as six to one; the advantage is the same whether the pullies be placed parallel, or under each other.

A, *plate* 5, *fig*. 4, is a different construction of the pulley; it consists of two pullies, one fixed, the other moveable; they are in the shape of a fusee or truncated cone: each pulley has six grooves, which are concentric to each other, and whose diameters are in arithmetical progression, the difference being equal to the diameter of the smallest pulley.*

When the extremity of the cord is fixed to the block of the upper pulley, if S, L, H, represent the diameters of the grooves of the lower fusee, and C, B, A, the diameters of the upper fusee, S should be to C : L : B : H : A as 1 : 2 : 3 : 4 : 5 : 6. But if the extremity of the cord be attached to the block of the lower fusee, and D, C, B, A be the upper grooves, and S, L, H, the lower, then the following are the proportions, D : S : C : L : B : H : A :: 1 : 2 : 3 : 4 : 5 : 6 : 7. The diameter of the rope is here included in the diameter of the groove.

The advantage gained here is as 12 to 1.

There are some systems of pullies, where the rule I have laid down does not take place ; but as they are seldom used, I shall only mention one, in which a separate string goes round each pulley ; one end thereof is fixed, while the other is joined to the moveable pulley next above it. Here there will be an equilibrium, when the power is to the weight as unity to that power of two, of which the index is the number of moveable pullies. See F, *plate* 3, *fig*. 4.

The weight is fastened to the lower pulley. The pullies do not here, as in the preceding systems rise together in one block with the weight, but act upon one another, so that each pulley doubles the power.

A power of one ounce will be in equilibrio with two ounces at A, with four at B, with eight at C, and with sixteen at D ; and the velocity of the power will be sixteen times that of the weight. Though this system be very powerful, yet as it raises weights very slowly, and takes up a considerable space, it is very seldom used.†

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* Camus.

† When the strings of a pulley do not draw parallel to each other, the power is to the weight, as radius to twice the co-sine of the angle which either string makes with the direction in which the weight acts.---E. EDIT.

OF THE INCLINED PLANE.

The inclined plane is generally reckoned among the mechanic powers, being used with advantage in raising weights by diminishing the weights laid thereon.

When hogsheads or pipes of wine, &c. are to be let down into a cellar, or brought up out of it, a plank is laid along the stairs, which in that case is an inclined plane, the only mechanical power that can be readily applied. So also, in making reservoirs for water, in gardening, in raising fortifications, &c. inaccessible by carts, inclined planes made of wood serve for the wheelbarrows to run on, to remove the earth from a lower to a higher place.

I have already so far explained to you the properties of the inclined plane, that I need do little more, in this place, than to show you, that the less the plane is inclined to the horizon, the easier a body may be rolled or forced up its surface.

If it were required to raise a heavy body to the height, CB, of the plane, *plate* 4, *fig*. 6, and perpendicular to the horizon, it is evident you must employ a power equal to that of the weight, and it would even then be very inconvenient ; but if an inclined plane, AB, be elevated to the height B, where the weight is to be raised, a less power than the weight will serve the purpose.

The weight is always most easily drawn or pushed in a line W w M, parallel to the plane, and passing through the centre of the weight : for if one end of the line be fixed at W, and the other end inclined towards B, the body would be drawn against the plane, and the power must be increased in proportion to the greater difficulty of the traction : if the line be carried above M, the power must be also increased ; but here only in proportion as it endeavours to lift the body off the plane.

As the power acts to the greatest advantage when the line of traction is parallel to the inclined plane, what I shall mention concerning this plane will principally apply to the line when in this situation. When so situate, there will be an equilibrium, when the power is to the weight as the height of the plane is to its base. In other words, the mechanical advantage is in proportion as the length of the plane exceeds its height.

Thus, if a weight of four ounces be laid on the plane RS, *plate* 5, *fig.* 2, whose length is to its perpendicular height as 2 to 1, it will be counterbalanced by a weight of two ounces, drawing the other in a direction parallel to the plane, by means of a pulley properly fixed at the end thereof. If the length of the plane were to its height as 4 to 1, it would be sustained by one ounce, hanging freely, as in the former instance.

Hence, in drawing a cart or waggon up hill, if the power of the horses be proportioned to the weight of the waggon, as the height of the hill is to its side, the waggon will not run back, and a little additional power will draw it up hill.

OF THE WEDGE.

The wedge is the next mechanical power. Those among you, who have seen men cleave wood, cannot be at a loss to know, that the wedge is a piece of wood or iron, thin at one end and thick at the back. The thin end is applied to the timber to be cleft, and the thick end struck upon by a hammer.

Mathematicians have long debated concerning the true theory of the wedge. *Hamilton*, *s'Gravesand*, *Desaguliers*, &c. assert, that the power is to the resistance, as half the back to the height. *Keil*, *Whiston*, *Nicholson*, &c. say, that it is as the whole of the back to the height.

There are many natural obstructions to a complete theory of the wedge: among these are to be reckoned the friction, the elasticity, and tenacity of the materials. Indeed the theory thereof will probably remain for a long time a matter of useless speculation, since it is connected with a part of physical knowledge of which we are at present wholly ignorant; I mean the nature and force of adhesion in bodies, their tenacity, and the flexibility of the fibres. All these circumstances, with many others, show, that the experiments where pressure is used as a power, cannot be adduced as illustrating the theory of this machine: for a wedge, when laden with a vast weight, will have scarce any effect in cleaving wood: whereas a blow upon the head of the wedge, will drive it easily into a hard body. *Percussion* puts all the parts of the wood into a tremulous motion; which by disuniting them, lessens the stickage, and thereby facilitates the motion of the wedge. It has been also thought, that the motion communicated to the materials may, in some degree, contribute to the force of the wedge.

I should therefore say no more on the subject of the wedge, if it were not necessary to explain to you that part of the mechanical apparatus, which is constructed to prove the effects of the wedge. You must, however, perceive, from what I have just now mentioned, that this part of the apparatus cannot be adequate to the establishing of any theory. Experiments made therewith will show the power of the wedge according to the hypothesis of the maker, but will bear very little semblance to the natural action thereof.

The two pieces of brass EH and GH, *plate* 5, *fig*. 4, are joined at H, by a hinge, and are drawn together by the threads h i, f g, going over the pullies i and g, having weights at the lower end of the cords. If a light wedge be drawn into this frame, by means of the weight L, hanging at the cord D e D, fastened to the wedge, and going over the pulley e; if the weight, L, be in the same proportion to the two weights R, I, as the half back of the wedge is to either of its sides, the weight which acts as a power for making the wedge go into the frame will balance the weight which pulls the side of the cleft, as a resistance against the sides of the wedge.

Thus, sixteen ounces on each side will balance, or be equal to eight ounces of power at the back of the wedge, when the back is equal to the sides.

But when the back is only half the length of one of the sides, four ounces of power will be equal to sixteen ounces of resistance on each side.

Plate 4, *fig*. 5, is another apparatus for illustrating the nature of the wedge. IKLM, and LMNO, are two flat pieces of brass joined together by a hinge at LM; P is a graduated arc of brass, on which the said pieces of brass may be opened to any angle not exceeding sixty degrees, and then fixed at the given angle by means of the two screws a, b; IKNO represents the back of the wedge, and IKLM, LMNO, the two sides of the wedge against which the wood acts in cleaving. By means of the arc P, the wedge may be so opened, as to adjust the thickness of the back in any proportion to the length of either of its sides, but not to exceed that length; and any weight, as P, may be hung to the wedge from the hook at the bottom thereof, which weight, together with that of the wedge, is the impelling power. AB, and CD, are two cylinders, to the edges whereof two flat plates are fitted, to prevent the wedge from slipping off edgewise from between them. A small cord goes over the pivot at the end of each cylinder; the cords, S, T, belonging to the cylinder AB, go over the fixed pullies W and X, and are fastened at the other end of the bar *w x*, on which any weight, Z, may be hung. In like manner the cords R, Q, from the cylinder DC, go over the fixed pullies u v, to the bar *v u*, from which a weight Y, equal to Z, may be hung. These weights, by drawing the cylinders towards one another, are to be considered as the resistance of the wood acting equally against the opposite sides of the wedge. The cylinders move on two bars, which are supported by the pillars E, F, G, H, when the back of the wedge is of the same length as either of the sides. The wedge, with a weight P, sufficient to make it equal to forty ounces, will be in equilibrio against forty ounces hanging at the lines belonging to each cylinder. If the back of the wedge be only half the length of the sides, it will be in equilibrio with eighty ounces at each cylnder.

To the wedge are referred the axe or hatchet, the spade, chissels, needles, &c. in a word, all kinds of instruments which, beginning from edges or points, grow gradually thicker. A saw is a number of chissels fixed in a line; a knife may be considered as a chissel, when employed in splitting; but if attention be paid to the edge, it is found to be a fine saw, as is clear from its producing a much greater effect by drawing a stroke, than would have followed from a direct action of the edge.

One considerable use of the wedge, is to raise up the beam of a house, to underprop it, when a floor gives way, by reason of two great a burden laid upon it; and so much force may be applied in this way, that some thousand tons may be raised together with the floor, and all secured by means of this small machine, which for this purpose is more convenient than either the screw or the lever.*

OF THE SCREW.

The mechanic power next for our consideration, is the screw; a machine of great efficacy in raising weights, or pressing bodies close together.

The screw consists of two parts; the first is called the male or outside screw, being cut in such manner, as to have a prominent part going round the cylinder in a spiral manner, which prominent part is called the thread of the screw; the other part, which is called the female, or inside screw, is a solid body, containing a hollow cylinder, whose concave surface is cut in the same manner as the convex surface of the male screw, so that the prominent parts of the one may fit the concave parts of the other.

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* There is no mechanical power without a theory, the wedge is, therefore, subject to mathematical investigation, although the various bodies that it is used to sever, may resist its sides in such different directions, as to vary its law of action at every blow.

The following is the description of an apparatus to illustrate its theory, on the neatest and most simple principle that I have ever seen, and we have made it from the directions as communicated to us by the Rev. Mr. *Vince*, of Cambridge.

Plate 4, *fig*. 8, is a representation of it as in use. A B C, is a brass frame or wedge, with the back, A B, made in any desired proportion to the length of one of the sides, A C. The two ends of a fine silk line, *a a*, are fixed to the sides, and the line turns over two small brass pullies, with the weight, D, suspended to it. On a hook at the vertex C, of the wedge the power, P, is suspended. The line *a a*, acts perpendicularly to the sides of the wedge, and represents the external resisting power: the weight of the wedge, with that of the power, P, together, represent the whole power. Suppose the two sides, A C, B C, to be six inches each in length, the back, A B, four inches, and the power P, with the wedge, two ounces, it will suspend a weight, D, representing the resisting body of six ounces, proving that *the power is to the weight as the back to the sum of the sides*.---E. EDIT.

One part is commonly fixed, whilst the other is turned round; and in each revolution the moveable part is carried, in the direction of the cylinder, through a space equal in length to the interval between two contiguous threads; whereby the body to be moved is carried through a space equal to that interval, which therefore expresses the velocity of the weight or resistance, while the circumference, which is described by the power whereby the moveable part of the screw is turned round, expresses the velocity of the power.

You may conceive how the screw is made, by cutting a piece of paper into the form of an inclined plane, and then wrapping it round a cylinder. The inclined plane, rising round the cylinder in a spiral manner, forms what is called the threads of the screw. Now it is evident, that the screw must be turned once round, before the resistance can be moved from one spiral winding to another.

A screw is seldom used without the application of a leaver to assist in turning it; consequently, as much as the circumference of a circle, described by this handle or winch, is greater than the intervals or distance between the spirals, so much is the force of the screw.

Thus, supposing the distance between the spirals to be half an inch, and the length of the winch to be twelve inches, the circle described by the handle of the winch, where the power acts, is nearly 76 inches, or 152 half inches, and consequently 152 times as great as the distances between the spirals; and therefore a power at the handle where the power acts, of no more than one pound, will balance 152 pounds acting against the screw; and as much additional power as is sufficient to overcome the friction, will raise the 152 pounds.

There is, therefore, an equilibrium on the screw, when the power is to the weight, as the distance between two contiguous threads is to the circumference described by the power.

A very considerable degree of friction always acts against the force of the power in a screw, but this is fully compensated by other advantages; for on this account the screw continues to sustain a weight even after the power is removed, or ceases to act, and presses the body against which it is driven. Hence the screw will sustain very great weights ; so that several screws, properly applied, would support a large building, while the foundation was mending, or renewed.

OF THE ENDLESS SCREW.

Sometimes a screw is applied to a wheel, so as to serve as a pinion ; it is then called an endless screw, being one that turns on an axis, and may be turned perpetually. The teeth, or threads, of this screw fit exactly into the teeth of a wheel, which are cut obliquely to answer thereto ; so that, as the axis turns round, the threads of the screw take hold of the teeth of the wheel, and, by turning it round, raises the weight.

The small apparatus before you, see *plate* 5, *fig.* 3, which is furnished with an endless screw, will be sufficient to illustrate the nature of its force, or mechanical advantage.

The wheel, C, has a screw, on its axis, working in the teeth of the wheel D, which suppose to be 48. It is plain, that for every time the wheel C, and screw, are turned round, the wheel, D, will be moved one tooth by the screw, and therefore, in 48 revolutions of the screw, the wheel will have turned once round.

The diameter of the groove in the circle C, is equal to the diameter of the groove round the wheel D ; but the velocity of the wheel C, is 48 times the velocity of the weel D. Consequently, if a line goes round the groove of the equal wheel F, and has a weight of 48 oz. suspended from its lower point, it will be balanced by a weight of one ounce suspended from a string going round the groove of the wheel C.

If the line, instead of going round the groove of the wheel F or D, goes round its axis, the power of the machine will be as much increased as the circumference of the groove exceeds the circumference of the axis ; which, if it were six times, one ounce, at C, would balance six times 48, or 288 ounces suspended from the axis of the wheel C ; so that the power or advantage gained is 288 to 1. That is to say, a man, who by his natural strength could lift a hundred weight, would be able to raise 288 hundred, or $14\frac{8}{20}$ ton weight by this engine.

The teeth of the wheel must be cut obliquely to answer the threads of the screw, supposing it to lie in the plane of the wheel, and therefore the wheel will be acted on obliquely by the screw. To remedy this, the screw may be placed obliquely to the wheel, in such a position, that when the teeth are cut straight or perpendicular to its plane, the teeth of the screw may coincide with and fit them, and the force be directed along the plane of the wheel.

At *plate* 4, *fig.* 3, is represented another apparatus for showing the power of the screw. Here the transverse arm is placed over the beam; at KLE is a screw, M the female screw; the upper part of the axis of the screw passes through a hole, which prevents the screw from any irregular motion; G is a wheel attached to the screw; a string is wound round this wheel, and goes off on that side of the wheel which is next to the transverse arm, and so as to be a tangent to, and go over the pulley H, at the other end of the tranverse arm, and hanging down, any weight, as a power to force down the screw, may be suspended. The point of the screw presses upon one end of the beam, while the resistance, or counterpoise, is placed at the other.

The screw is of excellent use, not only in itself, for raising great weights, and other purposes, but in the construction of several sorts of compound engines.

The screw, in a moving body, acts like an inclined plane. For it is just the same as if an inclined plane were forced under a body to raise it; the body being prevented from flying back, and the base of the plane being driven parallel to the horizon.

The use of this power is very great. It is of great service for fixing several things together by the help of screws; it is likewise very useful for squeezing or pressing things close together, or breaking them; also for raising or moving large bodies. The screw is used in presses for wine, oil, or for squeezing the juice out of any fruit. The very friction of this machine has its particular use; for when a weight is raised to any height, if the power be taken away, the screw will retain its position, and hinder the weight from descending again by its friction, without any other power to sustain it.

In the common screw, such as is here supposed, the threads are all one continued spiral, from one end to the other; but where there are two or more spirals, independent of one another, as in the worm of a jack, you must measure between thread and thread of the same spiral, in computing the power.

Of a micrometer screw. The screw is applied to so many uses, that it would be impossible to enumerate them all. Among other purposes, it is applied with great advantage for measuring or subdividing small spaces; when used for this purpose, it is called a micrometer.

For this purpose, a screw is so fitted to an instrument, that it may be turned round, or moved on its axis, without moving backwards or forwards lengthwise. A circular plate is fitted upon, and at right angle to the screw. There is a female screw fitting on the other, which carries a piece with a line to mark out the divisions it passes over on the bar.

Suppose the screw to have fifty turns or threads in an inch, the edge of the plate is divided into a hundred equal parts. Suppose the division on the plate of the female screw to be situate between two of the divisions on the bar, by the micrometer its distance from either is easily discovered.

To effect this, turn the screw till the line on the female screw exactly coincide with one of the divisions, and the number of turns on the screw, in producing the coincidence, will give the fiftieth parts of an inch, and the odd divisions on the plate will show the number of hundredth parts of one turn; that is to say, the hundredth parts of one fiftieth of an inch, or the five thousandth part of an inch.

A GENERAL REMARK.

From what has been shown, and proved to you, it follows, that the machanic powers depend on one easy principle; a principle by which you may compute the power not only of simple, but of compound engines.

If we consider bodies in motion, and compare them together, we may do it either with respect to the quantities of matter they contain, or the velocities with which they are moved. The heavier any body is, the greater is the power required, either to move or stop its motion. And again, the swifter it moves, the greater is its force. So that the whole momentum, or quantity of force, in a moving body, is the result of its quantity of matter, multiplied by the velocity with which it is moved; and when the products arising from the multiplication of the particular quantities of matter, in any two bodies, by their respective velocities, are equal, the moments or entire forces are so too.

Example. Suppose A to weight forty pounds, and to move at the rate of two miles in a minute; and B to weigh only four pounds, and to move twenty miles in a minute; the entire forces with which these two bodies will strike against any obstacle will be equal to each other, and it would, therefore, require equal powers to stop them; for, 40 multiplied by 2 gives 80, the same as is produced by multiplying 4 by 20.

This principle extends through the whole of mechanics. When two bodies are suspended by any machine, so as to act contrary to each other, if the machine be put in motion, and the perpendicular ascent of one body, multiplied into its weight, be equal to the perpendicular descent of the other body, multiplied into its weight, these bodies, however unequal in their weights, will balance one another in all situations; for, as the whole ascent of the one is performed in the same time as the whole descent of the other, their respective velocities must be directly as the spaces they move through; and the excess of weight in one body is compensated by the excess of velocity in the other: so that you may compute the power of any mechanical engine, by finding how much swifter the power moves than the weight; that is, how much farther in the same time; and just so much is the power increased by the help of the engine.

THE PROPERTIES OF THE LEVER APPLIED TO INVESTIGATE THE STRENGTH OF BODIES.*

When the two arms of a lever are not in a right line, but contain an invariable angle, as at C, *plate* 1, *fig*. 16, the law of equilibrium is the same as I have already explained; that is, if the power, P, be applied to the arm, C B, and the weight, W, by means of a pulley in the perpendicular direction A M, on the arm C A, the power and weight will sustain each other, if P be to W as C A to C B.

If several powers act upon the arm C A, find their centre of gravity, A, on the arm, C A, and suppose all the powers to be united there; and if the power, P, be to their sum as C A to C B, it will sustain them.

The sum of the powers being given, it is manifest, that the farther their centre of gravity is removed from their centre of motion, C, the greater the resistance they will oppose against the power P, and it will require the greater force in the power to overcome them.

From this *Galileo* justly concludes, that the bones of animals are stronger for being hollow. Their weight being given, if C B F represent their length, the circle C H D a section perpendicular to that length, and P any power applied along their length, tending to break them; then the strength or force of all their longitudinal fibres, by which the adhesion of the parts is preserved, may be conceived to be united in A, the centre of gravity of the circle C H D, which is the common centre of gravity of those forces, whether the section be a circle or a ring. But it is plain, that the distance, C A, of the centre of gravity is greater when the section is a ring, than when it is a circle without any cavity; consequently, the power with which the parts adhere, and which resist against P, that endeavours to separate them, is greater in the same proportion.

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* *Maclaurin's* Sir *Isaac Newton's* Philosophical Discoveries, p. 168. *Emerson's* Mechanics.

For the same reason, the stalks of corn, the feathers of fowls, and hollow spears, are less liable to accidents that tend to break them, than if they were of the same weight and length, but solid without cavity.

In this instance, we find, therefore, that art only imitates the wisdom shown in nature.

In similar bodies, engines, or animals, the greater are more liable to accidents than the smaller, and have a less relative strength; that is, the greater have not a strength in proportion to their magnitude. A great column, for example, is in much more danger of being broken by a fall than a similar small one. A man is in greater danger from accidents of this kind than a child; an insect can bear a weight many times greater than itself; whereas a large animal, as a horse, can hardly bear a burden equal to his weight.

To account for this, it is sufficient to show, that in similar bodies of the same texture, the force which tends to break them, or to make them liable to hurtful accidents, increases in the greater bodies in a higher proportion than the force which tends to preserve them entire or secure against such accidents.

Thus let us suppose these cylindric beams ABDE, FGHK, *plate* 1, *fig.* 18, fixed in the immoveable wall, I L, and let us consider them abstract, in the present instance, from any other force that may tend to break them, besides their own weight. Bisect A B in C, and F G in M, and their weights may be conceived to be accumulated at the points C and M, which are directly under the centre of gravity. To facilitate the computation, let A B be equal 2 F G; consequently, the weight of the beam, A B D E, will be eight times greater than the weight of the similar beam, F G H K; and the weight of the former being conceived to be accumulated in C, and that of the latter in M, and A C being double the distance of F M, it follows, that the force which tends to break the former at A, being eight times greater than that which tends to break the latter at F, and at the same time acting at a double distance, it must on both these accounts exert an effort sixteen times greater than that of the latter.

Now, to compare the forces which tend to preserve those beams entire and fixed in the wall, let A R E be a section of the greater beam, and I S K a section of the smaller one, perpendicular to their lengths at the points A and F. Bisect A E in *p*, and F K in q; then the number of the longitudinal fibres, whose adhesion tends to preserve the beams entire, or rather the quantity of this adhesion in the greater beam, will be to the quantity of adhesion in the smaller beam, as the area of the section A R F to the area of the section F S K; that is, in the present case, because the figures are similar, as the square of A E to the square of F K, or as 4 to 1. But the adhesion of the parts that are in contact with each other in the section A R E, may be conceived to be accumulated at p, their centre of gravity; and the adhesion of the parts in contact with each other, in the section F S K, is for the same reason to be conceived as accumulated in q. The adhesion, therefore, which tends to preserve the greater beam entire, is quadruple that which tends to preserve the smaller, and at the same time is to be conceived as acting at a double distance from the centre of motion, because Ap is equal to 2 F q; so that the effort which preserves the greater beam from breaking, is eight times greater than that which tends to preserve the smaller beam.

But we have found that the effort which tends to break the greater beam at A, is sixteen times greater than that which tends to break the smaller beam at F; but that the effort, on the other hand, which tends to preserve the adhesion of the greater beam entire, is only eight times greater than that which tends to preserve the adhesion in the smaller beam.

In general it will be found, in the same manner, that the efforts tending to destroy the adhesion of beams from their gravity only, increase in the quadruplicate ratio of their lengths; but that the opposite efforts, tending to preserve their adhesion, increase only in the triplicate ratio of the same lengths. From whence it follows, that the greater beams must be in greater danger of breaking than the smaller similar ones; and that though a smaller beam may be firm and secure, yet ae greater similar one may be made so long as necessarily to break by its own weight.

Hence *Galileo* justly concludes, that what may appear very firm, and succeed very well in models, may be very weak and infirm, or even fall to pieces by its weight, when it comes to be executed on large dimensions according to the models.

From the same principle, we may also infer, that there are necessarily limits in the works of nature and art, which they cannot surpass in magnitude. Thus, were trees of a very enormous size, their branches would fall by their own weight. Large animals have not strength in proportion to their size; and if there were any land animals much larger than those we know, they could hardly move, and would be perpetually subjected to most dangerous accidents. As to the animals of the sea, the case is different, as the gravity of the water in a great measure sustains these animals. Nor does it avail to advance against this, that bones have been found, which were supposed to have belonged to giants of an immense size, such as the skeletons mentioned by *Strabo* and *Pliny*, of which one was 60, the other 46 cubits high; for, natural historians from just grounds have concluded, that in some cases the bones were those of elephants, and in others those of whales, brought to the places where they were found, by some of those revolutions that have happened in past ages.

From what has been said, it clearly follows, that to make bodies, engines, or animals of equal relative strength, the greater ones must have grosser proportions. Thus, in order that the great cylinder A B D E, *plate* 1, *fig.* 16, may be as firm and secure against accidents as the small cylinder, F G H K, the section, A R E, and its diameter, A E, must be increased, till the effort arising from the adhesion of the parts bears as great proportion to the effort that tends to overcome this adhesion in the greater as in the smaller cylinder: and this sentiment being suggested to us by continual experience, we naturally join the idea of greater strength and force with the grosser proportions, and of agility with the more delicate ones.

In architecture, where the appearance of solidity is no less regarded than real firmness and strength, this is particularly considered, in order to satisfy a judicious eye and taste, the various orders of the columns serving to suggest different ideas of strength. But by the same principle, if we should suppose animals vastly large, from the gross proportions a heaviness and unwieldiness would arise, which would make them useless to themselves, and disagreeable to the eye. In this, as in all other cases, whatever generally pleases taste, not vitiated by education, or by fabulous and marvellous relations, may be traced till it appear to have a just foundation in nature; though the force of habits is so strong, and their effects upon our sentiments so quick and sudden, that it is often no easy matter to trace by reflexion the grounds of what we find pleasing.

If several pieces of timber be applied to any mechanical use where strength is required; not only the parts of the same piece, but the several pieces in regard to one another, ought to be so adjusted for size, that the strength may be always proportional to the stress they are to endure.

This proposition is the foundation of all good mechanism, and ought to be regarded in all sorts of tools and instruments we work with, as well as in the several parts of any engine. For who that is wise will overload himself with his work tools, or make them bigger and heavier than the work requires? Neither ought they to be so slender as not to be able to perform their office. In all engines, it must be considered what weight every beam is to carry, and proportion the strength accordingly. All levers must be made strongest at the place where they are strained the most; in levers of the first kind, they must be strongest at the support; in those of the second kind, at the weight; in those of the third kind, at the power, and diminish proportionally from that point. The axles of wheels and pullies, the teeth of wheels, which bear greater weights, or act with greater force, must be made stronger; and those lighter that have light work to do. Ropes must be so much stronger or weaker, as they have more or less tension; and in general, all the parts of a machine must have such a degree of strength as to be able to perform their office and no more. For, an excess of strength in any part does no good, but adds unnecessary weight to the machine, which clogs and retards its motion, and makes it languid and dead; and on the other hand, a defect of strength where it is wanted, will be a means to make the engine fail in that part, and go to ruin. So necessary it is to adjust the strength to the stress, that a good mechanic will never neglect it; but will contrive all the parts in due proportion, by which means they will last all alike, and the whole machine will be disposed to fail all at once. And this will ever distinguish a good mechanic from a bad one, who either makes some parts so defective, imperfect, and feeble, as to fail very soon; or makes others so strong or clumsey, as to outlast all the rest.

From this general rule it follows, that *in several pieces of timber of the same sort, or in different parts of the same piece, the breadth multiplied by the square of the depth, must be as the length multiplied by the weight to be borne;* for then the strength will be as the stress.

The breadth multiplied by the square of the depth, and divided by the product of the length and weight, must be the same in all.

One observation more, before I quit the doctrine of equilibrium or science of statics, which is to inform you that the writings of the Platonists and Pythagoreans cannot be properly understood without a knowledge thereof. Their allusions to arithmetic, geometry, and statics, are very numerous; arithmetic they considered as the science of mind; geometry, as the science of ethics or moral virtue; and under statics they couched judiciary and political knowledge.

They expressed their notions of the *principles* of all things in numbers; and *measure*, a geometrical term, was applied to moral action and virtue, as setting *due bounds* to the passions of the soul, marking out the lines of every duty, and ascertaining the exact medium between the too little and the too much, in every affection, energy, and action. Mankind in general have agreed with these philosophers to use metaphors taken from statics, to express their opinions of the different degrees of *merit* and *demerit*, the subject of political justice. Thus the terms *heavy* and *light* are every where applied to *crimes* and *punishments*. Thus likewise an important service to the state is termed *weighty* and *momentous*, whilst all little ones are termed *light* and of *small moment*. Hence, in all ages the *balance* has been acknowledged as a fit emblem of *distributive* or *civil* justice, and the *rectangular rule* or *plumb-line* has been understood to be a symbol or mark aptly representative of *communicative* justice. And the worth or *weight* of each part of the political system is to be estimated by the greater or less importance or *moment* of it, to the preservation of the system; and by its greater or less propensity to adhere to the centre of all civil union, the laws of the civil constitution.

OF COMPOUND ENGINES.

You have, no doubt, been often astonished at the vast variety of composition of which seven simple notes are susceptible, and have contemplated with immense pleasure the great quantity of words formed from the twenty-four letters of the alphabet, and the infinity of numbers produced from unity. In the same manner, from the simple machines I have described to you, compounded ones are formed by various combinations, so as to serve for different purposes, and yet the same general law takes place that I have already described to you, namely, that the power and weight sustain each other when they are in the inverse proportion of the velocities they would have in the direction wherein they act, if they were put in motion.

The mechanical powers, according to their different structure, serve for different purposes; and it is the business of the skilful mechanic to choose or combine them in the manner that may be best adapted to produce the effect required. Besides raising of weights and overcoming resistances, one of the most useful problems in this science, is that of making a regular movement that may serve to measure time with accuracy. It may, however, be proper to observe to you here, that machines seldom owe their origin or improvement to considerations deduced from the laws of motion; they are derived from other sources. It is from long experience of repeated trials, errors, deliberations, corrections, &c. continued throughout the lives of individuals, and by successive generations of them, that the practical sciences derive their gradual advancement from aukward beginnings to their most perfect state of excellence.

To be a good mechanist requires the labour of a whole life; it is an art rather perfected by practice than theory; the principles of mechanism may be learned in books, but the art must be acquired by experience.

The knowledge of machines, in a theoretical view, consists in knowing how to employ a given power, so as to produce a required effect. To this end, it is necessary in the first instance to know accurately the nature and value of the power to be employed, and then to consider the means and manner of applying this power to the bodies to be put in motion, so as to obtain most advantageously the desired effect.

If this effect can be obtained by simple machines, it is useless to employ those that are compound; if a simple one be not sufficient to answer the purpose, you must then have recourse to those that are compounded.

TO COMPUTE THE POWERS OF COMPOUND ENGINES.

As notwithstanding the combination of simple machines they still preserve their properties, the principles I have already laid down will be found sufficient for computing the effects. 1. To discover the mechanical power of any engine it will be sufficient to measure the spaces described in the same time by the power and the resistance; for the power always balances the weight, when it is in the same proportion as the velocity of the weight to the velocity of the power. Or, 2. By setting down the ratios of the power to the weight on each mechanical power, the sum of these will be the ratio of the power to the weight; for when the ratio of the power to the weight is equal to the sum of the ratios expressing the power and weight, there will be an equilibrium on each mechanic power. Or,

1. Divide the machine into all the simple ones of which it is formed; then begin at the power and call it one, and by the properties of the mechanical powers find the forces, in numbers, which the first simple machine exercises upon the second. Call this force one, and find the force in numbers with which it acts upon the third; and putting this force as one, discover its action on the fourth in numbers, and so on to the last. Then multiplying all these numbers together, and the product will be the force of the machine, supposing the first power one.

2. *For wheel work*, take the product of the number of teeth in all the wheels that act upon or drive others, for the power, and the product of the teeth in all the wheels moved by them, for the weight: or, instead of the teeth, take the diameter. Otherwise thus:

In wheel work, there are always two wheels fixed upon one axis, or else one wheel and a pinion, or a barrel which supplies the place of a wheel.

Of these, call that wheel the leader, which is acted upon by the power, or some other wheel; and call the other, which is upon the same axis, the follower, driving another wheel forward. The leader receives the motion, the follower gives it.

Then, having the number of teeth, or the diameter of each, take the product of all the leaders for the weight, and the product of all the followers for the power.

The product of all the leaders gives also the velocity of the power; the product of all the followers, the velocity of the weight.

The principles being clear, it will be superfluous to enter into the examination of a great number of machines; a few will be sufficient to render you masters of the subject.

OF SMEATON'S PULLIES.

I have already noticed to you some combinations of pullies. The construction of this mechanical engine is so simple, that it is of little importance whether it be considered under the head of simple or compound engines. Some variations in the construction thereof have been already noticed: I shall now lay before you the construction of a tackle of pullies, contrived by the late excellent mechanic Mr. *Smeaton*, and which is not so much known as it deserves to be.

The pullies are in general arranged in their blocks, by being placed one by the side of the other on their pin, or one directly under the other upon separate pins; but in either of these modes an inconvenience arises where more than three pullies are framed in one block.

If, according to the first method, three pullies be placed by the side of another, as the last line by which the draught is made (or the fall of the tackle, as it is commonly called), must necessarily be upon the outside pulley, the difference of their friction will give it so great a tendency to draw the block awry, that as much will be lost by the rubbing of the pullies against the block on account of its obliquity, as will be got by increasing the number of lines.

The second method is free from this objection; but, as the length of the two blocks, taken together, must be equal to the sum of the diameters of the six pullies, besides the intervening spaces for the ropes, and the necessary appendages of the framing, they would run out into such an inconvenient length, as to deduct very considerably from the height to which the weight might otherwise have been raised.

Hence it is evident, that no very great purchase can be obtained by the common tackles of pullies alone; to increase its power, a second tackle is sometimes fixed upon the fall of the first, but here the height to which the weight might have been raised by the first will be less, in the same proportion, as the purchase is increased by the second.

These impediments are obviated by Mr. *Smeaton's* contrivance; the pullies are here, *plate* 5, *fig.* 6, placed in each block in two tiers, several being placed upon the same pin, as in the first method; every one having another under it, as in the second.

In this method all the lines are clear of one another, and the blocks are kept parallel. The model before you consists of twenty pullies, five on each pin; with this model you may raise six hundred weight;* but with a tackle of this sort, properly executed in large, one man may easily raise a ton. Twenty is the largest number of pullies that answers well in practice.

The pullies are placed in two tiers in each block, several being upon the same pin as in the first method, and every one having another under it, in the second method; when the tackle is in use, the two tiers, that are the most remote from each other, are so much larger in diameter than those that are nearest, as to allow the lines of the former to go over the lines of the latter without rubbing.

This construction gives a new method of reeving the line upon the sheeves; for here, let the number of sheeves be what they may, the fall of the tackle will always be upon the middle sheeve, or that next the middle, according as the number of pullies on each pin is odd or even.

To reeve it, let the line be fixed to some convenient part of the upper block, and brought round the middle sheeve of the large tier of the under block; from thence round one of the same sort next to the middle one of the upper block; and so on, till the line comes to the outside sheeve, where the last line of the larger sheeve falls upon the first sheeve of the smaller, and being reeved round those till it comes at the opposite side, the line from the last sheeve of the smaller tier rises again to the first of the larger, whence it is conducted round till it ends on the middle sheeve of the upper block on the larger tier.

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* A large tackle was tried on board one of his Majesty's ships, and though the rope was new, one man raised a ship's gun and carriage of 2700 weight, there being a person to hold on to prevent the rope from slipping.

OF A COMPOUND ENGINE.

Plate 5, *fig.* 1, represents a compound engine. The endless screw is turned round by the wheel A B, the screw takes into a wheel C D; on the axis of this wheel is a pinion E, that moves the wheel F G; to the axis, H, of this wheel the running rope of the pullies, I K, is fixed.

The diameter of the fly-wheel, A B, or lever, is twice that of the wheel C D; the wheel, C D, has 64 teeth, the pinion, E, on the axis of this wheel has 10 teeth, the wheel, F G, has 80 teeth.

To calculate the force of this engine, multiply the diameter 2, of the fly, by 64, the number of teeth in C D, which gives 128, and this product by 8, the number of times the pinion, E, moves round while the wheel, F G, moves once round, which gives 1024; multiply this by 3, because the diameter of the wheel, F G, is three times that of its arbor H, and you obtain 3072; which multiplied by 4, the advantage gained by the pullies, gives 12288 to 1 for the power of this engine, or 768 pounds to one ounce.

A B C D E F, *plate* 5, *fig.* 3, is a smaller compound engine; the wheel or fly, C, has a screw on its axis, which works into the teeth of the wheel D, having 48 teeth, and the diameters of these two wheels are equal; but the wheel, C, moves 48 times round, while the wheel, D, is moving once round; and the diameter of the wheel, D, is 6 times that of its arbor, on which the chord is coiled that raises the weight; consequently, the power is as 6 times 48 or 288 to one, or one ounce will support 288 ounces.

On the upper part of *fig.* 2, *plate* 5, and *fig.* 2, *plate* 4, is represented a compound engine, consisting of three levers acting one on the other; they are all of the first kind. In the first, R, the proportion is as 1 to 5; in the second, S, as 1 to 4; in the third, T, as 1 to 6; or as 1 to 5×4×6, equal to 120; consequently, if the power be to the weight as 1 to 120, they will counterpoise each other.

OF WHEEL WORK.

Wheels with teeth are used in such a variety of ways, that it will be necessary to explain in some measure their nature and action. Although the computation of the numbers is not a part of mechanics, yet it is useful in the construction of a variety of engines. From the nature of the wheel and axle it is clear that the power or force on the pinion is to that on the circumference of the wheel on the same axis, as the diameter of the wheel is to that of the pinion.

Whether a wheel drives a pinion, or a pinion drives a wheel, the number of turns of the wheel, multipled by the number of its teeth, is equal to the number of turns that the pinion makes in the same time multiplied by its teeth; so that the number of cotemporary turns of a wheel and pinion are reciprocally proportional to their number of teeth. Thus, if the number of teeth in a wheel be 80, and those in the pinion 8, then the pinion will turn round 10 times to once of the wheel; therefore, the quotient of the wheel, divided by the pinion it drives, is the ratio of turns to unity; thus, $\frac{80}{8}$ gives 10. Wheels and pinions are therefore generally expressed by fractions, the numerator being the wheel, the denominator the pinion.

Hence, if there be any number of wheels acting upon so many contiguous pinions, and you divide the product of the teeth in the wheels by those in the pinions, the quotient will be the number of turns of the last pinion, in one turn of the last wheel.

The number of turns may, therefore, be expressed by a fraction, whose numerator is the number of teeth in the leading wheel, multiplied by its number of revolutions, and the denominator the number of teeth in the wheel that is driven. Thus a pinion of 6 acting on a wheel of 52, will make it turn 3 times while it turns 26 times itself $\frac{6 \times 26}{52}$ equal 3.

If, therefore, you have a piece of wheel work, consisting of several wheels and pinions, *ex. gr.* if a wheel of 48 acts on a pinion of 8, on whose axis there is a wheel of 40 taking into a pinion of 6 carrying a wheel of 36, which moves a wheel of 36, on whose axis there is an index. The number of turns made by the index, while the first wheel goes once round, will be expressed by this fraction, $\frac{48}{8}\times\frac{40}{6}\times\frac{36}{6}=\frac{69120}{288}=240$.

Hence it is evident, that any number of teeth on the wheels and pinions, having the same ratio, will give the same number of revolutions to an axis at a given distance from the first, thus $\frac{64}{10}\times\frac{50}{8}\times\frac{36}{6}$ give 240 like the preceding combination; it is therefore left to the skill of the artist to determine what numbers will best suit the general design and circumstances of his machine.

Thus, the same motion may be performed either by one wheel and pinion, or many wheels and pinions, provided the number of turns of all the wheels bear the same proportion to all the pinions which that one wheel bears to its pinion.

These principles conduct us to a rule, whereby the number of wheels and pinions being given, to find the number of teeth to be cut on them, in order to make an index turn a certain number of times, while the wheel to which the moving power is applied only turns once round. Thus, suppose it were required to make the index turn 3600 times, by means of 4 wheels and 4 pinions.

Rule. Find all the component primes of the given number or prime numbers, which are divisors thereof; these you will find to be 2, 2, 2, 2, 3, 3, 5, 5.[*]

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[*] *To reduce a given Number to its component Primes.*

Divide the given number by 2 continually, as often as possible, without a remainder; and the result by 3, as often as possible; and then by 5, and so on by 7, 11, 13, &c. all the prime numbers, till there arise a quotient less than the divisor; and then the divisors and the last dividend are the primes composing the given number.

Example.

Reduce 122760 to its component primes.

122760	2.
61380	2.
30690	2.
15345	3.
5115	3.
1705	5.
341	11.
31	

Answer 2. 2. 2. 3. 3. 5. 11. 31.

2. Distribute these divisors into separate parcels in any order, to form as many parts as you have wheels to use: thus make four parcels of the given primes, 2, 2,—2, 3,—2, 5,—3, 5.— Thirdly, of the product of each of these parcels form so many numerators of fractions, to each of which let 1 be the denominator; thus, $\frac{4}{1}$, $\frac{6}{1}$, $\frac{10}{1}$, $\frac{15}{1}$. Fourthly, multiply the numerator and denominator of each fraction by any number that you design for the teeth of your pinions, thus, $\frac{4}{1}\times 7=\frac{28}{7}$,—$\frac{6}{1}\times 8=\frac{48}{8}$, $\frac{10}{1}6=\frac{60}{6}$, $\frac{15}{1}\times 5=\frac{75}{5}$. Lastly, form a compound fraction of the product of these fractions, and choose therefrom the numerators to which you will give for a pinion one of the denominators, thus, $\frac{28\times 48\times 60\times 75}{7\times 8\times 6\times 5}$, which you may distribute thus, $\frac{75}{8}$, $\frac{60}{7}$, $\frac{48}{6}$, $\frac{28}{5}$; or you might have arranged your parcels at first in a different manner, as $2\times 2\times 5$;—2×3;—2×3;—5.— Or, $\frac{20}{1}$, $\frac{6}{1}$, $\frac{6}{1}$, $\frac{5}{1}$, from whence you would have obtained another set of wheels producing the same result as the former.

By the same rule you may find what change you must make in the numbers of a wheel and pinion, in order to preserve the other wheels, and yet make the index perform a greater number of turns; thus, if you wished to alter the wheel work $\frac{28}{5}$, $\frac{48}{6}$, $\frac{60}{7}$, $\frac{75}{8}$, to another giving 4000 turns, preserving all the pieces but the wheel and pinion $\frac{48}{6}$: you will then find $\frac{28}{5}\times\frac{60}{7}\times\frac{75}{8}\times\frac{x}{y}$, $=4000$, consequently, $\frac{x}{y}=\frac{4000}{450}=\frac{80}{9}$, for the wheel and pinion to be substituted in the room of that of $\frac{48}{6}$.

In the operation, the division of 7 is omitted, because it would not measure 341; so also dividing by 13, 17, because they are no aliquot parts of 31.

If you multiply the numbers 2, 2, 2, 3, 3, 5, 11, 31 together in any order, the product will be the original number; and when there is a great number of these component primes to be multiplied, you may divide them into as many parcels as you please, and after having multiplied each parcel by itself, you will find that on multiplying these respective products together, you will still obtain the original number.

OF THE ADVANTAGES GAINED BY MACHINES.

I have shown you, in treating of simple machines, that as much as was gained in force so much was lost in time, so that the effect produced by a given power in a given time is always the same, whatever machine is used; you have also seen, that this proposition holds good in all compound engines, that is, in whatever proportion the power is less than the weight, in the same proportion will the weight be slower than the power.

In order, therefore, to obtain a just idea of the advantages gained by machines, suppose a man by a fixed pulley raises a beam to the top of a house in two minutes, it is clear, that he will be able to raise six beams in twelve minutes; but by means of a tackle, with three lower pullies, he will raise the six beams at once with the same ease as he before raised one, but then he will be six times as long about it, that is twelve minutes; thus the work was performed in the same time, whether the mechanical power was used or not. But the convenience gained by the power is very great, for if the six beams be joined in one, they may be raised by the tackle, though it would be impossible to move them by the unassisted strength of one man.

Consequently, if by any power you are able to raise a pound with a given velocity, it will be impossible by the help of any mechine whatever, with the same power, to raise two pounds with the same velocity; yet, by the assistance of a machine, you may raise two pounds with half that velocity, or even one thousand with the thousandth part of that velocity; but still there is no greater quantity of motion produced when a thousand pound is moved than when one pound, the one thousand pounds moving proportionably slower. The power of machines consists only in this, that by their means the velocity of the weight may be diminished at pleasure, so that with a given force any given resistance may be overcome.

The motion of the weight is not at all increased by any engine, its velocity is only thereby so much diminished, that the quantity of motion of the weight may not exceed the quantity of motion in the power.

By machines, a power, otherwise incapable of communicating motion to, or supporting the pressure of a body, may effect its purpose by transferring a part of the weight upon a fulcrum, distributing it amongst a number of pullies, or placing it upon an inclined plane or screw; and by this artifice a power may keep a weight suspended, which exceeds it in any assigned proportion, though without any acquistion of moment in a given direction; for motion is only communicable according to the established natural relations subsisting between matter and motion, and the magnitudes of two powers in equilibrio are always inversely as their velocities.

It is, therefore, in vain to think of moving a greater weight by a small power, and with the same velocity as with a greater power. No real gain of force is acquired by mechanical contrivance: on the contrary, force, from friction, &c. is always lost, the advantages obtained are confined to convenience; for instance, by machines we are enabled to give a convenient direction to the moving power, and to apply its action at some distance from the body to be moved, which is a circumstance of almost infinite importance. By machines we can also so modify the energy of the moving power, as to obtain effects which it would not produce without modification.

Mechanics do not teach us to make, but to apply powers, such as we find them in nature; for we deceive ourselves if we think, that by means of any engine one man shall do the work of two in the same time, supposing him to employ the same strength.

In the performance of several works, where we have sufficient strength, we often want time; and where we have time to spare we want strength. In these cases the mechanic has an opportunity of directing the application of the powers according to time.

Thus, in making harbours, carrying on dykes, moles, or banks, where at every tide the sea may damage the work, and a spring tide overset it, the greatest number of hands must be employed that can work one by another. In some cases, as raising great blocks of marble or other heavy goods out of a ship to lay them on a wharf, many hands cannot be employed; here then an engine may be used, where one man may do the work of ten or twenty men, but he shall be ten or twenty times longer in performing it. And the engine is absolutely necessary, because without it the work could not be performed; a sufficient time must therefore be employed, without which a great strength would be of no use. In draining mines you are generally confined to time, because the subterraneous springs supply the water while a force is employed to draw it out; here then the intensity of the power must be superior to the quantity to be raised in a certain time.

When great manufactures are carried on, mechanism may often be employed to great use; thus, suppose that in winding silk fifty men being employed, should only move a weight equal to half a pound, while they carry their hand round; whereas one man can easily raise twenty-five pounds with the same velocity as his hand moves round in this instance, and that for ten hours in a day: here a machine may be applied, whereby one man employing his whole strength shall do the work of fifty men in the same time. In other cases, machines may afford great profit by rendering effectual the force or intensity of the power or powers employed, as in the fire-engine, water-wheel, &c.

OF THE REGULATION OF MOTION BY FLY-WHEELS.

A uniform motion is continued by applying the heavy wheel, (or a cross bar loaded with equal weights called a fly) to the machine; this being made to revolve about its axis, keeps up the force of the power, and distributes it equally in all parts of its revolution, by going on at the same rate it makes the motion uniform; for, on account of its weight, a small variation in force does not sensibly alter its motion, while its friction and the resistance of the machine prevents it from accelerating.

When the motion of the machine slackens, it helps it forward; and when it tends to move too fast, it will keep it back.

Every regulating wheel should be fixed upon that axis where the motion is swiftest, and should be heavy when the motion is designed to be slow, and light where it is to be swift. In all cases, the centre of motion should coincide with the centre of gravity of the wheel. The axis may be either perpendicular or parallel to the horizon.

Most mechanical operations consist either in communicating motion to quiescent bodies, or in overcoming resistance; in which operations the actions of the moving force, and its effects, are not always cotemporary; but motion is frequently first generated, and at subsequent times, employed in producing the result. Thus great mechanical effects may be produced by means of small velocities, generated in ponderous bodies by inconsiderable moving forces. Suppose a heavy cylinder, of iron or lead, moveable about its axis, and in a vertical plane; a small force being applied to turn the cylinder, will, if long continued, generate such a force as will produce effects in raising weights by no means obtainable by the moving force immediately applied.

Mr. *Atwood* has shown, that a force of twenty pounds, applied for thirty-seven seconds to the circumference of a cylinder of ten feet radius, and weighing 4713 pounds, would, at the distance of one foot from the centre, give an impulse to a musket-ball equivalent to what it receives from a full charge of gunpowder.

The same effect would be produced in six minutes ten seconds, by a man turning the cylinder of a winch one foot long, on which he constantly exerted a force of twenty pounds.

This accumulation of mechanic force appears extraordinary at first sight, and may suggest wrong notions concerning the subject, unless fully considered; when it will appear, that the cylinder has no principle of motion in itself, and cannot have any more than it receives.

An accumulation of motion in ponderous wheels, is, as I have just observed, of great service when applied to practical purposes. Thus mechines to lift weights, to grind corn, &c. as well as many others, having a large ponderous wheel, termed a fly, fixed to some part of their machinery, so that it shall revolve round its axis, have been found to produce much greater mechanical effects than could have been produced without such an additional mass of matter.

Here, however, I must beg you to observe, that there is no increase of power communicated to any engine by flies, ballast-wheels, &c. they act upon quite another principle.

In all machines in which flies are used, either a considerably greater force must be applied than what is necessary to move the machine without it, or the fly must have been set in motion some time before it is applied to the machine. It is this superfluous power, which is, as it were, collected by the fly; and serves as a reservoir, by which the machine may be supplied when the effort of the power is lessened.

Suppose a wheel and axle, with a ponderous wheel affixed to it, moveable round an axis by means of a handle, which is impelled by muscular force. Now here the use of a ponderous wheel is evident; for, suppose a certain uniform motion generated therein, this will continue for some time to raise the weight, although the moving force of the arm be discontinued, which must in some degree happen when the arm is ascending. Now, if there were no motion in the ponderous wheel, to continue the ascent of the power, the resistance would begin to preponderate as soon as the moving power was at all diminished; from which it is manifest how much motion would be lost without the fly.

It is a reservoir to supply the machine when the animal slackens his efforts; and this is always the case with animals, for none are able to exert a great power with absolute constancy. When he begins to move the machine, he is vigorous, and exerts a great power; which not only overcomes the resistance of the machine, but communicates a considerable degree of power to the fly. Hence the machine, when moving, yields for a time to a small impulse; during which time the fly acts as a moving power, and the animal recovers his strength. By degrees, however, the motion of the machine decreases, and the animal is then obliged to renew his efforts. In this case, if it were not for the fly, the velocity of the machine would be considerably increased; but it now acts as a resisting power, and the greater part of the superfluous motion is lodged therein, so that the increase of velocity in the machine is scarcely perceptible; and the animal again acquires time to rest himself, till the machine requires an increase of impulse; and so on alternately.

It is the same with a machine moved with water, or by a weight; for though the strength of these does not exhaust itself like muscular force, yet the yielding of the parts of the mechine renders the impulse much less after it begins to move: hence its velocity is accelerated for some time, till the impulse becomes so small, as to require an increase of power to keep up the necessary motion; but when the impulse slackens, the fly communicates part of its motion, so as to render the motion equable. The truth of what has been advanced, will be rendered sensible by considering the inequality of motion in a clock, when the pendulum is off; and how very regularly it goes when regulated by the pendulum, which acts as a fly.

Flies are particularly useful in any kind of work which is done by alternate strokes; as the lifting of large pestles, pumping of water, &c. In this case, the weight of the wheel employed is a principal object; and it is calculated by a comparison with the weight to be raised.

Thus, suppose it were required to raise a pestle of thirty pounds weight, to the height of one foot, sixty times in a minute. Let the diameter of the fly be seven feet, and suppose the pestle to be raised one at every revolution of the fly, we must then consider what weight, passing through twenty-two feet in a second, will be equivalent to thirty pounds, moving through one foot in a second. This will be 30 divided by 22, or $1 \frac{4}{15}$

pounds. Were a fly of this kind to be applied, and the machine set a going, the fly would be able to lift the pestle once after the moving power was withdrawn; but by increasing the weight of the fly to ten, twelve, or twenty pounds, the machine, when left to itself, would make a considerable number of strokes, and be worked with much less labour than if no fly had been used. It is certain, however, that the fly does not communicate any absolute increase of motion to the machine; for if a man, or any animal, be not able to set any mechanical engine in motion without a fly, he will not be able to do it though a fly be applied, nor will he be able to keep it in motion, though set a going with a fly, by means of a greater power.*

LECTURE XXXII.

OF THE DIFFERENCE BETWEEN PRACTICE AND THEORY IN MECHANICS.

I HAVE, in what I have already said on these subjects, paid very little attention to the physical properties of the materials of which machines are composed, or of the alterations these properties occasion in their effects. The propositions that are demonstrated with the utmost mathematical rigour, are not found to answer in practice, and the difference can only be allowed for, and estimated by experimental investigation.

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* Encyclopedia Britannica, vol. x. part ii.

In establishing the theory, I have supposed that machines did not rub against each other, and so interrupt their usual workings; I have supposed that all the planes on which they moved were even, all the levers inflexible, and that the air gave no resistance; but this is not the case in practice; all these are impediments.

Whenever motion is communicated to a body, a certain resistance must have been overcome by the moving force. This resistance is of various kinds: 1. The inertia of the mass moved. 2. That of its weight, or other absolute force, opposed to the action of the moving power. 3. Obstacles by which the moving body is retarded in its progress. Many circumstances relative to the moving forces which are exerted on bodies, in order to produce motion, must be attended to in order to proportion the means to the end, and to produce the desired effect, with all the advantages of which it is susceptable. It is a due observation of these and other particulars, which contributes to render mechanic instruments perfect, and the neglect of them defective in their construction.

It will, therefore, be necessary for you, in applying theory to practice, to consider all the physical circumstances which are capable of producing any diversity in the effects, how far they diminish motion, and to determine by experiments the laws that govern these circumstances, and the abatements to be made to make practice coincide with theory.

Even here, however it will be necessary to consider the machines as well made, as no rules can ever apply to the ignorance or errors of bad workmen.

Among the various physical causes which occasion a difference between the theory and practice of machines, you may consider two as the most important, and the most general: 1. The weight of the parts of which the machines are framed. 2. The friction of one part of the machine against another.

OF FRICTION.

However plane and smooth bodies appear to the eye, yet, if you examine their susfaces with a microscope, you will discover numberless inequalities. When a body is moved upon a plane, the prominent parts of the body must necessarily fall into each other's cavities, and thereby create a resistance to the motion of a body; for, the body cannot be moved, unless the prominent parts thereof be continually raised above the prominent parts of the surface whereon it slides; and this cannot be done, unless the body be at the same time lifted up, and, as it were, raised on an inclined plane equal to those protuberant parts, which will impede and diminish the effects of the moving power.

It is not possible to adopt any theory, that will take in all the circumstances that occur in practice, for the quantity of friction between the same bodies will vary under different circumstances, as their inequalities may be varied *ad infinitum;* not only by the nature of the bodies, but also by the degree of perfection they may receive from art. An experimental investigation, therefore, on the mechines themselves, will always be more satisfactory than any deductions from theory. Among the later writers on this subject, Mr. *Vince*[*] and Mr. *Coulomb*[†] claim the most attention, particuly the latter; whose tables, showing the frictions of different substances, may be found of considerable use; but, as they are too long for our purpose, I shall lay before you the contents of Mr *Vince*'s paper.

The experiments of Mr. *Vince* were made to determine, 1. Whether friction be a uniformly retarding force. 2. The quantity of friction. 3. Whether the friction varies in proportion to the pressure of the weight. 4. Whether the friction be the same, on whichsoever of its surfaces a body moves.

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[*] *Vince*, Phil. Transactions.

[†] *Coulomb*, Memoires de l'Academie des Sciences.

To resolve these questions, a plane was adjusted parallel to the horizon; at the extremity was placed a pulley, which could be elevated or depressed, so as to render the string which connected the body and the moving force parallel to the plane. A divided scale was placed by the pulley perpendicular to the horizon; the moving force descended by the side of the scale. A moveable stage was placed upon the scale, which could be adjusted to the space through which the moving force descended in any given time; which time was measured by a well-regulated pendulum, vibrating seconds.

Now, if friction be a uniform force, the difference between it and the given force of the moving power must be also uniform, and therefore, the moving body must descend with a uniformly accelerated velocity, and, consequently, the spaces described must be as the squares of the times, just as when there was no friction, only they will be diminished on account of the friction.

A body was placed upon the horizontal plane, and a moving force applied, which from repeated trials was found to descend $52\frac{1}{2}$ inches in 4″. The stage was then removed to that point to which the moving force would descend in 3″, upon the supposition that the spaces described by the moving power were as the squares of the times, and it was found to agree very accurately with the time. The stage was then removed to that point to which the moving force ought to descend in 2″, and was found to agree exactly with the time. The same was tried for 1″, and the coincidences were still exact. To find whether the difference in the time of descent could be observed by removing the stage a little above, or a little below the positions which corresponded to the above times, the experiment was tried, and the descent was always found too soon in the former, and too late in the latter case; which proves that the fore-mentioned space corresponded exactly with the times. Each descent, for greater certainty, was repeated eight or ten times. The experiment was also tried with different moving forces; in all which the spaces coincided with the times.

A great number of experiments were made with hard bodies, or those whose parts so firmly cohered, as not to be moved *inter se* by the friction; and in each experiment, bodies of very different degrees of friction were chosen, but the results all agreed. We may therefore conclude, that *the friction of hard bodies in motion is a uniformly retarding force.*

Experiments were made to determine whether the same law obtained for bodies when covered with cloth, woollen, &c. and it was found in all cases that the retarding force increased with the velocity; but, upon covering the bodies with paper, the consequences agreed with those already related.

The next question is to determine whether friction, *cæteris paribus*, varies in proportion to the weight or presure. Now, if the whole quantity of the friction of a body measured by a weight without inertia equivalent to the friction, increase in proportion to the weight, it is manifest that the retardation of the body, arising from friction, will not be altered; for the retardation varies as $\frac{\text{quantity of friction}}{\text{quantity of matter}}$: hence, if a body be put in motion upon the horizontal plane by any moving force, if both the weight of the body and the moving force be increased in the same ratio, the acceleration arising from that moving force will remain the same, because the accelerative force varies as the moving force, divided by the whole quantity of matter, and both are increased in the same ratio; and if the quantity of friction increase also as the weight, then the retardation from the friction will, from what has been said, remain the same, and therefore the whole acceleration of the body will not be altered; consequently, the body ought upon this supposition still to describe the same space in the same time. Hence, when both the body and moving force are increased in the same ratio, you may determine whether the friction increases in proportion to the weight.

A body weighing 10 oz. by a moving force of 4 oz. described in 2″ a space of 51 inches; by loading the body with 10 oz. and the moving force with 4 oz. it described 56 inches in 2″; and by loading the body again with 10 oz. and the moving force with 4 oz. it described 63 inches in 2″.

A body whose weight was 16 oz. by a moving force of 5 oz. described a space of 49 inches in 3″; loading it with 64 oz. and the moving force of 20 oz. the space described was 64 inches.

From these experiments, and many others of the same kind, it appears, that the space described is always increased by increasing the weight of the body and the accelerative force in the same ratio; and as the acceleration arising from the moving force continues the same, it is manifest that the retardation arising from friction must have been diminished, for the whole accelerative force must have been increased, on account of the increase of the space described in the same time; and hence, as the retardation from friction varies as the $\frac{\text{quantity of friction}}{\text{quantity of matter}}$, the quantity of friction increases in a less ratio than the quantity of matter or weight of the body.

The third question Mr. *Vince* proposed to determine was, Whether the friction varies by varying the surface on which the body moves?

Let us call the greater surface of the body, A; its lesser surface, *a*. Now, the weight on every given part of *a* is much greater than the weight on an equal part of A, as A is greater than *a*. If, therefore, the friction were in proportion to its weight, it is manifest, that the friction on *a* would be equal to the friction on A; the whole friction on A being, on such a supposition, as the weight on any given part of each surface multiplied into the number of such parts, or the whole area, which products, from the above proportion, are equal.

But, from the last experiments it has been proved, that the friction on a given surface increases in a less proportion than the weight; consequently, the friction on any given part of *a*, has a less ratio to the friction on any equal part of A, than A has to *a*; and hence the friction on *a* is less than that on A; that is, the smallest surface has always the least friction, contrary to what has been generally concluded.

In trying the experiments on this head, care must be taken to choose such bodies as have exactly the same roughness, with a regular grain, without knots; or an equal degree of roughness may be obtained by pasting some fine rough paper on each surface.

A body was taken, whose surface was to its edge as 22 to 9; with the same moving force it described on its flat surface $33\frac{1}{2}$ inches in $2''$, on its edge 47 inches in the same time.

One whose surfaces were as 32 to 3, and with the same moving force, described on its flat side 32 inches in $2''$, and on its edge 37 inches in the same time.

From these and many more experiments of the same kind, it appears that the smallest surface had always the least friction; we may, therefore, conclude, that the friction of a body does not continue the same when it has different surfaces applied to the plane on which it moves, but that the smallest surface will have the least friction.

The experiments which have been heretofore made on this subject, have been instituted to find what moving force would *just* put a body at rest in motion; and they concluded from thence, that the accelerated force was then equal to the friction: but it is manifest, that any force, which will put a body in motion, must be greater than that which opposes its motion, otherwise it would overcome it; and, consequently, if there were no other objection, the friction could not be accurately ascertained: but there is another objection, which totally destroys the experiment, so far as it tends to show the quantity of friction, which is the adhesion of the body to the plane while it lies at rest; and this is confined by experiments.

Thus a body of $12\frac{3}{4}$ oz. was laid upon a horizontal plane, and then loaded with a weight of 8lb. and such a moving force was applied, as would, when the body was just put in motion, continue that motion without any acceleration; and in which case the friction may be just equal to the accelerative force. The body was then stopped; when it appeared that the same moving force, which had kept the body in motion before, would not put it in motion: it appears, therefore, that this body, when laid upon the plane at rest, acquired a very strong cohesion to it.

A body, whose weight was 16 oz. was laid at rest upon the horizontal plane, and it was found that a moving force of 6 oz. would just put it in motion; but that a moving force of 4 oz. would, when it was just put in motion, continue that motion; and therefore the accelerative force must then have been equal to the friction, and not when the moving force of 6 oz. was applied.

These experiments show how very considerable the cohesion was, in proportion to the friction, when the body was in motion; it being in the latter case almost one-third, and in the former, very near the whole friction. All these conclusions, therefore, deduced from experiments instituted to determine the friction from the force necessary to put a body in motion, have been manifestly false, as such experiments only show the resistance which arises from cohesion.

I shall conclude this article with a few general observations on friction; and advising you in all cases to have recourse to experiments, and on the engines themselves, to know the resistance occasioned by friction.

To lessen the quantity of friction, the parts are often made of different materials; as iron axes are made to work in brass or bell-metal collars, as from the diversity in the pores of the different substances, they are less apt to adhere or stick together. Care is also taken to polish the parts, and make them as smooth as possible.

Wood, and all metals, when greased and oiled, have nearly the same friction. Where wood acts against wood, grease makes the motion twice as easy. Wheels greased or tarred go four times easier than when wet.

The friction increases with the weight, though not in the same proportion: it is also augmented, except in a very few cases, by an increase of velocity. A great surface also occasions somewhat more friction with the same weight and velocity: yet friction may be increased by two little surface to move on; as upon clay, &c. where the body sinks in.

The friction of a lever is very small. Pullies are subject to great friction, because their diameters in general are very small in proportion to their axes, because they very often bear against the blocks, and from the wear of the holes and axles.

The friction of the wheel and axle is as the weight upon it, and the diameter of the axis, the stiffness of cords, &c.

In a screw there is a great deal of friction. Those with sharp threads have more friction than those with square threads, and endless screws have more than either.

Some writers have considered bodies moving one over the other in two different manners. Thus we may, 1. Apply the same parts of one to different parts of the other; as when you make a book slide on a table, as in the experiment of Mr. *Vince*. Or, 2. You may make different parts of one surface successively touch the different parts of the other; as when you make a ball roll upon a billiard table.

This latter species of friction is never so powerful in retarding motion as the former kind; of which you have an evident and familiar proof, in the custom of dragging the wheels of carriages when the descent is too deep; for when they are afraid that a coach or waggon should go too fast down a hill, they hinder the wheels from going round their axis: consequently, the same point of the circumference drags successively upon a number of points on the ground, which considerably retards the motion of the carriage. It is not so when every wheel turns in the usual way about its axis.

This may be further illustrated by the apparatus with friction wheels, which I used in a former lecture, in illustrating the theory of accelerated motion. To this end I place the axis of the pendulum in these two pivots, and then put it in motion. It is now a friction of the first kind, as the cylindrical axis passes successively over the lower part of each of the holes.

Let us now place the axis of the pendulum upon the intersection of the two pair of rollers, and put the pendulum in motion, which motion will cause the rollers to turn, and thus be only a friction of the second kind, the circumference of one rolling on the circumference of the other, the part which touched before now touching no longer, the parts being also in a favourable direction for disengaging themselves from each other, and you will find that it will now go a much longer time than before.

Mr. *Garnett*, now of Newbrunswick, in the State of New Jersey, has obtained a patent for a new and ingenious application of this principle. Between the axle and nave of any wheel work, a space is left to be filled up by solid equal rollers, nearly touching each other; these have axes, which are inserted in a circular ring at either end, whereby the relative distances to each other are preserved, and they are kept parallel to each other, because the circular rings are fastened together by small supports which pass between the rollers.

OF MAN, CONSIDERED AS AN ARTIFICAL MACHINE.*

Man has been considered by anatomists, as a system of all the artificial machines united in the human fabric; they have found the lever, the pulley, the axle in the wheel, the wedge, and even the screw, or at least something resembling each of them, in his person: thus his arms have been likened to levers; his head to a wheel turning upon its axle; the diagrastic muscle, that assists his swallowing, to a rope running over its pulley; the glands as lifting up their fluids in the manner of an artificial water-screw; and his teeth have been compared to wedges. But some have not stopped here; they have gone on not only to please themselves with the resemblance, but to estimate the force of man through all his vital and involuntary motions; such as the running of the blood through his veins, the drawing his breath, and such like, by the inflexible laws of mechanism. They have even applied geometrical rules to measure the objects constantly in change, and built theories upon proportions they were unable to discover. Thus, when *Borelli* once got the hint of comparing the muscles or fleshy parts to chords, he then readily built his theory, and calculated the human force by considering the thickness of the cords, and and the length of the lever. Thus, when another found the similitude between the blood running through its channels, and water spouting through pipes, he pursued the speculation, till he at last was taught to believe, that vomits would cure a spitting of blood, and bathing in warm water would be a remedy for the dropsy: happy, however, had his theory never been put into practice.

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* *Goldsmith*, Survey of Experimental Philosophy, vol. i. p. 245 to 369.

It is as impossible to determine the muscular force of any man by the bare inspection or admeasurement of his muscles, as it is to measure the swiftness of the circulation of his fluids by the spouting of his blood from a vein. Neither can be done, though *Cheyne* has pretended to demonstrate, that if we compare the muscular strength of two animals, that animal whose fluids circulate twice as swift, will be six times as strong. *Friend* and *Wainwright* adopted his demonstration, for he called it a demonstration; and indeed it was drawn up with a sufficient degree of mathematical parade. *Martine*, however, in a treatise entitled *De Similibus Animalibus*, has demonstrated, that *Cheyne*'s demonstration was false; but it was in order to establish another demonstration of his own. He asserted, that the force in similar animals was as the cube roots of the fourth power of the limb put in motion. You do not, perhaps, understand the precise meaning of these words; but it is no matter, for his demonstration is as false as the former.

From the mere dimensions of the muscles in two similar animals, it is impossible to determine their force. The strength of the muscle is generally more in proportion to the exercise it has been employed in, than to its size: the legs of a chairman are stronger by use; it is the same with the arms of a smith: in short, to use the words of a bully in a Spanish comedy, who mistook his man and was beaten, we can never know the strength of the muscles till we experience their effects.

But though we cannot determine, with any precision, of two men which are the stronger, yet in the same man we can compare the force of his muscles with rather more precision. This at least can be said, with great certainty, that those muscles which are inserted into the bone, nearest to the place where it moves upon another, overcome the greatest resistance, and consequently act with the greatest force.

All our flesh is composed of muscles, which (if I may use a vulgar similitude) are like red ribbands, and almost all have one of their ends fixed into one bone, and the other end into some other bone. Thus, if we feel the great ham-string, which is made up of many muscles, we shall find, that at one end it is fixed into the bones of the leg, just under the knee, and at the other end it runs upwards, partly to be fixed in the great bone of the thigh. The muscles being thus stretched from one bone to another, have a wonderful power of contracting and shortening themselves at pleasure; and when we choose to put them into action, they swell in the middle, somewhat in the shape of a ninepin. As these muscles thus contract, they must necessarily draw the two bones into which they are inserted their own way. The ham-string, when it contracts, for instance, draws the leg backward towards the thigh. When we want to make the limb straight, there are muscles inserted under the fore-part of the knee, that, contracting, answer this purpose; while, in the mean time, the ham-string suffers itself to be relaxed, in order to let the opposing muscles take effect. This being understood, it will follow, that if we consider any one of the bones, the arm-bone for instance, as a beam, and the muscles that raise it, and put it into motion, as the power that agitates and works the instrument, the whole will give us the idea of the third kind of lever, where the prop is at one end, the weight to be sustained at the other, and the strength is applied between them both. Thus, for instance, if I stretch out my arm, the prop is in the joint of my shoulder, the weight is my hand, and the raising power is the muscles, which are fixed into the arm-bone near the shoulder, and go from thence to be inserted into the bones of the trunk of my body. Now, the nearer the shoulder these muscles are inserted into the arm-bone, it is evident that the longer will be the lever against which they are to act, and consequently the greater will appear the weight which they are to sustain. To make this quite plain, suppose a ladder were laid flat on the ground: and suppose that I, standing at one end, take the nearest round of the ladder in both my hands, and thus pulling back, attempt to raise the farther end keeping the nearer end still steady to the ground. Would not this require immense strength to effect? Pretty similar is the force that the muscles of the arm exert in raising the whole length of the arm, and the weight of the hand beside. They are inserted into the bone close to the shoulder, and support the whole length of the arm in the desired direction. But, what is more, they do not only act upon the lever at so disadvantageous a distance, but also they act upon it in a direction the most oblique, and consequently at a greater disadvantage still. Suppose I attempt to raise the distant end of the ladder, by pulling the round nearest me; this, as I said, will be very disadvantageous: but suppose yet farther, that I should first lie upon my back, and then, by drawing the next round to me of the ladder, I should attempt to raise the distant end, the force that would be capable of effecting this would be incredible. Yet in this very manner it is that the muscles of the shoulder act in raising the arm. They are not only inserted at the greatest distance from the weight, but they exert their power the most obliquely. The force they exert in keeping the hand and arm extended is great; the force they exert in keeping it extended, while the hand holds a weight of about twenty pounds, is astonishing. Some say, that these muscles, upon equal terms, would lift a weight ten thousand times greater. What has been here said of the muscles of the arm is true, in a greater or less degree, of all the muscles of the body; so that this natural machine, thus fashioned by the Great Workman, is infinitely more powerful than any artificial machine that man could form, though it took up four times the space.

The muscles, as we said, are supported by bones; these make altogether a single pillar or column, which, though not perfectly straight, but with about five different curvatures or bendings, yet when perfectly balanced upon itself, would actually support weights that would surprize the inexperienced. *La Hire* and *Desaguliers* give us several accounts of the amazing weight some people have sustained, when they are able to fix the pillar of their bones directly beneath it. The latter tells us of a German, who showed several feats of this kind at London, and who performed before the king and a part of the royal family. This man, being placed in a proper situation, with a belt which rested upon his head and shoulders, and which was fixed below to a cannon of four thousand weight, had the props which supported the cannon taken away, and, by fixing the pillar of his bones immoveably against the weight, supported it with seeming unconcern. There are few who have not seen those men, who, catching a horse by the tail, and placing themselves in direct opposition to the animal's motion, have thus stopped the horse, though whipped by his rider to proceed. In all such cases, the pillar of the bones is placed in direct opposition to the weight; they support each other, and are prevented from rubbing or cracking by elastic gristles fixed between them: these give way a little upon great pressure, and restore themselves almost instantly, when that is removed. Besides these, there is a viscous or slimy liquor that is squeesed in, as if from a sponge, between every joint, which keeps these gristles smooth, moist, and pliant. By means of this fluid all the joints move easily, and obey the impulse of the muscles with greater dispatch. This fluid and the gristles (or cartilages, as anatomists call them) contribute not a little to the strength of the animal; they resist the burden with an elastic force, and conform themselves to the inequality of the pressure. In old age both are diminished, the gristles become hard, and this liquor, which anatomists call the synovia, is squeezed out in less quantities. The man, therefore, in old age, becomes more stiff and more weak, chiefly upon this account, though partly because his muscles become then also more rigid, hard, and less fleshy, as it is usually called, as those who have eaten the flesh of old animals know. While we are at rest, this fluid, or synovia above-mentioned, oozes out between the joints, to fit them for the hour of action; when in exercise, the ends of the bones press against their gristles, and these are separated, in some measure, by the synovia or fluid. But there is still another liquor, of an oily nature, which is pressed at the same time from a small fleshy sponge, placed in every joint; and this, mixing with the synovia, makes all supple and fit for business. I said that the synovia, or viscid liquor, oozes out between the joints in the hour of rest; it is therefore in greatest quantity between them in the morning, after we have taken our rest the preceding night. So great is the quantity usually separated during sleep between the joints of the back-bone, that some men are an inch taller in the morning than at night, and all men are somewhat taller, as may be quickly found by any who may choose to make the experiment upon themselves.

From what has been said it appears, that, in carrying large burdens, the whole art consists in keeping the column of the body as directly under the weight as possible, and the body as upright under the weight as we can: for if the centre of gravity in the burden fall without this column, it will go near to fall; in fact, if the supporter were an inanimate machine, it would fall inevitably; but human power, in some measure, catches the centre while yet beginning to descend, and restores the balance which it had lost the moment before. A man balancing under a weight, resembles one of those people whom we usually see walking upon a wire; they totter from side to side, for a moment lose the centre of gravity, but by throwing forward a limb, or distorting their bodies, they recover it again, to the great amusement of every spectator. It is thus that he who carries a weight is obliged to act; on whatever part of his body the weight is placed, he balances it by throwing as much of his column beneath the load as he can. Could the weight be laid and evenly balanced upon him, standing in his natural posture, he could, as we observed before, support an incredible burden; and though he could not move under what he could thus support, yet he could carry a much greater load, than if the burden were laid in any other manner. The weight a man could support, when thus evenly laid on his shoulders, would break the back of the strongest horse in the world. The reason is obvious: in a man the whole column of bones support the weight directly; in a horse, the weight is laid upon the column cross-wise. The porters of Constantinople are known to carry each a weight of nine hundred pounds: they lean upon a staff while loaded, and are unloaded in the same manner. The porters of Marseilles in France are found to carry yet more; their manner is this: four of them carry the burden between them, each having a sort of hood that covers the temples and head down to the shoulders; to this is fastened the cords that support the frame or bier on which the weight is laid. By this contrivance, the whole column of the bones acts directly against the load, and an immense weight is thus sustained.

We now, therefore, at length see the reason why two men, carrying a load, can sustain a greater weight than what either could separately carry, if it were divided into two equal parts. The reason is, that two men can bear the load each more upright, and with the column of their bones more opposed against it.

As a man bears a weight the better the more upright he stands against it, it must follow necessarily, that the more bendings he makes in supporting weights, the less will be his power. There are three principal bendings in the human column; the first at the hams, the second at the hips, and the third along the back-bone; which resembles the ozier in pliancy, though it is stronger than the oak. A man of ordinary stature and strength, upon an average, has been computed to weigh a hundred and sixty pounds; he can support, as we said before, an immense weight, if his column act directly against it; if he bend a little at the hams, such a man may raise from the ground about a hundred and seventy pounds, provided the weights be placed to the greatest advantage. If he bend at the hips and back, he will lift thirty pounds less. If a weight be placed upon his head, and he be put between the rounds of a ladder placed horizontally, and breast high, he can lift thirty pounds by the strength of the muscles of his shoulders and neck alone.

From this we see, that human strength is not the fourth part as great when the body is bent, as when it is upright. From this also we see, that when a man draws a load after him, as in that case all his muscles act in oblique direction, he can exert but very little force, when compared to other animals. *Desaguliers* pretends to say, that a horse can draw as much, upon an average, as five English workmen. The French writers say, Dr. *Barthes* in particular, that a horse can draw as much as six Frenchmen, or seven Dutchmen; but if the load were to be placed upon the shoulders, two men will be found to be as strong as a horse. A London porter will carry three hundred weight at the rate of three mile an hour; two chairmen carry a hundred and fifty pounds each, and walk at the rate of four miles an hour; whereas a travelling horse seldom carries above two hundred weight, and a day's journey, with such a load, would be apt to disqualify him from travelling the day following.

Man's greatest force, therefore, is directly upward; if he draw a load, he must act at a great disadvantage. A man, however, when obliged to draw a load, a rolling-stone for instance, hath two methods of doing it; he may either turn his back to the stone, and, pushing the frame with his breast, thus go onward, while the stone rolls after; or he may turn his face to the stone, and go backward, drawing the stone with him. This last method may be the most inconvenient, but it gives the workman much the greater share of power, and that for two reasons: in the first place, by inclining farther back, he can give a greater column of his body to the draught; and, in in the next place, a greater number of muscles will come into action; particularly the two great deltoid muscles of the arms, the force of which is very great. It is for this reason that men who row a boat, more usually draw the ore to them, than push it from them.

OF THE FEATS OF STRONG MEN.

Dr. *Desaguliers* motions a German, in his time, who passed for a man of uncommon strength, and got considerable sums of money by the daily concourse of spectators. After having seen him twice, the Doctor guessed at the manner in which he performed his tricks, and, as soon as he had prepared a proper frame, preformed all of them, and usually exhibited them afterwards at his lectures ; proving that any person of ordinary strength could easily do all that the German was accustomed to perform.

The German sat on the inclined board of a frame, with his feet against an upright immoveable prop, strongly counterbalanced, with a girdle round him a little below his hips. To the iron rings of the girdle a rope was fixed ; the rope went between his legs and through a slit of the prop. Several men, or even two horses, could not by pulling move him out of his place : he seemed to pull with his hands, but those were of no anvantage to him.

The same man, having fixed a rope to a strong post, passed it through a fixed iron eye, then hooked it on his girdle, and setting his feet against the post, near the said eye, raised himself from the ground by the rope ; which he broke by suddenly stretching out his legs, and fell backwards on a feather-bed laid on the ground to catch him.

He lay down on the ground, with an anvil on his breast, upon which another man hammered with all his force a piece of iron with a sledge hammer.

He put his shoulders upon one chair, and his heels upon another, and supported one or two men standing upon his belly, raising them up and down as he breathed ; making with his back-bone, thighs, and legs, an arch ; the abutment being on the chairs.

He lay down on the ground, a man standing upon his knees ; he then drew his heels towards his breech, and so raised his knees till they were perpendicular under the man ; when he raised up his own body, and putting his arms about the man's legs, rose with him, and set him down on a low table. This he sometimes did with two men.

OF WHEEL CARRIAGES.

By what we have seen of man, considered as a machine, it is easy to observe that his frame is not adapted to drawing carriages; while, on the contrary, in that of an animal upon all fours, the column of whose bodies, and the situation of whose muscles, act almost directly upon bodies placed behind them; they are perfectly fitted by nature for this kind of service. Horses are usually employed in the draught in England; mules, oxen, and other animals, are sometimes used in other parts of the world. It might incur ridicule, if we pretended to inform the learner, that each of these will draw a weight or carriage in proportion as they are strong: but notwithstanding this is generally the case, yet we are going to mention what will seem a paradox; namely, that two horses may be found, one stronger than the other, and also better skilled in the draught, yet the weaker shall draw a weight, with the very same carriage, which the stronger one could not remove! This will be effected, if the weaker horse be the heavier; and if he exceed his antagonist more in weight than he is exceeded in strength. I have before observed, that the weight re-acts and pulls back the horse, as much as the horse acts upon the weight to pull it forwards. Now, the horse has two sources of power in drawing the weight along; his strength, which gives him velocity, and his weight, which added, gives force; and it is evident, that the horse which hath both in the greatest proportion will draw the heaviest weights. If we should imagine both horses raising an equal weight from a deep pit, and this weight still increased, so as to overcome their strength, it is plain that the lighter horse would soonest be drawn in. We have several instances, in ordinary practice, of the great benefit of increasing the horse's weight to promote his draught.

Horses have little or no power to draw, but what they have from gravity or weight; otherwise they could take no hold of the ground, and then they must slip and draw nothing.

Common experience will inform you, that when a horse is to convey a certain weight, he ought, that he may draw the better, to have a proportionable weight on his back or shoulders. A horse in a two-wheeled cart, in which there is a ton weight, when it is in equilibrium will not be able to draw it; but when there are fifty or sixty pounds bearing on his back, he will draw it with ease. If it be two or three tons, when he bears one hundred or two hundred pounds on his back, he will be able to draw the load, because the wheels of a cart are very high.

When a horse draws hard, he bends forward, and brings his breast nearer the ground; and then, if the wheels be very high, he is pulling the carriage against the ground.

A horse, tackled in a waggon, will draw two or three tons weight, because the line of traction is below his breast.

It is very common, when one horse is drawing a heavy load, to see his fore-feet rise from the ground, and he will nearly stand an end. It is usual in this case to add a weight on his back, to keep his fore-feet down, by a person mounting on him, which will enable him to draw the load he could not move before.

The case is nearly the same in applying the strength of a man in wheeling a load in a wheelbarrow. When most of the load lies on the wheel, he will slip and not be able to get forward; but when bringing the weight nearer his arms, he will be able to drive it forward. In drawing a heavy garden-roll, if the axis of motion were even with that part of his body where his arms are extended, he could not be able to draw it along; but will draw it easily if the line of traction be low.

In a loaded cart which hangs nearly in equilibrio, if two men were to take it by the shafts, then they would not be able to move it; but one of them in the shafts, and the other behind the cart, pushing the breech upward as well as forward, he lays a load on the first man's back, and so pressing both the feet against the ground, they will easily draw the load.

In a long team, where only the hind horse bears on, if you take off half the number, and fix them to a lower point of traction, they will be able to move a much greater weight.

Sledges were probably the first machines used in carrying loads; we find them thus employed in *Homer*, in conveying wood for the funeral pile of *Patroclus*. There are some countries also, that preserve their use to this day. However, men early began to find how much more easily a machine could be drawn upon a rough road that run upon wheels, than one that thus went with a sliding motion. Though indeed, if all surfaces were smooth and even, bodies could be drawn with as much ease upon a sledge as upon wheels: and in *Holland*, *Lapland*, and other countries, they use sledges upon the smooth surface of the ice; for, as every surface upon which we travel is usually rough, wheels have been made use of, which rub less against the inequalities than sledges would do. In fact, wheels would not turn at all upon ice if it were perfectly smooth, since the cause of the wheels turning upon a common road is the obstacles they continually meet. For, if we suppose the wheels to be lifted from the ground and carried along in the air, the wheels in this case would not turn at all, for there would be nothing to put one part in motion rather than another; in the same manner, if they were carried along upon perfectly smooth ice, they would meet nothing to give a beginning to the rotatory motion, and all their parts would rest equally alike. But, if we suppose the wheel drawn along a common road, then the parts will receive unequal obstructions, for it meets with obstacles that retard it at bottom, therefore the upper part of the wheel, which is not retarded, will move more swiftly than the lower part, which is; but this it cannot do, unless the wheel moves round. And thus it is, that the obstacles in the rough road cause this rotatory motion in the wheel.

The utility of wheels arises therefore, from their turning upon their axes, the resistance arising from friction being very much diminished, and the draught thereby rendered more easy; and you will find by experiment, that it requires considerably less force to draw a carriage when the wheels are free to turn about their axis, than when they are chained together and cannot turn. According to *Helsham*, a carriage with four wheels will be drawn with five times as small an effort as one that slides on the same surface in a sledge. From the foregoing experiment, it not only appears that the friction is very much lessened, but that this diminution does not arise from the wheels touching the plane in a few points, but from their rotation on their axis.

A sledge passing over a plane undergoes a friction, or rubbing of its parts against the plane equal to the distance through which it moves; but if an axis be applied whose circumference is six inches, and on that a wheel be place whose circumference is eighteen feet, it is evident, that in moving the carriage eighteen feet over a plane, the wheels will make but one revolution; and, as there is no sliding of the parts between the plane and the wheels, but only a mere change of surface, by one part of the wheel rising and the other descending nearly perpendicular to the plane, no friction will take place there, the whole being transferred to the nave acting on the axis; which nave having made but one revolution in the same time, there has been only a sliding of the parts equal to the circumference of the hole in the nave, here supposed to be about six inches, so that the friction is lessened about as one to thirty-six; besides the advantage gained by confining it to so small a surface, whereby the parts are more easily kept smooth and fitted to each other, and substances applied and retained to lessen the remaining friction.*

By the application of wheels to a carriage, the friction is lessened in proportion of the diameters of the axis, and concave part of the naves, to those of the wheels.

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* *Artice's* Remarks on the comparative Advantages of Wheel Carriages, p. 21, 40, 41, 42, &c.

When a carriage is drawn up hill, or any regular plane ascent without wheels, you have not only the friction to overcome, but the power must also be sufficient to overcome that proportion of the weight of the carriage, which the perpendicular part of the inclined plane bears to that proportion of the plane.

Wheels applied to a carriage moving up a regular plane of ascent appear only to act as removing the friction; for, though they may be considered as levers, yet, as each arm of the lever is lengthened in proportion to the size of the wheels, the power will be only augmented as far as the ascent can be considered as a mechanical power for raising the wheels, carriage, &c. to the top of the hill.

Large wheels have the advantage of small ones in overcoming obstacles, because they act as levers in proportion to their sizes. And in general the centre of gravity should be as near as may be to the axis of the wheel; and where safety is particularly considered, the nearer that centre is to the ground the better.

If the suspension be below and the body be turned forwards, as is the case with two-wheeled carriages deacending hills, then will the greater part of the weight be thrown before the axis, and must be partly borne up by the horse that draws; in ascending, the same proportion will be thrown backwards and tend to lift the animal. If the body be suspended above the centre of gravity, the disadvantages will be equal, but the effect will be reversed.

The latest experiments on this subject have been made by the Rev. Mr. *Vince*. Some results thereof which have been published in his excellent Plan of a Course of Lectures on the Principles of Natural Philosophy, I shall now lay before you; besides these, there are several curious observations on this subject by Mr. *L. Edgworth*, published in the Transactions of the Royal Irish Academy, which are well worth your attention; of which, however, I shall only mention that which relates to the use of springs when applied to carriages.

MR. VINCE, ON WHEEL CARRIAGES ON PLANE HARD GROUND.

If the wheels be all equal and narrow, it requires the same weight to draw the carriage, whether it be loaded before or behind.

If broad wheels be put on, of the same size and weight, it requires the same weight to draw the carriage as for the narrow wheels, at whatever part it is loaded.

If two wheels be low, and two high, it requires a greater weight to draw the corriage than when all are high.

In this case it makes no sensible difference which go before. The common opinion, therefore, that the high wheels drive on the lower when they go forward is not true.

If the wheels be all equal, it requires a greater weight to draw the carriage, the less the wheels are.

The disadvantage of small wheels arises from hence, that the resistance of the ground, which turns the wheels about, more easily overcomes the friction at the axle in a large than a small wheel, because it acts at a greater distance. For, the mechanical advantage of wheels is, that the resistance which must be overcome by a force more than equivalent to it, if the wheels could not turn, is overcome by a less force in the proportion of the radius of the wheel, to the radius of the axle, when the wheels do turn. Hence the disadvantage of laying the load upon the low wheels, as it increases the friction where there is the least power to overcome it. Where the load is but small, and consequently the friction but small, there is but a small difference between the small and large wheels; but when the load is great, the difference becomes considerable.

ON HARD GROUND WITH OBSTACLES.

If W be the weight of the carriage, and the centre of gravity be in the middle; also if r=the radius of the wheel, and x=the height of the obstacle, then the power P acting parallel to the horizon which is just sufficient to balance the carriage at the obstacle without drawing it over $= \frac{W \times \sqrt{(2rx - xx)}}{2r - 2x}$.

For the power may be conceived to be drawing a weight up an inclined plane, which is a tangent to the circle at the point where it touches the obstacle; and as when that end rises, the other rests upon the horizontal plane, the power has to elevate a weight only equal to $\frac{1}{2}W$.

Experiments of this kind are subject to inaccuracies which cannot be accounted for. The power will sometimes hang for some time without moving the carriage, and then it will suddenly draw the carriage over the obstacle. Sometimes there will be a difference of half an ounce out of about ten ounces in drawing the same carriage over the same obstacle, although every care is taken to have all the circumstances accurately the same. Many of the experiments, however, answer very nearly to the theory, nor do any of them differ from it very materially.

The use of high wheels in going over obstacles is very manifest from this proposition, and as carriages are continually going over obstacles, high wheels will always have the advantage. Moreover, in sinking into holes, they have a double advantage; first, they do not sink so deep as low ones would; and secondly, after sinking, they ascend again with less power. As, when the centre of gravity is in the middle of the carriage, the power has but half its weight to elevate in going over an obstacle; therefore, when the load is not in the middle, it throws the centre of gravity towards one end, and, therefore, when that end goes over an obstacle the power has more than half the weight to raise, the pressure upon each wheel being inversely as the distance of the centre of gravity from them. Hence, every carriage should be loaded most towards the higher wheels, by which means less than half the weight will be thrown upon the lower wheels, and thus each pair of wheels may be made to require the same power to draw them over an obstacle. The same power, however, that may be necessary for one obstacle, will not be sufficient for another.

If the height of the obstacle be inconsiderable in respect to the radius of the wheel, which is the case with the common obstacles, as stones, &c. which carriages usually meet with, then $P = W \times \sqrt{\frac{2r}{x}}$. Now as each pair of wheels has the same obstacles to go over, x is given, and that P may be given, or that it may require the same power for each pair, W must vary as $\sqrt{r}$; now the weight supported by each wheel is inversely as its distance from the centre of gravity. Hence to overcome small obstacles, the distance of the centre of gravity from the great wheels : its distance from the small : : the square-root of the radius of the small wheel: the square-root of the radius of the large wheel. The diameters of the wheels of a common waggon are about five feet eight inches, and four feet eight inches, and the distance of the wheels, when narrow, about six feet six inches; hence the centre of gravity of the load of a waggon ought to be about three feet six inches nearer to the higher than to the lower wheels. For a broad wheel waggon, where the distance of the wheels is about seven feet ten inches, the centre of gravity ought to be about four feet two inches nearer to the higher than to the lower.

It appears also that when W and x are given, and x is very small, P varies inversely as the square-root of the radius of the wheel. Hence the advantage of a wheel to overcome a small obstacle varies as the square-root of the radius of the wheel. This resistance of the obstacle causes the wheel to turn, but this resistance is not friction; the friction arises from the rubbing of the parts of one body against those of another, whereas where the wheel only turns upon a point, the friction therefore only takes place at the axle, where the parts rub one against another. There is therefore no friction at the ground, unless when the wheels slide, which is the case when they are chained together, as is frequently done to prevent them from running too fast down a hill.

UPON SAND.

It requires a less force to draw a narrow than a broad wheel carriage upon sand.

The disadvantage of the broad wheels seems to arise from their driving the sand before them.

If two wheels be high and two low, it requires a greater force to draw the carriage than when all the wheels are high.

If all the wheels be low, it requires a greater force to draw the carriage than in the last case.

In all these cases it requires a less force to draw the carriage when loaded behind than before.

Whatever permits the load to rise gradually over an obstacle, without obstructing the velocity of the carriage, will tend to facilitate its draught, and the application of springs has this effect to a very considerable degree; the same weight of four pounds being drawn over the same obstacles, when springs were put between the load and the carriage, by four pounds instead of fourteen. This remarkable difference points out the great advantage of springs in rough roads, an advantage which might be obtained for heavy waggons as well as for other carriages, by a judicious application of the same means.

It appears from the Memoirs of the French Academy, that the idea of applying springs to carriages had occurred to M. Thomas, in the year 1703, who has given a drawing of a carriage constructed upon this principle many years before it was attempted to be put in execution. So little expectation had he of success, that he expressly mentions it as a theory which could not be reduced to practice; he had, however, no notion of applying springs to facilitate the draught, but merely for the convenience of the rider; and I apprehend that it is not at present commonly imagined that springs are advantageous for this purpose; nor would it at first sight appear credible, that, upon a rough paved road, such as are common in Cheshire, and other parts of England, a pair of horses could draw a carriage mounted upon springs with greater ease and expedition than four could draw the same carriage if the springs and braces were removed, and the carriage bolted fast down to the perch.

The reason why springs so much facilitate the draught of carriages seems to be, not only that they allow the wheels to pass more gradually over the obstacles, but that by their elasticity they make the carriage bound upwards every moment for a small way; thus its gravity is for that moment in a great measure counteracted, and the progressive motion which it has already acquired is at liberty to act more freely in pushing it forward; for were it possible very suddenly to take away the horses from a carriage mounted on springs, and moving with a considerable velocity, it would continue for some time to move of itself; the weight in this case acting as a fly upon any mechanical engine, by means of which the machine accumulates a certain quantity of power, and will keep itself in motion for a considerable time after the hand is taken away from it. The weight of all carriages, indeed, has some effect of this kind, otherwise the draught would require an intolerable exertion of strength: and it is to be observed, that this tendency to proceed in the direction in which it is once set a going, is remarkable in all great quantities of matter, and very perceptible even when weights are pulled directly upward; for, in raising great weights by a crane, the burden is lifted with considerable more ease when near the top than at the bottom, even after making every necessary allowance for the weight of the rope.

I have now finished the lectures on mechanics, a science of whose importance you cannot be ignorant, as it is to the manufacturer and the merchant that this nation owes its power and glory. The manufacturer supplies the merchant, and the merchant supplies the world with all its wealth. It is thus that industry is promoted, arts invented and improved, commerce extended, superfluities mutually vended, and wants mutually supplied; that each man becomes a useful member of society; that country is tied to country, and clime to clime; that the remotest regions are brought to neighbourhood and converse. It is thus the labour of individuals knits into one family, and weaves into one web the affinity and brotherhood of mankind.

Manufacturers bear so intimate a relation to the mechanical powers, as to be incapable of subsisting in any degree of vigour without their support; the operations would be tedious, expensive, and imperfect, if performed by the mere efforts of human strength or manual skill, unassisted by mechanical aid. The mechanic powers facilitate their performance, and render their productions more complete.

Most of our manufactures are, under Providence, entirely dependent on our machines. Of the importance of the steam-engine, and the application of its active and potent principles to the brewery; to machines for spinning cotton and grinding corn, &c. &c. you are well acquainted, and must therefore be sensible of the importance of mechanical knowledge, which will be of the highest value as long as industry and commerce are the natural, the living, the never-failing fountains, from whence alone the wealth of this world can be taught to flow.

Dr. *Smith*, in his Wealth of Nations, has shown, that the productive powers of labour, and the greater part of skill, dexterity, or judgment with which it is any where directed or applied, seem to have been the effects of the division of labour: what is the work of one man in a rude state of society being generally that of several in an improved state. To make this more easily understood, by considering some particular manufacture, he instances that of a pin-maker; a workman not educated to this business, nor acquainted with the machinery used in it, which the division of labour has rendered necessary, could scarce, perhaps, with his utmost industry, make twenty pins in a day. But in the way which this business is now carried on, not only the whole work is a peculiar trade, but it is divided into a number of branches, of which the greater part are likewise peculiar trades.

One man draws out the wire, another straightens it, a third cuts it, a fourth points it, a fifth grinds its top for receiving the head; to make the head, requires two or three distinct operations; to put it on is a peculiar business; to whiten the pins is another; and it is even a trade by itself to put them into paper. And the important business of making a pin is in this manner divided into about eighteen distinct operations, which in some manufactories are all performed by distinct hands, though in others the same man will perform two or three of them. Dr. *Smith* mentions a small manufactory, where ten only were employed, and where some of them consequently performed two or three distinct operations; but though they were poor, and therefore but indifferently accommodated with the necessary machinery, they could, when they exerted themselves, make among them about twelve pounds of pins in a day. There are in a pound upwards of 4000 pins of a middling size, they could therefore make among them upwards of 48,000 pins in a day; each person, therefore, making a tenth part of 48,000, might be considered as making 4,800 pins a day. But if they had all wrought separately and independently, and without having been educated to this peculiar business, they certainly could not each of them have made twenty pins a day, perhaps not one, that is certainly not the 240th, perhaps not the 48,000th, part of what they are at present capable of performing, in consequence of a proper division and combination of their different operations.

This great increase in the quantity of work, which, in consequence of the division of labour, the same number of people are capable of performing, is owing to three different circumstances: 1. To an increase of dexterity in every particular workman. 2. To the saving of time, which is commonly lost in passing from one species of work to another. 3. To the invention of a great number of machines, which facilitate and abridge labour, and enable one man to do the work of many.

This view of the advantages of mechanics points out clearly the difference in the order of Providence between man and the brute creation. In every forest, in every field, God has spread the carpet, and prepared the repast for those whom he has instructed to prepare it for themselves. To man alone he gave reason, invention, and a social nature; he gave him weakness and wants as a fertile though humble bed, wherein he sowed the seeds of every human virtue. By those weaknesses and wants he compels him to associate; and from society he derives all the conjugal, paternal, and filial endearments; the friend, the neighbour, the citizen, countryman, and all the charities that unite the great brotherhood of man.

God suffers not any man, and rarely any nation to be sufficient of itself, or of ability to supply its own occasions; the wealthiest, the strongest, the wisest individual, though of power to extend his service through his country and through the world, cannot singly provide for his personal subsistence.

Thus man, who in himself is the most impotent and pitiable of all living creatures, becomes the most powerful and formidable through community; from his weakness he gathers strength, and from wants he gathers plenty. For here, and here alone, God hath laid the foundation of all the wealth and dominion that ever rose upon earth. The innumerable wants, infirmities, and diseases that are incident to human nature demand innumerable provisions from the various arts, sciences, manufactories, and occupations that are exercised and cultivated throughout the earth; for from these alone arises the universal wealth of the world; whatever is necessary, useful, commodious; whatever conduces to the convenience, delight, or happiness of mankind. Hence the buzz of wheels, reels, and looms; the sound of hammers, files, and forges; the shouts of vintage, and the songs of harvest.

Though man's life consisteth not in the abundance of his possessions, yet industry, which is the natural parent of opulence, is as well a blessing as a duty to man, ever since he was appointed to earn his bread with the sweat of his brow. Many temporal benefits follow in the train of industry, its works are the works of peace, and it tends to open the avenues where the virtues may walk; and all legislators are called upon to encourage the mechanic arts, that they may promote ingenuity and industry, and enable the manufacturer to persevere in his labour. When the good householder walked out to the market-place, and found labourers loitering there, when it was now toward the evening, he asked them, "Why stand ye here idle?" and when they answered, "because no man hath hired or given us employment, he took this for a sufficient apology; he had compassion upon them, and he supplied them with the divinest kind of charity,—the means of earning their own bread."

APPENDIX TO LECTURE XXXII.

BY THE E. EDITOR.

CONTAINING A DESCRIPTION OF TWO WHIRLING-TABLES, OR MACHINES, FOR EXHIBITING AND DEMONSTRATING BY EXPERIMENTS THE NATURE OF CENTRAL FORCES, OR THE PRINCIPAL LAWS OF GRAVITATION, OF THE PLANETARY MOTIONS IN THEIR ORBITS, THE DOCTRINE OF TIDES, &c. *Plate* 6.

THE whirling-table may be considered by a lecturer as an esseential part of his mechanical apparatus; and as it illustrates some of the laws in mechanics already given by our author, I have judged, that it would be acceptable to many readers to have a description of two, of the most approved sort.

Fig. 1, *plate* 6, is the representation of a whirling-table, constructed in the most portable and simple manner by the late Mr. *James Ferguson*, and described by him in Lectures on Select Subjects, first published many years ago, and from which the following description is extracted.

" A A is a strong frame of wood, B a winch or handle fixed on the axis, C, of the wheel, D, round which is the catgut string F, which also goes round the small wheels G and K, crossing between them and the great wheel D. On the upper end of the axis of the wheel G, above the frame, is fixed the round board *d*, to which the bearer, M S X, may be fastened occasionally, and removed when it is not wanted. On the axis of the wheel, H, is fixed the bearer N T Z: and it is easy to see, that when the winch, B, is turned, the wheels and bearers are put into a whirling motion.

" Each bearer has two wires W X, and Y Z, fixed and screwed tight into them at the ends by nuts on the outside. And when these nuts are unscrewed, the wires may be drawn out in order to change the balls U and V, which slide upon the wires by means of brass loops fixed into the balls; thus keeping the balls up from touching the wood below them. A strong silk line goes through each ball, and is fixed to it at any length from the centre of the bearer to its end, as occasion requires, by a nut-screw at the top of the ball; the shank of the screw goes into the centre of the ball, and presses the line against the under side of the hole that it goes through. The line goes from the ball, and under a small pulley fixed in the middle of the bearer; then up through a socket in the round plate, see S and T, in the middle of each bearer; then through a slit in the middle of the square top, O and P, of each tower, and going over a small pulley on the top, comes down again the same way, and is at last fastened to the upper end of the socket fixed in the middle of the abovementioned round plate. These plates, S and T, have each four round holes near their edges for letting them slide up and down upon the wires, which make the corners of each tower. The balls and plates being thus connected, each by its particular line, it is plain, that if the balls be drawn outwards, or towards the ends, M and N, of their respective bearers, the round plates, S and T, will be drawn up to the top of their respective towers O and P.

" There are several brass weights, some of two ounces, some of three, and some of four, to be occasionally put within the towers O and P, upon the round plates S and T: each weight having a round hole in the middle of it, for going upon the sockets or axes of the plates, and is slit from the edge to the hole, for allowing it to be slipt over the foresaid line, which comes from each ball to its respective plate. See *fig.* 2.

" The experiments to be made by this machine are as follows:

" 1. *The propensity of matter to keep the state it is in.* Take sway the bearer MX, *fig.* 1, and take the ivory ball, *a*, to which the line or silk cord, *b*, is fastened at one end; and, having made a loop on the other end of the cord, put the loop over a pin fixed in the centre of the board *d*. Then turning the winch, B, to give the board a whirling motion, you will see, that the ball does not immediately begin to move with the board, but, on account of its inactivity, it endeavours to continue in the state of rest which it was in before. Continue turning until the board communicates an equal degree of motion with its own to the ball, and then turning on, you will perceive, that the ball will remain upon one part of the board, keeping the same velocity with it, and having no relative motion upon it, as is the case with every thing that lies loose upon the plane surface of the earth, which having the motion of the earth communicated to it, never endeavours to remove from that place. But stop the board suddenly by hand, and the ball will go on, and continue to revolve upon the board, until the friction thereof stops its motion: which shows, that matter being once put in motion, will continue to move for ever, if it meet with no resistance. In like manner, if a person stands upright in a boat, before it begins to move he can stand firm; but the moment the boat sets off, he is in danger of falling towards that place which the boat departs from: because, as matter, he has no natural propensity to move. But when he acquires the motion of the boat, let it be ever so swift, if it be smooth and uniform, he will stand as upright and as firm as if he were on the plane shore; and if the boat strike against any obstacle, he will fall towards that obstacle, on account of the propensity he has, as matter, to keep the motion which the boat has put him into.

" 2. Take away this ball, and put a longer cord to it, which may be put down through the hollow axis of the bearer M X, and wheel G, and fix a weight to the end of the cord below the machine; which weight, if left at liberty, will draw the ball from the edge of the whirling board to its centre.

" *Bodies moving in orbits have a tendency to fly out of these orbits.* Draw off the ball a little from the centre, and turn the winch; then the ball will go round and round with the board, and will gradually fly off farther and farther from the centre, and raise up the weight below the machine; which shows, that all bodies revolving in circles have a tendency to fly off from these circles, and must have some power acting upon them from the centre of motion, to keep them from flying off. Stop the machine, and the ball will continue to revolve for some time upon the board; but as the friction gradually stops its motion, the weight acting upon it will bring it nearer and nearer to the centre in every revolution, until it brings it quite thither. This shows, that if the planets met with any resistance in going round the sun, its attractive power would bring them nearer and nearer to it in every revolution, until they fell upon it.

" 3. *Bodies move faster in small orbits than in large ones.* Take hold of the cord below the machine with one hand, and with the other throw the ball upon the round board as it were at right angles to the cord, by which means it will go round and round upon the board. Then observing with what velocity it moves, pull the cord below the machine, which will bring the ball nearer to the centre of the board, and you will see, that the nearer the ball is drawn to the centre, the faster it will revolve; as those planets which are nearer the sun revolve faster than those which are more remote; and not only go round sooner, because they describe smaller circles, but even move faster in every part of their respective circles.

" 4. *Their centrifugal forces shown.* Take away this ball, and apply the bearer M X, whose centre of motion is in its middle at *w*, directly over the centre of the whirling-board *d*. Then put two balls, V and U. of equal weights upon their bearing wires, and having fixed them at equal distances from their respective centres of motion, *w* and *x*, upon their silk cords, by the screw-nuts, put equal weights in the towers O and P. Lastly, put the catgut strings, E and F, upon the grooves, G and H, of the small wheels, which, being of equal diameters, will give equal velocities to the bearers above, when the winch, B, is turned: and the balls, U and V, will fly off towards M and N; and will raise the weights in the towers at the same instant. This shows, that when bodies of equal quantities of matter revolve in equal circles with equal velocities, their centrifugal forces are equal.

" 5. Take away these equal balls, and instead of them, put a ball of six ounces into the bearer, M X, at a sixth part of the distance, *w z*, from the centre, and put a ball of one ounce into the opposite bearer, at the whole distance *x y*, which is equal to *w z*, from the centre of the bearer; and fix the balls at these distances on their cords, by the screw nuts at top; and then the ball U, which is six times as heavy as the ball V, will be only a sixth part of the distance from its centre of motion; and, consequently will revolve in a circle of only a sixth part of the circumference of the circle in which V revolves. Now, let any equal weights be put into the towers, and the machine be turned by the winch; which, as the catgut string is on equal wheels below, will cause the balls to revolve in equal times; but V will move six times as fast as U, because it revolves in a circle of six times its radius; and both the weights in the towers will rise at once. This shows, that the centrifugal forces of revolving bodies, or their tendencies to fly off from the circles they describe are in direct proportion to their quantities of matter multiplied into their respective velocities; or into their distances from the centres of their respective circles. For, suppose U, which weighs six ounces, to be two inches from its centre of motion *w*, the weight multiplied by the distance is 12: and supposing V, which weighs only one ounce, to be 12 inches distant from the centre of motion x, the weight 1 ounce, multiplied by the distance 12 inches, is 12. And as they revolve in equal times, their velocities are as their distances from the centre, namely, as 1 to 6.

If these two balls be fixed at equal distances from their respective centres of motion, they will move with equal velocities; and if the tower, O, have six times as much weight put into it, as the tower, P, has, the balls will raise their weight exactly at the same moment. This shows that the ball U, being six times as heavy as the ball V, has six times as much centrifugal force, in describing an equal circle with an equal velocity.

"6. *A double velocity in the same circle is a balance to a quadruple power of gravity.* If bodies of equal weights revolve in equal circles with unequal velocities, their centrifugal forces are as the squares of the velocities. To prove this law by an experiment, let two balls, U and V, of equal weights be fixed on their cords at equal distances from their respective centres of motions w and x; and then let the catgut string, E, be put round the wheel K, whose circumference is only one half of the circumference of the wheel H or G, and over the pulley, s, to keep it tight; and let four times as much weight be put into the tower P, as into the tower O. Then turn the winch B, and the ball, V, will revolve twice as fast as the ball U, in a circle of the same diameter, because they are equidistant from the centres of the circles in which they revolve; and the weights in the towers will both rise at the same instant; which shows, that a double velocity in the same circle will exactly balance a quadruple power of attraction in the centre of the circle. For the weights in the towers may be considered as the attractive forces in the centres, acting upon the revolving balls; which, moving in equal circles, is the same thing as if they moved in one and the same circle.

"7. *Kepler's problem.* If bodies of unequal weights revolve in unequal circles, in such a manner that the squares of the times of their going round are as the cubes of their distances from the centres of the circles they describe; their centrifugal forces are inversely as the squares of their distances from those centres. For, the catgut string remaining as in the last experiment, let the distance of the ball V, from the centre *x*, be made equal to two of the cross divisions on its bearer; and the distance of the ball U, from the centre *w*, be three and a sixth part; the balls themselves being of equal weights, and V making two revolutions by turning the winch, in the time that U makes one: so that if we suppose the ball, V, to revolve in one second, the ball, U, will revolve in two seconds, the squares of which are one and four: for the square of 1 is only 1, and the square of 2 is 4; therefore the square of the period, or revolution of the ball V, is contained four times in the square of the period of the ball, U. But the distance of V is 2, the cube of which is 8, and the distance of U is $3\frac{1}{6}$, the cube of which is 32 very nearly, in which 8 is contained four times; and therefore, the squares of the periods of V and U are to one another as the cubes of their distances from *x* and *w*, which are the centres of their respective circles. And if the weight in the tower, O, be four ounces, equal to the square of 2, the distance of V from the centre *x;* and the weight in the tower, P, be 10 ounces, nearly equal to the square of $3\frac{1}{6}$, the distance of U from *w;* it will be found, upon turning the machine by the winch, that the balls, U and V, will raise their respective weights at the same instant of time. Which confirms that famous proposition of *Kepler*, *viz.* That the squares of the periodical times of the planets round the sun are in proportion to the cubes of their distances from him; and that the sun's attraction is inversely as the square of the distance from his centre: that is, at twice the distance, his attraction is four times less; and thrice the distance, nine times less; at four times the distance, sixteen times less; and so on, to the remotest part of the system.

"8. *The absurdity of the Cartesian vortexes.* Take off the catgut string, E, from the great wheel D, and the small wheel H, and let the string, F, remain upon the wheels D and G. Take away also the bearer, M X, from the whirling-board *d*, and instead thereof put the machine, A B, upon it, fixing this machine to the centre of the board by the pins *c* and *d*, *fig.* 3, in such a manner, that the end, *e f*, may rise above the board to an angle of 30 or 40 degrees. In the upper side of this machine are two glass tubes, *a* and *b*, close stopped at both ends; and each tube is about three quarters full of water. In the tube, *a*, is a little quicksilver, which naturally falls down to the end, *a*, in the water, because it is heavier than its bulk of water; and on the tube *b*, is a small cork, which floats on the top of the water at *e*, because it is lighter; and it is small enough to have liberty to rise or fall in the tube. While the board, *b*, with this machine upon it, continues at rest, the quicksilver lies at the bottom of the tube *a*, and the cork floats on the water near the top of the tube *b*. But, upon turning the winch, and putting the machine in motion, the contents of each tube will fly off towards the uppermost ends, which are farthest from the centre of motion, the heaviest with the greatest force. Therefore the quicksilver in the tube, *a*, will fly off quite to the end *f*, and occupy its bulk of space the e, excluding the water from that place, because it is lighter than quicksilver; but the water in the tube, *b*, flying off to its higher end, *e*, will exclude the cork from that place, and cause the cork to descend towards the lowermost end of the tube, where it will remain upon the lowest end of the water near *b;* for the heavier body having the greater centrifugal force, will therefore possess the uppermost part of the tube; and the lighter body will keep between the heavier and the lowermost part.

"This demonstrates the absurdity of the Cartesian doctrine of the planets moving round the sun in vortexes: for, if the planet be more dense or heavy than its bulk of the vortex, it will fly off therein, farther and farther from the sun; if less dense, it will come down to the lowest part of the vortex, at the sun: and the whole vortex itself must be surrounded with something like a great wall, otherwise it would fly quite off, planets and altogether. But, while gravity exists, there is no occasion for such vortexes; and when it ceases to exist, a stone thrown upwards will never return to the earth again.

"9. *If one body move round another, both of them must move round their common centre of gravity.* If a body be so placed on the whirling-board of the machine, *fig.* 1, that the centre of gravity of the body be directly over the centre of the board, and the board be put into ever so rapid a motion by the winch B, the body will turn round with the board, but will not remove from the middle of it; for, as all parts of the body are in equilibrio round its centre of gravity, and the centre of gravity is at rest in the centre of motion, the centrifugal force of all parts of the body will be equal at equal distances from its centre of motion, and therefore the body will remain in its place. But, if the centre of gravity be placed ever so little out of the centre of motion, and the machine be turned swiftly round, the body will fly off towards that side of the board on which its centre of gravity lies. Thus, if the wire, C, with its little ball, B, *fig.* 4, be taken away from the demi-globe, A, and the flat side, *e f*, of this demi-globe be laid upon the whirling board of the machine, so that their centres may coincide; if then the board be turned ever so quick by the winch, the demi-globe will remain where it was placed. But, if the wire, C, be screwed into the demi-globe at *d*, the whole becomes one body, whose centre of gravity is now at or near *d*. Let the pin, *c*, be fixed in the centre of the whirling-board, and the deep groove, *b*, cut in the flat side of the demi-globe, be put upon the pin, so that the pin may be in the centre of A, see *fig.* 5, where this groove is represented at *b*, and let the whirling-board be turned by the winch, which will carry the little ball B, *fig.* 4, with its wire C, and the demi-globe A, all round the centre pin *c i;* and then, the centrifugal force of the little ball B, which weighs only one ounce, will be so great, as to draw off the demi-globe A, which weighs two pounds, until the end of the groove, at *e*, strikes against the pin *c*, and so prevents the demi-globe, A, from going any farther: otherwise, the centrifugal force of B would have been great enough to have carried A quite off the whirling-board. Which shows, that if the sun were placed in the very centre of the orbits of the planets, it could not possibly remain there; for the centrifugal forces of the planets would carry them quite off, and the sun with them; especially when several of them happened to be in any one quarter of the heavens. For the sun and planets are as much connected by the mutual attraction that subsists between them, as the bodies A and B are by the wire C, which is fixed into them both. And even if there were but one single planet in the whole heavens to go round ever so large a sun in the centre of its orbit, its centrifugal force would soon carry off both itself and the sun. For, the greatest body placed in any part of free space might be easily moved: because if there were no other body to attract it, it could have no weight or gravity of itself; and, consequently, though it could have no tendency of itself to remove from that part of space, yet it might be very easily moved by any other substance.

"10. As the centrifugal force of the light body, B, will not allow the heavy body, A, to remain in the centre of motion, even though it is 24 times as heavy as B; let us now take the ball A, *fig.* 6, which weighs six ounces, and connect it by the wire, C, with the ball B, which weighs only one ounce; and let the fork, E, be fixed into the centre of the whirling-board: then hang the balls upon the fork by the wire, C, in such manner, that they may exactly balance each other; which will be when the centre of gravity between them, in the wire at *d*, is supported by the fork. And this centre of gravity is as much nearer to the centre of the ball A, than to the centre of the ball B, as A is heavier than B, allowing for the weight of the wire on each side of the fork. This done, let the machine be put into motion by the winch; and the balls, A and B, will go round their common centre of gravity *d*, keeping their balance, because either will not allow the other to fly off with it. For, supposing the ball, B, to be only one ounce in weight, and the ball, A, to be six ounces; then, if the wire, C, were equally heavy on each side of the fork, the centre of gravity, *d*, would be six times as far from the centre of the ball, B, as from that of the ball A; and, consequently, B will revolve with a velocity six times as great as A does; which will give B six times as much centrifugal force as any single ounce of A has: but then, as B is only one ounce, and A six ounces, the whole centrifugal force of A will exactly balance the whole centrifugal force of B: and, therefore, each body will detain the other so as to make it keep in its circle. This shows, that the sun and the planets must all move round the common centre of gravity of the whole system, in order to preserve that just balance which takes place among them. For, the planets being as unactive and dead as the above balls, they could no more have put themselves into motion than these balls can; nor have kept in their orbits without being balanced at first with the greatest degree of exactness upon their common centre of gravity, by the Almighty Hand that made them and put them in motion.

" Perhaps it may be here asked, that since the centre of gravity between these balls must be supported by the fork, E, in this experiment, what prop it is that supports the centre of gravity of the solar system, and consequently bears the weight of all the bodies in it; and by what is the prop itself supported? The answer is easy and plain; for the centre of gravity of our balls must be supported, because they gravitate towards the earth, and would therefore fall to it: but, as the sun and planets gravitate only towards one another, they have nothing else to fall to; and therefore have no occasion for any thing to support their common centre of gravity: and if they did not move round that centre, and consequently acquire a tendency to fly off from it by their motions, their mutual attractions would soon bring them together; and so the whole would become one mass in the sun: which would also be the case if their velocities round the sun were not quick enough to create a centrifugal force equal to the sun's attraction.

" But after all this nice adjustment, it appears evident, that the Deity cannot withdraw his regulating hand from his works, and leave them to be solely governed by the laws which he has impressed upon them at first. For if he should once leave them so, their order would in time come to an end; because the planets must necessarily disturb one another's motions by their mutual attractions, when several of them are in the same quarter of the heavens; as is often the case: and then, as they attract the sun more towards that quarter than when they are in a manner dispersed equably around him, if he were not at that time made to describe a portion of a larger circle round the common centre of gravity, the balance would then be immediately destroyed; and, as it could never restore itself again, the whole system would begin to fall together, and would in time unite in a mass at the sun. Of this disturbance we have a remarkable instance in the comet which appeared lately; and which, in going last up before from the sun, went so near to Jupiter, and was so affected by his attraction, as to have the figure of its orbit much changed; and not only so, but to have its period altered, and its course to be different in the heavens from what it was last before.

"11. Take away the fork and balls from the whirling-board, and place the trough, AB, *fig.* 7, thereon, fixing its centre to the centre of the whirling-board by the pin H. In this trough are two balls, D and E, of unequal weights, connected by a wire, *f*, and made to slide eaisly upon the wire, C, stretched from end to end of the trough, and made fast by nut-screws on the outside of the ends. Let these balls be so placed upon the wire C, that their common centre of gravity, *g*, may be directly over the centre of the whirling-board. Then turn the machine by the winch, ever so swiftly, and the trough and balls will go round their centre of gravity, so as neither of the balls will fly off; because, on account of the equilibrium, each ball detains the other with an equal force acting against it. But if the ball, E, be drawn a little more towards the end of the trough at A, it will remove the centre of gravity towards that end from the centre of motion; and then, upon turning the machine, the little ball, E, will fly off, and strike with considerable force against the end A, and draw the great ball, B, into the middle of the trough, Or, if the great ball, D, be drawn towards the end, B, of the trough, so that the centre of gravity may be a little towards that end from the centre of motion, and the machine be turned by the winch, the great ball, D, will fly off and strike violently against the end, B, of the trough, and will bring the little ball, E, into the middle of it. If the trough be not made very strong, the ball D, will break through it.

" 12. *Of the tides.* The reason why the tides rise at the same absolute time on opposite sides of the earth, and consequently in opposite directions, is made abundantly plain by a new experiment on the whirling-table. The cause of their rising on the side next the moon every one understands to be owing to the moon's attraction : but why they should rise on the opposite side at the same time, where there is no moon to attract them, is perhaps not so generally understood.

For it would seem, that the moon should rather draw the waters, as it were, closer to that side, than raise them upon it, directly contrary to her attractive force. Let the circle *a b c d*, *fig.* 8, represent the earth, with its side, *c*, turned towards the moon, which will then attract the waters so, as to raise them from *c* to *g*. But the question is, why should they rise as high at that very time on the opposite side, from *a* to *e?* In order to explain this, let there be a plate AB, *fig.* 9, fixed upon one end of the flat bar DC ; with such a circle drawn upon it as *a b c d*, in *fig.* 8, to represent the round figure of the earth and sea ; and such an ellipsis, as *e f g h*, to represent the swelling tide at *e* and *g*, occasioned by the influence of the moon. Over this plate, AB let the three ivory balls, *e*, *f*, *g*, be hung by the silk lines *h*, *i*, *k*, fastened to the tops of the crooked wires HIK, in such a manner, that the ball, at *e*, may hang freely over the side of the circle *e*, which is farthest from the moon, M, at the other end of the bar ; the ball, at *f*, may hang freely over the centre, and the ball, at *g*, hang over the side of the circle *g*, which is nearest the moon. The ball, *f*, may represent the centre of the earth, the ball, *g*, some water on the side next the moon, and the ball, *e*, some water on the opposite side. On the back of the moon, M, is fixed the short bar, N, parallel to the horizon, and there are three holes in it above the little weights *p*, *q*, *r*. A silk thread, *o*, is tied to the line, *k*, close above the ball *g*, and passing by one side of the moon M, goes through a hole in the bar N, and has the weigh, *p*, hung to it. Such another thread *n*, is tied to the line *i*, close above the ball *f*, and passing through the centre of the moon, M, and middle of the bar N, has the weight, *q*, hung to it, which is lighter than the weight. A third thread, *m*, is tied to the line, *h*, close above the ball *e*, and passing by the other side of the moon M, through the bar N, has the weight, *r*, hung to it, which is lighter than the weight *q*.

The use of these three unequal weights is to represent the moon's unequal attraction at different distances from her. With whatever force she attracts the centre of the earth, she attracts the side next her with a greater degree of force, and the side farthest from her with a less. So, if the weights be left at liberty, they will draw all the three balls towards the moon with different degrees of force, and cause them to make the appearance shown in *fig.* 10; by which means they are evidently farther from each other than they would be if they hung at liberty by the lines *h*, *i*, *k*; because the lines would then hang perpendicularly. This shows, that as the moon attracts the side of the earth which is nearest her with a greater degree of force than she does the centre of the earth, she will draw the water on that side more than she draws the centre, and so cause it to rise on that side: and as she draws the centre more than she draws the opposite side, the centre will recede farther from the surface of the water on that opposite side, and so leave it as high there, as she raised it on the side next to her. For, as the centre will be in the middle between the tops of the opposite elevations, they must of course be equally high on both sides at the same time.

But, upon this supposition, the earth and moon would soon come together: and undoubtedly they would, if they had no motion round their common centre of gravity, to create a degree of centrifugal force sufficient to balance their mutual attraction. This motion they have; for as the moon goes round in her orbit every month, at the distance of 240,000 miles from the earth's centre, and of 234,000 miles from the centre of gravity of the earth and moon, so does the earth go round the same centre of gravity every month at the distance of 6000 miles from it; that is, from it to the centre of the earth. Now as the earth is, in round numbers, 8000 miles in diameter, it is plain, that its side next the moon is only 2000 miles from the common centre of gravity of the earth and moon; its centre 6000 miles distance therefrom: and its farthest side from the moon 10,000. Therefore, the centrifugal forces of these parts are as 2000, 6000, and 10,000; that is, the centrifugal force of any side of the earth, when it is turned from the moon, is five times as great as when it is turned towards the moon. And as the moon's attraction, expressed by the number 6000, at the earth's centre, keeps the earth from flying out of this monthly circle, it must be greater than the centrifugal force of the waters on the side next her; and, consequently, her greater degree of attraction on that side is sufficient to raise them; but as her attraction on the opposite side is less than the centrifugal force of the water there, the excess of this force is sufficient to raise the water just as high on the opposite side. To prove this experimentally, let the bar, DC, *fig.* 9, with its furniture, be fixed upon the whirling-board of the machine, *fig.* 1, by pushing the pin, P, into the centre of the board; which pin is in the centre of gravity of the whole bar with its three balls, *e*, *f*, *g*, and moon M. Now, if the whirling-board and bar be turned slowly round by the winch, until the ball, *f*, hang over the centre of the circle, as in *fig.* 11, the ball, *g*, will be kept towards the moon by the heaviest weight *p*, *fig.* 9, and the ball *e*, on account of its greater centrifugal force, and the lesser weight *r*, will fly off as far to the other side, as in *fig.* 11. And so, whilst the machine is kept turning, the balls, *e* and *g*, will hang over the ends of the ellipsis *l f k*. So that the centrifugal force of the ball, *e*, will exceed the moon's attraction just as much as her attraction exceeds the centrifugal force of the ball *g*, whilst her attraction just balancing the centrifugal force of the ball *f*, makes it keep in its circle. And hence it is evident, that the tides must rise to equal heights at the same time on opposite sides of the earth. This experiment, to the best of my knowledge, is entirely new.

" *The earth's motion demonstrated.* From the principles thus established, it is evident, that the earth moves round the sun, and not the sun round the earth : for the centrifugal law will never allow a great body to move round a small one in any orb whatever ; especially when we find, that if a small body moves round a great one, the great one must also move round the common centre of gravity between them. And it is well known, that the quantity of matter in the sun is 227,000 times as great as the quantity of matter in the earth. Now, as the sun's distance from the earth is at least 81,000,000 of miles, if we divide that distance by 227,000, we shall only have 357 for the number of miles that the centre of gravity between the sun and earth is distant from the sun's centre. And as the sun's semidiameter is one-fourth of a degree, which, at so great a distance as that of the sun, must be no less than 381,500 miles, if this be divided by 357, the quotient will be $1268\frac{2}{3}$, which shows, that the common centre of gravity between the sun and earth is within the body of the sun ; and is only the $1068\frac{2}{3}$ part of his semidiameter from his centre towards his surface.

" All globular bodies, whose parts can yield, and which do not turn on their axes, must be perfect spheres, because all parts of their surfaces are equally attracted toward their centres. But all such globes which do turn on their axes will be oblate spheroids ; that is, their surfaces will be higher, or farther from the centre, in the equatoreal than in the polar regions. For, as the equatoreal parts move quickest, they must have the greatest centrifugal force ; and will therefore recede farthest from the axis of motion. Thus, if two circular hoops, AB and CD, *fig.* 12, made thin and flexible, and crossing one another at right angles, be turned round their axis EF, by means of the winch *m*, the wheel *n*, and pinion *o*, and the axis be loose in the pole or intersection *e*, the middle parts A, B, C, D, will swell out so as to strike against the sides of the frame at F and G, if the pole *e*, in sinking to the pin E, be not stopt by it from sinking farther: so that the whole will appear of an oval figure, the equatoreal diameter being considerably longer than the polar. That our earth is of this figure is demonstrable from actual measurement of some degrees on its surface, which are found to be longer in the frigid zones than in the torrid: and the difference is found to be such as proves the earth's equatoreal diameter to be thirty-six miles longer than its axis. Seeing then, the earth is higher at the equator than at the poles, the sea, which like all other fluids naturally runs downward, or towards, the places which are nearest the earth's centre, would run toward the polar regions, and leave the equatoreal parts dry, if the centrifugal force of the water, which carried it to those parts, and so raised them, did not retain and keep it from running back again toward the poles of the earth."

The apparatus of Mr. *Ferguson* to illustrate the theory of the tides is rather incorrect and inconsistent. I should not have inserted his description of it in this work, but that some uninformed readers might be possessed of a whirling-table with such an apparatus. It is an appendage that may hereafter be dispensed with. Mr. *Ferguson* would soon have been convinced of his mistake, had he considered that the earth and moon are unnaturally represented, when attached, as in this apparatus, to the end of a bar or rod; for, the earth does not always present the same face to the moon. There remains yet to be contrived a new apparatus to illustrate the theory of the tides. It has been clearly demonstrated that the equilibrium of the waters of the ocean is distributed by the unequal gravitation of their different particles to the sun and to the moon; and this equilibrium cannot be restored till the waters come in from all quarters, and rise up around the line connecting the centres of the earth and of the luminary. The spherical ocean must acquire the form of a prolate spheroid, generated by the revolution of an ellipse round its transverse axis. The waters will be highest in that place which has the luminary in its zenith, and in the antipodes to that place; and they will be most depressed in all those places which have the luminary in their horizon.

Fig. 13 represents a whirling-table constructed upon a more elegant and accurate principle than the foregoing. The axes A, A, upon which the bearers and other apparatus are turned, are of iron, and of much greater length than those of Mr. *Ferguson's;* consequently, the motion is quite easy and without any shake, to which the others are liable. Their lower ends turn upon two brass screws, fixed on projecting brass pieces, B, B. One of these long axes and bottom screw are made hollow, to admit of the line passing down as before described. The weights to represent the centrifugal and gravitating forces, are of the same shape as shown at *fig.* 14: for the former, they are applied to a pin on the brass carriage of the bearers, C; this carriage is supported on the screws by four small pullies, and it is moved with the least possible friction. A small pin is fixed under the carriage, which catches into a brass ratchet placed on the base of the bearer, and serves to keep the weight from being drawn to the vertical line or frame before action. The frame work, D, to carry the weights for gravitation, consists of two outside brass pillars for the frame-work, and two inner ones for the sliding piece, E, on which the weights are placed, there being a slit cut into them for that purpose. The silken line connected to the piece, E, goes over a pulley at the top, and under another at the bottom, along the bearer, to the carriage, C, to which there is a contrivance at the underside of a ratchet wheel admitting the line to be extended to the length desired in a ready manner. The various apparatus are fastened to the pullies by two milled brass screws, *a*, *a*. The band going upon the pullies is crossed and passes through two apertures in the front piece of the table F, where it is crossed again over a vertical pulley, and the large vertical mahogany wheel, G. By turning the handle, H, the operator, while standing, puts the machine into motion in the most convenient manner.

The wheel, G, is here represented as placed in a vertical position. In some cases it may be the most convenient for the experimenter; but, when attached to the table in a horizontal direction, there is less length of band, somewhat less friction, and less workmanship; on which account, this method is preferred by several operators.

For the convenience of package, the board, K, may be taken from the feet, and the feet, being joined by hinges, may be folded together, so as to occupy but little room.

The two brass screws, shown at *b*, *b*, are to extend the distance of the wheel, should the band at any time by use get slackened.

The forms of the rest of the apparatus are nearly similar to what has already been described of Mr. *Ferguson's*. The following is the only addition necessary to be described to the reader: I represents a glass globe of about seven inches in diameter, fixed by an axis to a brass ring; a semicircular ring fixed to this, and connected to a pulley on the point of the piece F; this sphere is to be filled with water and spirits, or oil of turpentine, tinged with a red or blue colour. When the wheel, G, is put into motion, the glass sphere will have a rapid one about its axis perpendicular to the piece F, and the oil of turpentine will be seen to separate into a number of curious globules, and settle into a position in the middle of the sphere, in a longitudinal direction, about the axis of motion. The cause of this effect must be evident to the reader, who has consulted that shown by the apparatus, *fig.* 3, being like that; from the lightest fluid body giving place to the heaviest in a centrifugal motion, and the centre of motion here being extended to an axis, the oil will of course move from the interior surface of the globe towards the axis of motion, and be in a longitudinal cylindrical form. A contrivance to this sphere has been added, to give at the same time a motion about an axis perpendicular to the other. The effect of this compound motion on the coloured oil, I have not been a witness to. A point on the surface of the sphere will describe a motion similar to a figure of 8. This experiment serves as another refutation of the vortices of *Descartes*, and is fully related in the Leçons de Physique par *Nollet*, 1784, Tome Second, p. 52, et seq.

The sphere is contrived to open by unscrewing one of the two suspending screws, to admit of the sphere being filled with other fluids, when desired.

LECTURE XXXIII.

HYDROSTATICS.

THE science describing the mechanical affection of fluids, and their efficacy in the production of those natural phenomena of which they are the immediate causes, is properly and usually called by foreign writers *hydrodynamics*, and is divided into two branches; *hydrostatics*, whose object is the relation between the weight of water and other bodies; and *hydraulics*, which treats of the motion of water through pipes, conduits, &c. Hydrostatics is now used by us with greater latitude to denote the science describing the properties of all fluids, but principally those of water; and explaining the motions thereof, whether in pipes, pumps, syphons, fire engines, jets d'eau, &c. and I shall endeavour to exhibit them to the eye and to the understanding in the most easy and familiar manner.

The principles of all knowledge are founded in mind; the mind of man, either animated by desire or pressed by necessity, puts in action its various energies, and unfolds the seeds of knowledge. We reason right when we apply to any subject only those ideas which are derived from its real nature; but we fall into every kind of error, when from prejudice or any other cause we conclude before we have reasoned, reason before we know, and presume that we know before we have examined. The more simple the object is in itself, the sooner the mind discovers its various relations, and is enabled to rise from one truth to another, till it reach those that are the most abstract. Hence it is, that greater progress has been made in geometry than in any other science purely intellectual, because the suppositions are few and simple, and only abstract properties are considered; but, when material objects are considered, of whose elementary parts we know neither the size nor the form, nor the laws by which they are actuated, and yet endeavour to foretel their effects, to calculate their efforts, and to determine their action, nature shows herself independent and faithful to the laws prescribed to her, and of which we are ignorant; she contradicts our views, disconcerts our projects, and renders our efforts useless.

It is thus in hydraulics; experience contradicts theory, and the ablest mathematicians, a *Newton*, a *Bernouilli*, and a *D'Alembert*, have not been able to make them coincide. Hence, our knowledge of this subject is very limited; for, though during a course of ages, many men of the greatest abilities have endeavoured to investigate this subject, it is still enveloped in obscurity and surrounded with difficulties. We are still ignorant of the true laws which govern the motions of water; even assisted by experiments for 150 years, we can hardly yet be said to have determined either the time, the quantity, or the actual velocity with which water spouts from a given aperture. We know still less concerning the uniform motion of those rivers and rivulets which water the surface of the earth. That you may form a more true idea of the little we know, I shall give you a short sketch of some of those things of which we are ignorant.

Were it necessary to calculate the velocity of a river, whose breadth, depth, and inclination are known, to determine the height its waters will be raised by the accession of another river, or how much they will be lowered by a cut from it; to fix the inclination necessary to give to the waters of an aqueduct a certain velocity, or determine the capacity of the bed with a given inclination that shall be sufficient to supply a certain town with water; so to trace the windings for a river, that it may not endeavour to change the bed in which it has been inclosed; to calculate the water expended by a conduit pipe, where the length, diameter, and charge are given; to determine how much the waters of a river, a canal, &c. will be raised by a bridge, a lock, &c. and how far the return of the water will be sensible; to assign the best form for the mouth of canals and rivers; to find the most advantageous figures for ships and boats, that they may cleave the water, and move therein with the least resistance; to calculate the force necessary to move a body floating on the water; all these questions, and an infinite number more of the same kind, cannot be solved in the present state of this science. It seems almost incredible, that we are still so far ignorant what the impulse of water striking directly against a plane surface is equal to; consequently still more so when it strikes against convex and other curved surfaces.

Nay, it is not clear, when we consider the immense aqueducts, &c. of the ancients, whether they did not know more than we do on these subjects. You have already seen that their knowledge of mechanics was greater than is generally believed, and to *Archimedes* we are indebted for the general principles of hydrostatical knowledge; he even goes so far in one of his works as to establish, that any point whatsoever of a fluid mass is equally pressed in all directions, and examines the conditions necessary to make a body floating on a fluid take and preserve an equilibrium of situation.

One-hundred years after him, *Ctesibius* and *Hero* of Alexandria invented several ingenious hydraulic engines; among these, we may reckon the fountain of compression, still known by the name of *Hero's* fountain, and the bent syphon so useful for decanting liquors: thus, without knowing distinctly the spring and weight of the air, they employed these agents with success.

S. I. Florentinus appears to be the first, who has given us any account of the motion of fluids; he was inspector of the public fountains under the Emperors

Nerva and *Trajan;* and in a work on this subject, has made some very valuable remarks, and communicated some excellent observations. From his time, to that of *Benedict Castelli*, the friend of *Galileo*, we find no traces of any further progress. The necessity of guarding against the continual damages from the overflowings of the rivers in Italy, induced *Urban* the Eighth, who had invited *Castelli* to Rome as a teacher of the mathematics, to request he would apply himself to this subject: the result of his inquiries is contained in his treatise entitled, *Della Mesura dell Acque correnti;* in this he was ably followed by *Guglielmini Frisi*, &c. But still we scarcely find one writer who acquiesces in the solution of another; even the great *Newton*, who paid particular attention to some of these subjects, is not very consistent with himself. Here, therefore, much remains for future experiments, in order that new axioms may be established,—new discoveries made; thus the great volume of nature is calculated by the Omniscience of its Author to afford scope to the virtuous, and honourable employment to sages and philosophers, till they be removed to a more perfect system, where the First will take place of second causes, when the film which is spread over the corporeal eye will be removed, new objects presented, and new scenes disclosed under the aspect and illumination of a brighter sun.

OF FLUIDITY.

A fluid is generally defined to be a body whose parts are so loosely connected together, that they easily yield to any force impressed upon them, and move freely amongst each other. In this sense, fire, air, mercury, water, &c. are considered as fluids.

In almost every physical speculation, wherever experiment can reach, the subject will admit of some illustration; where that is denied, the reasonings are in general vain and conjectural. We do not know the form of the parts of which fluids are composed, and can make no experiments to reduce them into the primary particles of which they are composed.

There is nothing more different in accuracy and truth, than that apprehension which is adequate to the purposes of the vulgar, and that which ought to satisfy the investigation of a philosopher. Thus, there is nothing more obvious to the vulgar than fluidity, yet the philosopher finds it a property difficult to be conceived, and which he could not give credit to, if it was not rendered familiar to him by custom and experience; it is a physical phenomenon which has not yet been explained, and of which it is very difficult to give a clear account. How, indeed, can we comprehend that a material and incompressible substance can be composed of parts so elementary, so moveable among themselves, and yet with so little adherence, as to assume immediately the form of any vessel into which it is poured; that its surface is always parallel to the horizon, or perfectly level; that in syphons, or when agitated by the wind, it makes isochrone vibrations, or undulations like a pendulum; that it runs off where favoured by the smallest descent, &c.

Fluidity is caused by a certain degree of fire, which, when employed for this purpose, disappears with respect to any other sensible or perceptible effect. It does not dilate the volume, but resists the particular attachment of the parts.

Some have endeavoured to give mechanical ideas of a fluid body, by comparing it to a heap of sand; but the impossibility of giving fluidity by any kind of mechanical comminution, will appear by considering two of the circumstances necessary to constitute a fluid body.

1. That the parts, notwithstanding the greatest compression, may be moved, in relation to each other, with the smallest conceivable force, or will give no sensible resistance to motion within the mass in any direction.

2. That the parts shall gravitate to each other, whereby there is a constant tendency to arrange themselves about a common centre, and form a spherical body; which, as the parts do not resist motion, is easily executed in small bodies. Hence the appearance of drops always takes place when a fluid is in proper circumstances.

Let us now see how far these qualities may be procured by mechanical operations, even executed without those imperfections that necessarily attend human performance.

A body of sand, the particles of which should be perfectly spherical and polished, or smooth, would only imitate a fluid in being able to spread itself upon a smooth plane, instead of lying in a heap, but would possess neither of the two qualities essential to a fluid body.

For, a heap of spherical bodies, if compressed, could not move by relation to each other, except by a force sufficient to balance that by which, in this case, they are necessarily retained in their places. Neither can the parts of the supposed body of sand cohere, either to themselves, or to other bodies, in the manner of fluids, as in each particle the mass of gravitating matter must be great in proportion to the point of contact by which they should cohere. If the cohesion of the particles of sand increased, the spreading quality would be diminished.

Many other differences might be pointed out; but supposing every thing else favourable to the mechanical theory, yet still there would remain to be explained the operation of fire, which is so essential to fluidity. This would lead us too far, as it would render it necessary for us to investigate the nature of that resistance by which the figure of bodies is preserved in their hardness.

By fire, hard bodies are made soft; but it is not properly that portion of loose fire which augments the volume of bodies that renders them fluid: their fluidity is occasioned by *a certain quantity of fire, which then disappears, with regard to any other sensible or perceptible effect.*

OF THE GRAVITY OF THE PARTICLES OF FLUIDS, AND ITS EFFECTS ON THE FLUIDS THEMSELVES.

Though no one finds any difficulty in allowing that water and other fluids are really ponderous, and do actually gravitate when considered as a whole body, being convinced by their own senses that a vessel weighs less when empty than when filled with any fluid, and weighs heavier the more it contains; yet, in the early times of philosophy, there were those that believed fluids did not gravitate *in proprio loco*, as they termed it; that is, when immersed in the same, or a different fluid. A simple experiment will show you that they were mistaken, and that fluids lose nothing of their weight *in proprio loco*. Here is a hollow glass ball, *plate* 2, *Hpdrostatics*, *fig.* 5, furnished with a brass stop-cock, and made so heavy as to sink in water. Exhaust it of its air, and then shut the cock. I exhaust the air from it, to give room to a quantity of water equal in bulk to the exhausted air. Suspend it now from the end of this balance, so that the bottle and the stop-cock may be under the surface of the water in the jar, and then counterpoise it by a weight in the opposite scale. Now open the cock, the water will then rush in, and the ball will preponderate, and bear down the beam on which it hangs; clearly proving, *that the parts of water retain their gravity in water*, so as to press and bear down upon the parts beneath them, otherwise the phial would not become heavier upon the admission of the water; and you will find that the ball overbalances the counterpoise, as much as the weight of the quantity of water in the ball.

To facilitate the explanation of hydrostatic phenomena, it has been usual for the writers on this subject to consider the fluid in a vessel as cut into several horizontal planes or imaginary surfaces, and to consist of a vast number of small, equal, lubicrous, spherical globules. Thus, *plate* 1, *Hydrostatics*, *fig.* 2, A B C D may represent a vessel consisting of such globules; a b, c d, e f, imaginary horizontal surfaces. Besides this imaginary horizontal division of a fluid, they often consider it as divided into perpendicular columns, from the top to the bottom of the fluid, as at *plate* 1, *fig.* 3.

Though fluids are subject to the laws of gravity as well as solids, yet their fluidity occasions some peculiarities necessary to be noticed. The parts of a solid are so connected together as to form but one and the same whole;

their effort is as it were concentrated in a single point, called the centre of gravity. This is not the case with fluids; the particles here are all independent of each other, are extremely moveable, yielding to the least effort that tends to separate the one from the other.

The parts of a fluid gravitate independently of each other. This is a natural consequence of their fluidity, or their not adhering together; whereas the particles of a solid cohere together, and gravitate as one mass. It is clear, from this principle, that if a hole be made in a vessel full of water, the power necessary to prevent the fluid from running out, must be able to overcome the column of the fluid pressing on the hole, and that the weight to be overcome is the same, whether there be only this column of the fluid acting on the part stopping the hole, or whether the vessel be full.

This will be rendered clearer by an experiment. This cylindrical glass vessel, A B C D, *plate* 1, *fig.* 1, *Hydrostatics*, has a hole at bottom; a cylindrical tube of brass passes through, and is fitted to this hole; a small piston or plug is fitted to this tube; and being well greased, slides easily up and down; a long wire is fixed to this piston, to be hooked on to one arm of the balance E F. On the upper part of this short tube I can occasionally fit a glass tube, G H, which is exactly of the same diameter as the brass tube, and of the same height with the large vessel.

I fit the glass tube in its place, and pour water up to this mark. Now, I shall put weights into the scale at the opposite arm of the balance, till the piston just begins to rise; I then take away the glass tube, and fill the large vessel with water to the same height, and you see that the same weight as before overcomes the pressure. Now, as the same weight overcomes the pressure, whether a column of water be only the size of the piston, or whether the vessel be full of water, it is clear, that particles of water exercise their gravity independent of each other; but, if the mass of water contained in the outer vessel were changed into ice, to raise the piston you must use a weight equal to the weight of the whole column of ice.

The surface of a fluid which is contained in an open vessel, and free from all external impediments, will be level, or parallel to the horizon. No part of a fluid can stand higher than the rest; for, if any part be raised, it must descend by the force of gravity, and, in so doing, will spread and diffuse itself till it be on a level with the other parts; for, having gravity, and yielding easily to every impression, they obey the force of gravity, and slip down till they come to a level.

As the gravity of the particles reduces the upper surface to a level, so likewise it occasions a pressure on the lower part, greater or less in proportion to their depths below the surface, each part containing a pressure equal to the weight of all those that lie above it; consequently, the particles which are at equal depths below the surface are equally pressed. In other words, as the upper surface of the fluid is parallel to the horizon, and as the lower parts sustain the upper, and are pressed by them, this pressure will be in proportion to the incumbent matter, that is, to the height of the fluid above the particle that is pressed: but, as the upper surface of the fluid is parallel to the horizon, all the points of any surface that you may conceive within the fluid, parallel to the horizon, are equally pressed. Should this equality of pressure be at any time destroyed, and there be a less pressure on one part of the surface than on the other parts, the fluid yielding to any impression, this part will be moved, that is, will ascend till the pressure becomes equal.

To confirm this by experiment, take this glass tube, open at both ends, and, stopping one end with your finger, immerge the other in water. The water, you see, rises in the tube; but the tube being full of air, while you keep your finger upon the orifice, the rise is but small; but if you take away your finger, that the air which is compressed may escape, the water will rise up into the tube, and not be at rest till it attains the same height with the external water.

Solids, you know, make no effort but in the direction of gravity, or perpendicularly downwards; *but fluids exert a force of pressure equal to their gravity, in all directions, and in all equally.* This follows from the nature of a fluid, for its particles yield to any impression, and are easily moved; therefore no drop will remain in its place, if, whilst it is pressed by a superincumbent fluid, it be not equally pressed on all sides, because, being a fluid itself, it will yield to every impression, and begin to move, unless it be acted upon by equal forces, in all possible directions. But it cannot move, because the surrounding drops resist on all sides its motion with the same force that it endeavours to move, and consequently the drop must remain at rest; and what is thus proved of one drop, holds equally true of all; consequently, *all the parts of a fluid, at equal depths below the surface, are pressed equally in all directions.*

Take the several glass tubes, A, B, C, D, *plate* 1, *fig.* 4, which are open at both ends: immerge them in water to the same depth, their upper orifice being stopped by the finger. Upon taking away the finger, the water will rise to the same height in all the tubes, though it enters the lower end in very different directions: in A the pressure is directed upwards, in B downwards, in C sidewise, and in D obliquely, but the pressure is equal in each. If you pour a greater quantity of water into the vessel, it will rise equally in the tubes; *so that the fluids press in all directions, and that with a force proportionable to their heights.*

The same experiment is perhaps rendered still clearer by pouring some mercury into the tubes. The tubes for this purpose are smaller than those I used in the former experiment: some of them are straight, and others bent at various angles. Though the tubes are open at both ends, one of the extremities should be closed till after the immersion, to prevent the mercury from falling out.

On immerging the lower end of these tubes in water, you will find the mercury ascend towards the upper end of the tubes. It is to be remarked, concerning this experiment, that whatever be the angles at which the tubes are bent, and however they are inclined to the horizon, if before immersion the mercury in all the tubes be on a level, it will continue so after immersion, provided all the tubes be immersed to the same depth.

Consequently, when it has been proved, that the pressures of a fluid are as the surface pressed, and their depths from the surface of the incumbent fluid, it will follow, that the pressure of a fluid is not only propagated in all directions, but that the quantities of the pressure at the same depths, and on a given surface, are equal in all directions.

On a cursory view of the subject, you may consider it as a kind of mechanical paradox, that the pressure of a fluid upwards, or in a direction contrary to that of gravity, should be nothing more than a consequence of gravity itself; but it is very easy to show, from mechanical principles, that a force acting in a given direction may communicate pressure through a number of intermediate bodies, so that the last body shall be impelled in any direction whatever, even in that which is directly contrary to the original impulse; and this is the case in respect of the particles which compose fluids

From the foregoing experiments it will appear, I hope, very clear to you, *that the perpendicular pressure of any fluid column is, from some unknown connexion of the parts, diffused laterally in every direction; and at the same depth, the pressures estimated in any direction are equal to each other.* What has been proved of water, obtains in all other substances that are fluid, and under the influence of gravity.

OF THE ACTION OF FLUIDS AGAINST THE BOTTOMS, SIDES, AND TOPS, OF THE VESSELS IN WHICH THEY ARE CONTAINED.

The bottom and sides of a vessel containing a fluid (and the top also, when the fluid is raised above it in a tube) are pressed by the parts of the fluids which immediately touch them; and as action and re-action are equal, these parts all sustain an equal pressure.

But as the pressure of fluids is equal every way, the bottoms and sides of the vessels are pressed as much as the neighbouring parts of the fluid; but this action, you have seen, increases in proportion to the height of the fluid, and is every way equal at the same depth.

This pressure depends on the height, not the quantity, of the fluid; consequently, when the height of the fluid, and the area or surface pressed, remain the same, the action upon this surface will always be equal, however the figure of the vessel be changed. In other words, the pressure which the bottom of the vessel sustains from the fluid contained in it, whatever be the shape of the vessel, is equal to the weight of a pillar of the fluid, whose base is equal to the area of the bottom, and whose height is the same with the perpendicular height of the fluid.

That this is the case, in vessels that are equally wide from top to bottom, is plain and obvious, because the bottom of such a vessel does actually sustain such a column of fluid, a column in this case equal to the whole weight of the fluid. Here, the whole weight of the fluid contained in the vessel, and no other force besides, presses upon the bottom, and is consequently proportional to the quantity of matter contained in the vessel, which quantity is as the surface of the bottom, and the perpendicular height above it.

But that the case should be the same in irregular vessels is not so easy to conceive; for instance, that in a vessel which from a large bottom grows narrower as it rises, the bottom should bear the same pressure when the vessel is filled, as it would were the vessel equally wide throughout from bottom to top, seems strange and surprizing, yet it is what necessarily follows from the nature of fluidity.

Before I proceed to illustrate this proposition by experiment, I shall endeavour to explain it to you by diagrams; considering it, 1. When the vessel is narrower at the top than the bottom; 2. When it is wider at the top than the bottom.

1. If the vessel M N F T, *plate* 1, *fig.* 6, be smaller at the top than at the bottom, the pressure upon the bot-

tom, E T, is as great as the pressure upon the bottom of a cylindrical vessel, A B C D, *plate* 1, *fig*. 7, of equal base and height, when they are both filled with water, or any other fluid, notwithstanding there will be considerably more water in the cylinder than in the cone.

Make F G, O R, in the cylinder, *plate* 1, *fig*. 7, equal to O R, the base of the column, M N O R, of the cone, *plate* 1, *fig*. 6. Now, as these columns of water are equal, it is evident, that O R in the cylinder, and O R in the cone, sustain an equal weight, and consequently an equal pressure.

It is also evident, from what has been explained at the beginning of this article, that every part equal to O R, at the bottom of the cylinder, is pressed just as much as O R. But I have to prove, that every such part of the bottom of the cone is pressed equal to O R at the bottom of the cylinder; for instance, the part, F I, *fig*. 6, is pressed just as much as O R is. I have shown you, that all equal parts of a fluid, at equal depths from the surface, are pressed equally; but the drops contiguous to F I and O R are at equal depths from the surface; therefore these drops, and consequently the psrts F I and O R are equally pressed. Now, as every part equal to O R, in the bottom both of the cone and cylinder, is pressed as much as O R, and since one bottom is equal to the other, it follows, that the whole pressure upon F T is equal to the whole pressure upon C D.

You may still, though you have seen that the proposition is true, have a difficulty in discovering the reason why it is true; for it certainly does not seem likely, at first view, that F I, with no more water over it than fills the space F E I, should be pressed as much as O R, which sustains the whole column M N O R.

But it must be remembered, that the water, F E I, presses upwards against F E, as well as downwards against F I; and if a hole were made at F E, and a tube soldered therein, the water, by the pressure upwards, would be sustained in the tube at the same height that it stands in the vessel: therefore this pressure is equal to the weight of as much water as would fill the tube C A F E.

Now, the same pressure which would support the water in such a tube acts upon F E ; but the re-action of F E downwards is equal to the action upwards against it: that is, E F keeps the water down with a force equal to that with which it endeavours to rise, equal to the difference of weight between F E I and M N O R : and as F I sustains both the weight of the water F E I, and the action or force with which the water is kept from rising, but O R sustains only the weight of water M N O R, the pressure upon F I will be equal to the pressure upon O R, and the same may be proved of any other column. Therefore the bottom of the cone is as much pressed by the weight of water which fills the cone, and this re-action together, as the same bottom would be pressed by the weight of as much water as would fill up the whole cylindrical space C B F T ; that is, *the pressure upon the bottom of a conical vessel, is equal to the pressure upon the bottom of a cylindrical one of the same base and height.*

The same mode of reasoning may be applied to the vessel D B L P, *plate* 1, *fig*. 8, which consists of two cylindrical parts N M L P, a great cylinder at the bottom, and D B I V a lesser one at the bottom. For the pressure upon L P, when the vessel is full of water, will be as great as if the vessel was as wide at top as at bottom ; that is, as great as it would be upon the same bottom L P, supposing the vessel was a uniform cylinder, whose base was L P, and height L F.

L A, and O R, two equal drops at the same depth, are pressed equally ; and O R, having as much water to sustain, is as much pressed as if the vessel was a uniform cylinder. Therefore L A, or C P, or any other equal part at the bottom, and consequently the whole bottom is as much pressed in one case as it would be in the other. Indeed L A, or C P, have less water to sustain than O R ; but the column, N T L A, presses upwards against N T with a force equal to the difference between this column and D B O R, or to the weight of as much water as would fill the space F E N T ; for if a hole was made at N T, and a tube, F E N T, soldered into it, the pressure against the bottom of the tube would support water in it to the height N T, the same height it stands at in the tube D B I V. Now, the re-action of N T downwards is equal to the action upwards against it, that is, the force with which N T keeps the water below it, down against L A, is equal to the force with which this water presses against N T.

L A is therefore pressed down, not only by the weight of the water N T L A, but likewise by the re-action of N T, which is equal to the weight of as much water as would fill F E N T, and make N T L A equal D B R O; from which it follows, that the weight and re-action together on L A, are equal to the weight on D B R O, by which O R is pressed; and the same may be proved of every other equal portion of the whole bottom and cover, and therefore, by the weight and re-action, L P is as much pressed as if it were at the bottom of a cylindrical vessel F H L P, having the same dimensions at the top as at the bottom, and filled with water to the height L F.

Though the pressure upon F T, *plate* 1, *fig.* 6, is equal to the pressure upon C D, when both vessels are filled with water to the same perpendicular height; yet, if they were filled with ice, or any other solid substance, instead of water, C D would be more pressed than F T.

For C D, whether the vessel be filled with ice or water, sustains the whole weight of the body which rests upon it, and no more; but F T, which, besides the weight M N F T, sustains the re-action of the sides M N F T, when the vessel is filled with water, has only the weight to sustain when it is filled with ice; for ice, or any other solid body, does not press upwards. This is a property, which, as it only rises from the nature of a fluid, belongs to fluids only; F T will therefore be only pressed by the weight of the ice, and consequently will be less pressed than C D, in proportion as the cone is less than the cylinder, when their bases and heights are equal.

For the same reason L P, *plate* 1, *fig.* 8, if it were full of ice, would be as much less pressed than when it is full of water, as the quantity of matter contained in the compound vessel, N M L P, is less than the quantity of matter contained in a cylindrical vessel, whose base is L P, and height L F.

The second case of the proposition is when the vessel A B O R, *plate* 1, *fig.* 9, is wider at top than bottom. For here also the pressure of any fluid upon the bottom, O R, of it, is the same as in a cylindrical vessel, S T O R, of an equal base, and filled with the same sort of fluid to the same height.

For the bottom O R, in either case, sustains just the same quantity of fluid, and consequently the same quantity of matter. If it be the bottom of a cylinder, then it sustains no more than the column S T O R, because the vessel holds no more. If it be the bottom of an inverted cone, as A B O R, then it sustains only the same column; for though that vessel holds more than this, yet all the rest of the fluid is supported by the sides A O, B R, and therefore does not press on the bottom.

Thus you see, that whether a vessel be narrower or wider at the top than the bottom, the pressure upon the bottom is the same as in a cylindrical vessel of the same base and height; for when it is narrower at the top than at the bottom, though it holds less water than the cylindrical one would, yet the pressure is not less, because the re-action of the sides supplies the defect; and when it is wider at the top than at the bottom, though it holds more water than the cylindrical one would hold, yet the pressure is not greater, because the sides support the excess.

Let us now confirm by experiment what I have endeavoured to render plain without it. The apparatus on the table, *plate* 1, *fig.* 10, is designed for this purpose. It is sometimes called the apparatus of *Paschal*, sometimes the apparatus for illustrating the *hydrostatic paradox*. It consists of three vessels, *fig.* 11 and 12, and A B C D, *fig.* 10, all of which are of the same size at bottom, and of the same height, and may be screwed alternately on the brass barrel E F, *fig.* 10, in which a piston slides up and down with ease. One of the vessels, *fig.* 12, is cylindrical; an other, A B C D, *fig.* 10, is an inverted cone, wider at top than bottom; the third, *fig.* 11, is a tube screwed to a plate, which makes the bottom the same size as that of the other two; it has a funnel at top, to prevent the water, in making the experiment, from splashing over.

I shall first screw the cylindrical vessel to the barrel, pushing down the piston as low as it will go, then hook the wire of the piston to the chain suspended from the short ends of the steelyards G H, I K. Now pour water into the cylinder up to the mark in the inside thereof, and find what weights, suspended from the longer arms of the steelyard, will raise the piston; then take the cylindrical vessel from the barrel.

Subsitute the vessel A B C D, *fig.* 10, which is like an inverted cone, in place of the former; fill it with water to the mark, as before, and hook on the wire of the piston to the steelyards; and though the quantity of water is now many times greater than what was in the cylinder, yet the same counterpoise will raise the piston.

Take off the conical vessel, and screw on the tubular one; and though this holds a much smaller quantity than either of the former, still it requires the same counterpoise.

The friction of the piston being the same in every case, makes no alteration in the experiment.

To show that the lateral pressure is equal to the perpendicular pressure upon a larger scale, and in a manner which relates more to the preceding experiment, here is an apparatus, *plate* 1, *fig.* 5, with different tubes, that communicate with each other. The middle one is a large glass tube or cylinder, A B; the lower end is firmly cemented into a strong brass hoop; to the sides of this hoop are soldered the brass tubes G, H, into each of which a glass tube is cemented. One of these, E F, is parallel to the large glass vessel A B; but the other, C D, is inclined thereto. The inclined tube is sometimes furnished with a joint, that the inclination may be varied at pleasure.

Pour water into the tube E F; this will run through G, into the larger vessel A B, and rise therein; and if you continue pouring water until it come to any height therein, as I K, and then leave off, you will find that the surface of the water in the small tubes EF, C D, is at the same height; the perpendicular altitude is the same in all the three tubes, however small the one may be in proportion to the other. This experiment clearly proves, that the small column of water balances and supports the large column; which it could not do if the lateral pressures at the bottom were not equal to each other. Whatever be the inclination of the tube C D, still the perpendicular altitude will be the same as that of the other tubes, though to that end the column of water must be much longer than those in the upright tubes.

Hence it is evident, that a small quantity of fluid may, under certain circumstances, counterbalance any quantity of the same fluid.

Hence also it is evident, *that in tubes which have a communication, whether they be equal or unequal, short or oblique, the fluid always rises to the same height.* Consequently, water cannot be conveyed by means of a pipe that is laid into a reservoir, to any place that is higher than the reservoir.

It has been asserted, that the ancients were ignorant of this principle, and knew not the use of pipes for conveying water up hills; but this assertion is not true; they did know the use of pipes, but chose to employ aqueducts in their stead, for reasons we cannot now with certainty ascertain.

Our next experiment proves, with great clearness, the hydrostatical parodox, that very great weights may be balanced by a very small weight of water without its acting to any mechanical advantage; but, more particularly, it also proves, that its pressure upwards is equal to its pressure downwards, and all this even to those who have no previous knowledge of hydrostatical principles. The apparatus, *plate* 2, *fig.* 1, consists of two large thick boards C D, E F, connected together by leather, like a pair of bellows; hence it is usually called the *hydrostatic bellows.* A long bras or glass pipe is fixed to the bottom board; so that water being poured in at the top will pass between the two boards. In the apparatus before you the boards are oval; the longest diameter is eighteen inches, the shortest one sixteen inches. I have poured water enough into the bellows to keep the boards asunder, and have put six hundred-weights on the top of the boards. I shall now pour water into the tube, to the height of three feet, and it will push up all the weights. Thus the water in the pipe, which weighs but a quarter of a pound, sustains six hundred pounds weight. Take off the weights, and try, by pressing upon the upper board, to force the water out at the upper tube; your strength, you find, is scarce sufficient for the pressure. Thus you see clearly how great a pressure upwards is exerted by the water.

Fig. 16, *plate* 1, represents another instrument for proving, that the pressure of fluids is in proportion to their perpendicular height, without any regard to their quantity.

A B C D is a box, at one end of which, as at *a*, is a groove from top to bottom, for receiving the upright glass tube I, which is bent to a right angle at the lower end, as at *fig.* 17; and to that end is tied the end of a large bladder K, *fig.* 17, which lies in the bottom of a box. Over this bladder is laid the moveable board M, *fig.* 18, in which is an upright wire. Leaden weights, N, N, *fig.* 16, to the amount of sixteen pounds, with holes in the middle, are put upon the wire, over the board, and press upon it with all their force.

The bar, p, is then put on, to secure the tube from falling, and keep it upright; and then the piece, E F G, is to be put on, to keep the weights in a horizontal position, there being a round hole at e. Within this box are four upright pins, to prevent the board at first from pressing on the bladder.

Pour water into the tube at top; this will run into the bladder; and after the bladder has been filled up to the board, continue pouring water into the tube, and the upward pressure of the fluid will raise the board with all the weight upon it, even though the bore of the tube should be so small, that less than an ounce of water would fill it.

Upon this principle mathematicians assert, *that the same quantity of water, however small, may produce a force equal to any assignable one, by encreasing the height and base upon which it presses.* Dr. *Goldsmith* mentions having seen a strong hogshead split by this means. A strong though small tube of tin, twenty feet high, was inserted in the bung-hole; water was poured into this to fill the hogshead, and continued till it rose within about a foot of the top of the tube; the hogshead then burst, and the water was scattered about with incredible force.

As the bottom of a vessel bears a pressure proportional to the height of the liquor, so likewise do those parts of the sides which are contiguous to the bottom, because the pressure of fluids is equal every way; and as the pressure which the lower parts of a fluid sustain from the weight of those above them exerts itself equally every way, and is likewise proportional to the height of the incumbent fluid, the sides of a vessel must every where sustain a pressure proportional to their distance from the upper surface of the liquor. Whence it follows, that in a vessel full of liquor, the sides bear the greatest stress in those parts next the bottom; and that the stress upon the sides decreases with the increase of the distance from the bottom, and in the same proportion; so that in vessels of considerable height, the lower parts ought to be much stronger than the upper, to be able to withstand the greater pressure.

OF THE ACTION OF FLUIDS ON BODIES IMMERSED IN THEM.

Archimedes, in his two books *De Insidentibus Humido*, is the first we know of who made inquiries concerning the sinking and floating of bodies in fluids; their relative gravities, their levities, their situations, and positions. He was also probably the first that ever attempted to determine in what proportion bodies differ from one another as to their specific gravities, and this he effected in order to discover the cheat of the workmen, who had debased king *Hiero's* crown; and though the means he employed was certainly much inferior to what would now be used, yet he was so pleased with his discovery, that not being able to contain his joy, like a madman leaping from the bath naked as he was, he is said to have run about the streets of Syracuse, crying out his Ευρηκα wherever he came. Before I proceed to explain this interesting subject, I must define some terms which I have only loosely explained to you.

The density of a body is the quantity of matter which it contains under a given bulk.

The density of a body is therefore measured by the proportion which its quantity of matter bears to its bulk; for the more numerous the particles of matter are in the same portion of space, the greater is the density of the body; and the fewer the particles, the less the density.

The specific gravity of a body is the weight of it when the bulk is given; or, the specific gravity of a body is its weight compared with another body of the same magnitude.

It is called the *specific gravity*, because it is the comparative weight of different species or sorts of bodies. Thus, when the specific gravity of gold is said to be to that of water as 19 to 1, the meaning is, that, bulk for bulk, or under equal dimensions, the weight of gold is to that of water as 19 to 1; or that a cubic inch of gold will weigh 19 times as much as a cubic inch of water.

The specific gravities of bodies are as their densities, for the specific gravity is the weight of a given bulk, and the weight of bodies is as their quantities of matter; therefore the specific gravity of a body is as the quantity of matter contained in a given bulk, that is, as its density.

The specific gravities of bodies are inversely as their bulks when their weights are equal. The specific gravities of bodies are, you have already seen, as their densities, and the densities of bodies are inversely as their bulks, when the weights are equal. Thus, if the specific gravity of gold be to that of silver as 19 to 11, and a cylinder of gold 11 inches high weigh a pound, a cylinder of silver having an equal base, and weighing a pound, must be 19 inches high; for since the specific gravities are 19 to 11, the bulks, that is, the heights must be as those gravities inversley, or as 11 to 19. If the specific gravity of mercury be to that of water as 14 to 1, and a cylinder of mercury of a certain weight be 30 inches high, then a cylinder of water of equal base must be 420 times as high, so that the height of the cylinder of water will be 14 times 30, or 420 inches, or 35 feet.

The *magnitude* of a body is expressed by a number denoting its relation to some criterion generally used, and similar to itself, as a cubical inch, foot, &c.

The *absolute weight* of a body is relative, being expressed by a number denoting its relation to some arbitrary or conventional standard, as 1 pound, 1 ounce, of which it is a multiple or aliquot part; and in the same sort of matter, supposed to be homogeneous, it depends upon and varies as the magnitude.

The specific weight or gravity of the same species of matter, whether its magnitude be great or small, as of A, 2 A, or 3 A, is the same, being according to the definition of the weight of a given bulk. The object therefore of specific gravities is to distinguish different species of matter from each other, in one of their most obvious qualities, weight of matter contained in a given space.

The *weight* of any portion of matter is easily ascertained, but it is not always easy to measure the space occupied by a body or its *magnitude*, and in some instances it cannot be effected without artificial methods.

It is found expedient to employ as a criterion some pure and homogeneous substance, as distilled water, whose specific gravity, or weight of a given bulk, is nearly the same at all times; and by comparing this with other substances, the ratio of their specific gravities may be discovered; and denoting the specific gravity of water by any number taken at pleasure, the numbers expressing the specific gravities of other bodies are hence obtained.

Thus the weight of one cubical foot of pure rain-water is equal to 1000 ounces avoirdupois; if its specific gravity be denoted by 1 or 1000, the weight of one cubical foot, or other measure of another substance, is hence found, and tables of the specific gravities of bodies constructed.

One ounce avoirdupois is equal 437.5 grains, and one ounce troy equal 480 grains; consequently, as 1 oz. avoirdupois is to 1 oz. troy, so is 437.5 to 480; and as 1 avoirdupois pound is to 1 pound troy, so is 437.5×16 to 480×12, or 1750 to 1440.

A cubic foot of water is equal 1000 ounces or 62.5 pounds avoirdupois, but 62.5 pounds avoirdupoise is to x (or number of troy pounds contained in a cubic foot of water) as $\frac{1}{1750}$ to $\frac{1}{1440}$, (or inversely as the number of grains in each) as 1440 to 1750; and, consequently, by the rule of three x is equal $\frac{6.25 \times 175}{144}$ lb. equal 75.95 lb. troy. One cubic inch of water equals $\frac{1000 \times 437.5}{1738}$ gr. 253.18 grains equal $\frac{253.18}{437.5}$, or 57869 parts of one avoirdupois ounce equals $\frac{253.18}{480}$, or 5274 parts of one troy ounce.

It follows, from what has been already proved to you in the foregoing part of this lecture, *that when a solid is immersed in a fluid, it is pressed by that fluid on all sides, and that pressure encreases in proportion to the height of the fluid above the solid:* but you may also prove it directly by experiment. Thus, tie a leather bag to the end of a glass tube, and fill it with murcury, immerge the bag in water, but so that the upper or open end of the tube may be always above the surface of the water; the pressure of the water against the bag will raise the mercury in the tube, and the ascent of the mercury will be in proportion to the height of the water above the bag.

When a solid is immersed in a fluid to a great depth, the pressure against the upper part differs very little from the pressure against the under part; whence bodies very deeply immersed, are as it were equally pressed on all sides; but a pressure which is equal on all sides, may be sustained by soft bodies without any change of figure, and by very brittle bodies without their breaking.

Take a piece of soft wax of an irregular figure, and an egg, and inclose them in a bladder full of water; place it in this box, and put on the moveable cover which will bear on the bladder; you may then place on this cover a weight of 100 or even 150 pounds, without breaking the egg, or anywise altering the figure of the wax.

You have already seen, that fluids press upon bodies to which they are contigious every way, and on all sides, but the pressure upon each part is not the same; the altitude of the fluid is every where the measure of its force, and the several parts of the same body being at different depths, must needs be differently affected: you have therefore to consider which of these impressions will prevail.

Now, it is evident that the lateral pressures do all balance each other, being equal, as arising from equal altitudes of the fluid, and opposite in their directions; so that from these the body is nowise determined to any motion.

But a body immersed in a fluid is pressed more upwards than it is downwards; for those parts of the fluid which are contiguous to the under surface have a greater altitude, and therefore a greater force than those that are contiguous to the upper surface; the body must, therefore, be more violently elevated by the former than depressed by the latter, and would therefore ascend by the excess of force, were it devoid of gravity.

For, when a solid body is immersed in a fluid, it presses down, and endeavours to descend by the force of its gravity; but it cannot descend, without moving as much of the liquid out of its place as is equal to it in bulk; it is therefore resisted, and pressed upwards by a force equal *to the weight of as much of the fluid as is equal in magnitude to the bulk of the body*, being the difference in weight of two columns of the fluid, whereof one reaches to the upper, the other to the under surface of the body.

I will endeavour to illustrate this by a diagram : when any hard body, as a piece of lead, is immersed in water, the lower part of it, m n, *plate* 1, *fig*. 13, must be continually pressed upwards just as much as the water itself in the same place with the lead is pressed upwards. Now, the force with which the water, m n, is pressed upwards, is exactly equal to the force with which it would be pressed downwards if the lead were out of the way ; for, every part of a fluid is pressed as much upwards as it is downwards. The force with which m n would be pressed downwards if the lead were out of the way, would be equal to the weight of the incumbent column, or of as much water as would fill the whole space E H m n; therefore, the force with which m n is pressed upwards, and, consequently, the force with which the piece of lead is pressed upwards, is equal to the weight of as much water as would fill the whole space E H m n, or the whole space H P n o, if this space be taken equal to E H m n.

Let us next consider the force with which this piece of lead is pressed downwards ; this force is just equal to the weight of as much water as is above it, that is, it is equal to the weight of the column E H r s.

The difference, therefore, of the two pressures will be the difference in weight between the two columns E H m n, E H r s ; for, the weight of the former is equal to the pressure upwards, and the weight of the latter is equal to the pressure downwards; consequently, the pressure upwards will be as much greater than the pressure downwards, as the weight of the water, E H m n, is greater than the weight of water E H r s.

But the difference between these two weights is just as much as would fill the space, r s m n, which the body fills ; for just so much water added to E H r s would make it equal to E H m n : consequently, *the body is pressed more upwards than it is downwards by a force equal to the weight of as much water as would fill the space taken up by the body*. In other words, the body is acted upon by two forces in contrary directions; but, the force with which the fluid acts upon it to make it ascend exceeds the force by which it presses it downwards;

and this excess is equal to the weight of as much of the fluid, whatever it be, as would fill the space taken up by the body.

The case will be the same, whatever be the figure of the body immersed; for, suppose it to be a cone, T S L V, *plate* 1, *fig*. 15, then as every equal part of a fluid at the same depth is pressed equally in all directions, if V I be equal to L V, it follows, that these two parts of a thin sheet of fluid, F E, will be pressed upwards by equal forces; but V I is pressed as much upwards as downwards, therefore L V is pressed as much upwards as V I downwards.

Now, the force that presses V I downwards is the weight of the fluid H P V I, that is above it; consequently L V, where the bottom of the body is placed, is supported by a force equal to the weight of the column H P V I, and this column is equal to M H L V. Therefore the body is pressed upwards with a force that is equal to the weight of as much of the fluid as would fill the whole space M H L V.

The same body is in the mean time pressed downwards by the weight of all that fluid that is above any part of it, that is by the weight L T S V H M, and not merely by the column O W T S, which reaches from the surface to the top of the body. From hence it follows, that the difference between the centre column M H L V, or such a column as this would be if the body were out of the way, and the column, L T S V H M, is the difference between the pressure upwards and the pressure downwards. But this difference is plainly equal to as much of the fluid as would fill up the space the body takes up; the force, therefore, by which the fluid acts upon the body to make it ascend, exceeds the force by which it presses downwards, and this excess is equal to the weight of as much of the fluid as would fill the space taken up by the body.

But as all bodies by the force of gravity tend downwards, it is clear from what has been said, that it depends upon the absolute weight of the immersed body whether it shall ascend or descend. 1. If the weight of the body exceed that of an equal bulk of the fluid, the excess of force will tend downwards. 2. If the weight of the body be less than an equal bulk of the fluid, the upward pressure will prevail, and it will ascend. 3. If both be precisely equal, the body will remain at rest in any part of the fluid.

First then, *a body immersed in a fluid will sink, if it be specifically heavier than that fluid;* for it endeavours to descend by its own weight, and is supported by a force equal to the weight of an equal bulk of fluid, or of as much fluid as will fill the space taken up by the body. If, therefore, the body be specifically heavier than the fluid, *i. e.* bulk for bulk heavier than the fluid, its weight will be greater than the pressure upwards of the fluid which is to support it; and, consequently, this pressure will not so support it, as to keep it from sinking.

If you throw a stone into the water, it sinks, for it is specifically heavier than the water; that is, where the bulks are equal, the weight of the stone is greater than the weight of water; therefore, the force with which it endeavours to descend is greater than the excess of pressure upwards, which being too weak to sustain it, the stone sinks to the bottom.

A body that is immersed in a fluid will rise to the surface, and swim upon it, if it be specifically lighter than the fluid. A piece of cork, when it is immersed in water, is pressed by the water both upwards and downwards; but the pressure upwards exceeds the pressure downwards, and this excess is equal to the weight of as much water as is of the same bulk with the piece of cork; therefore, as far as the action of the water is concerned, the cork ought to rise to the top, and the cork itself being also specifically lighter than water, the force with which it endeavours to sink is less than the force which buoys it up; it must, therefore, on this account rise till it comes to the surface. Hence you see why fir, oak, and elm, that are specifically lighter than water, will swim in it; while ebony and guaiacum, that are specifically heavier, will sink therein.

There is always a part of any body that floats on the water, below the surface; and this part is equal in bulk to as much of the fluid as would weigh what the body weighs.

Let p t e i, *plate* 1, *fig.* 14, be a piece of cork, then s n e i, the part below the surface, A B, of the water, will be equal in bulk to as much water as would weigh what p t e i, the whole cork, weighs.

The force with which the water at e i is pressed upwards, is exactly equal to the force with which it would be pressed downwards, if the cork, p t e i, were out of the way, because every part of a fluid is pressed equally in all directions.

But the force with which e i would be pressed downwards if the cork were away, is equivalent to the weight of as much water as would fill the space taken up by the part of the cork below the water; and consequently, the force with which e i, the bottom of the cork is pressed upwards, is equivalent to the weight of as much water as would fill up the space s n e i, or the part of the cork below the surface.

If, therefore, the part which is below the surface have the same bulk as a quantity of water that would weigh what the whole cork weighs, then the pressure upwards will be equal to the weight of the cork, and keep it from sinking.

A body that has the same specific gravity with the fluid into which it is immersed, will rest in any part of the fluid wherever it happens to be placed. For the body, you know, endeavours to descend by its own weight, and is prevented from descending by a force equal to the weight of an equal bulk of fluid; but when the body and the fluid are of the same specific gravity, equal masses of each are of the same weight; and consequently, the force with which the body endeavours to descend, and the force which opposes the descent, are equal to each other; and, as they act in contrary directions, the body will rest between them, so as neither to sink by its own weight, nor to ascend by the pressure of the fluid upwards.

It is manifest from these positions, that if by any contrivance the specific gravity of a solid can be varied so as to be one while greater, another less, and then equal to the specific gravity of the fluid, wherein it is immersed, the body will sink, or rise, or remain suspended, according to the variations of its specific gravity. And this is the case in the experiment with these little glass images which I am now going to exhibit to you: these are made to ascend, or descend, or remain suspended at pleasure; the reason of which I shall explain to you after you have seen the experiment.

The images being set to float on the water, the top of the vessel must be covered with a bladder closely bound about the neck of the vessel, to the end that the air which lies on the surface of the water may not force its way out when it is condensed by the hand. The images themselves are nearly of the same specific gravity with the water, but rather a little lighter, and consequently float near the surface; the images being hollow, are full of air, which, by means of small holes in their heels, communicates with the air without. When the air which lies beneath the bladder is pressed by the hand, it presses on the surface of the water; and as the pressure is propagated through all the water, those portions which are contiguous to the heels of the images are thereby forced into the holes, by which means the air within is condensed, and at the same time the weight of the images is increased by the weight of the influent water; and when so much water is forced in, as to render the specific gravity of the images greater than that of the water, the images descend and fall to the bottom, where they remain as long as the pressure above continues; but when that is taken off by the removal of the hand, the condensed air in the images dilates and expands itself, and in so doing drives out the water, upon which account the images become specifically lighter than water, and of course ascend. As the pressure on the bladder is greater or less, so must the quantity of water be which is forced into the images; and therefore whenever it happens, that during the ascent or descent of an image, such a pressure is made as suffices to force in just as much water as is requisite to reduce the image to the same specific gravity with the water, the image stops and remains suspended; upon increasing the pressure it descends, and ascends if it be lessened. Some of the images begin to descend sooner or rise later than others, either because they are specifically heavier, or because the cavities in their legs are greater in some images in proportion to their magnitudes, than they are in others.

This is but an experiment of mere amusement; many and more important uses are the result of our being able to determine the specific gravities of bodies: to this, therefore, we shall now proceed.

All bodies when immersed in a fluid, loose the weight of an equal bulk of that fluid; in other words, every body immersed in a fluid loses a part of its gravity equal to the weight of the fluid, which would fill the space taken up by the body.

A piece of lead, or of any other substance, when it is immersed into water, is not so heavy as when it is out of water; for the water, you know, presses it more upwards than downwards, and the excess of the pressure upwards will support part of the weight. But this excess was shown to be equivalent to the weight of as much water as has the same bulk with the lead; and, consequently, since the body immersed must lose as much of its weight as the fluid can support, the lead will loose the weight of an equal bulk.

Thus a cubic foot of lead, r s m n, hanging by the string, L l, *plate* 1, *fig.* 13, will weigh less in the water than it does out of it, because the water by its pressure upwards against the lead will support a cubic foot of water, or one thousand avoirdupoise ounces; for so much a cubic foot of water weighs, and consequently so much of its weight the lead must lose.

Again, a body endeavours to descend by its whole weight; when it is immersed in a fluid, it is supported by a force equal to the weight of the same bulk of that fluid: and since these two forces act in contrary directions, the weight which the body retains in the fluid will be the difference between them, or it loses the weight of an equal bulk of the fluid.

The following experiment which is plain and decisive, will render the position self-evident; the apparatus for it consists of a beam, a small hollow cylindric bucket, A B, *plate* 2, *fig*. 2, and another cylinder, C D, which precisely fits the capacity of the bucket A B ; only a portion of one arm, E F, of the beam is represented in this figure. I suspend the bucket at one end of the beam; at the bottom of the bucket is fixed a strong thread of silk with a loop on the lower end ; to this loop the close cylinder is suspended ; I counterpoise these by a wedge at the other end of the beam ; I then set a jar of water under the cylinder, and gently lower the beam, and you see it becomes lighter and lighter upon the beam as the cylinder descends ; now it is quite immersed, and you see the equipoise is destroyed by the descent of the weight of the other arm. To show you how much weight is lost by the cylinder, I shall add the weight of a quantity of as much water as is equal in bulk to the cylinder ; that is, I shall fill the bucket, which you know is exactly of the same size ; and as I do it gradually, you observe the equipoise is restored by degrees, till the bucket is full, and then the beam is truly horizontal as at first, the loss of weight being restored by the equal cylinder of water in the bucket.

From what has been said, you now see clearly whence the loss of weight proceeds ; we see now, it is no otherwise lost than as it is sustained by the action of a contrary force ; you may therefore now see why the weight of a bucket of water is not perceived while it is in the water,—not because that weight is destroyed, but because it is supported ; not because fluids do not gravitate when they are in fluids of the same sort, but because there is a pressure in a contrary direction which is exactly equal to their gravity.

Since the weight, which a body loses when it is immersed, is always the weight of as much of that fluid as is equal in bulk to itself, it follows, that the weight lost by the body cannot at all depend either on the depth of the fluid itself, or the depth to which it is immersed therein. An anchor loses no more of its weight when it is at the bottom, than when it is just below the surface, for in either case it loses the weight of as much water as is equal in bulk to itself. It is not more easy to swim in deep than in shallow water, provided the water be not so shallow as to prevent one from striking freely; for, whatever be the depth of the water, a man loses the weight of as much water as is equal in bulk to his own body; for which reason shallow water will buoy him up with as great force as deep water. Indeed, it is easier to swim in the sea than in a river, because salt water is specifically heavier than fresh; and as a man loses the weight of as much salt water as is equal in bulk to his body, and in the river loses only the weight of an equal bulk of fresh water, the weight lost here being greater, salt water will buoy him up with the greater force.

There are very few, if any animals, that are specifically heavier than common water. The substances, indeed, of both animals and vegetables are specifically heavier; the floating of either is therefore to be attributed to the cells or receptacles interspersed within them, which are filled with air, oil, &c. substances lighter than water; so that, taken together, they form a mass, lighter than an equal bulk of common water.

Further, the bulk of the body is increased by distending the chest in inspiration; this has been tried by an experiment on a fat man of an ordinary size, by finding what weight he could support so as to have the top of the head just above water. When his breast was full of air, he was found to rise with fourteen pounds of lead without striking out in the least, and two ounces more would have kept him under; but when his breast was not thus distended, he could only bring up eleven pounds.

All equal solids, though of different specific gravities, when they are immersed in the same fluid, lose an equal weight. The weight which gold, silver, lead, stones, or any other body, loses in water, does not at all depend upon the sort or figure of the body, but upon its bulk or size. The stone loses the weight of an equal bulk of water, so does the lead; when, therefore, they are of the same size, they each of them lose the weight of the same quantity of water, that is, they lose an equal weight: but if two bodies of the same sort differ in size, they will lose different weights in proportion to their size.

To prove that the loss of weight is not affected by the absolute weight of the body, but depends on its size, alter the weight of the cylinder, by adding to or diminishing the number of shot contained in it, and then repeat the experiment with it and the bucket as before, and you will find the event the same.

In the same manner, you show that all bodies, however different their specific gravity may be, if their magnitudes be equal, do suffer an equal loss of weight in the same fluid; thus a cylinder of block-tin, equal in dimensions to the brass cylinder, but specifically lighter, being immersed in water, will lose the same weight.

Bodies of the same weight, but of different specific gravities, lose unequal parts of their weights when they are immersed in the same fluid. Thus, this piece of gold in one scale weighs just as much as a piece of copper in the other scale, when they are weighed in air. Now, hang them by horse-hairs on the hooks of your balance, and let them down into the glass vessels, and as soon as they are immersed in the water, the equilibrium will be destroyed, and the gold will outweigh the copper; for, as they have equal weights, their bulks are as their specific gravities; that is, the gold will be as much less than the copper as the specific gravity of the gold is greater than that of copper; the gold, therefore, because it is the smaller of the two bodies, will lose less of its weight than the copper does, and will consequently outweigh the copper.

If the gold and the copper be made of the same weight, when they are under water, then by drawing them out of the water, the copper will become the heavier; for when they were under the water, each of them lost as much of its weight as the water could sustain, that is, each of them lost a weight in proportion to its bulk: but the copper being the bigger of the two, lost the greater weight, and as the weight which they lost in water is recovered upon their being drawn up in the air, the copper recovers more weight than the gold, and will therefore outweigh the gold.

The weight lost by a solid immersed in a fluid is communicated to the fluid. Though a solid loses part of its weight when immersed in a fluid, you are not to suppose that the weight so lost by the solid is actually destroyed, but that it is imparted to the fluid, the fluid constantly gaining what the solid loses; for, if you put the vessel with the water wherein the cylinders were immersed into a scale, and counterpoise it, you will find, upon the immersion of the cylinder, it will preponderate with exactly the same weight that the cylinder loses.

Before I instruct you how to discover the specific weights of bodies, it will be necessary to note two curious facts, to show how lead or gold may be made to swim upon water, or how a light body, like wood, may be made to remain sunk at the bottom of a vessel of water.

A body that is specifically heavier than a fluid, may be supported in it by the pressure upwards, if the pressure downwards be taken away. As bodies specifically heavier sink, because the force wherewith they press downwards exceeds the pressure from beneath which opposes their descent, and the force wherewith they descend is equal to the difference of those pressures; if by any contrivance those two forces can be reduced to an equality, then the bodies will not descend, but remain in the fluid. To show this, here is a circular brass plate exactly fitted so as to cover the lower aperture of a cylindrical tube; I keep it close to the tube by this string, and now immerse the tube perpendicularly in water, till the plate of brass is plunged therein somewhat more than eight times its own thickness; when at this depth, the piece of brass is supported by the pressure of the water, and does not sink though I let go the string.

The brass endeavours to descend by its own weight, it is pressed upward by a weight equal to that of a cylindrical column of water, having the same base with the brass plate, and being eight times thicker; and because brass is eight times specifically heavier than water, the weight of a cylindrical column of water which presses upward, and the weight of the brass by which it endeavours to descend, will be equal, and consequently the brass will be just supported in equilibrio.

Again, a body that is specifically lighter than the fluid in which it is immersed, will not rise if the pressure upward be prevented. To prove this, here is a bit of cork so fitted to the bottom of this vessel, that the surfaces are every where in contact; now, if you pour mercury into the vessel, you will find that the cork will not ascend till it be separated from the bottom of the vessel.

The effect of a fluid's pressure in a direction contrary to that of gravity is here evinced by a very decisive experiment; as long as the fluid is prevented from communicating with the under surface, the cork continues attached to the bottom of the vessel partly by its own weight, and partly by the pressure of the mercury on its upper surface.

From what I have already explained, you must necessarily have discovered the rationale of sinking and swimming. You saw that when a body was, bulk for bulk, heavier than the fluid, by being immersed it loses only the weight of an equal bulk of the fluid, and consequently the residual or remaining gravity of the solid must carry it down to the bottom, or make it sink.

On the other hand, when the solid has less weight in the same bulk than the fluid, then it cannot by its weight displace or raise upwards its whole bulk of the fluid, but only so much of it as is equal to its own weight; and from this deficiency in weight it will be but partly immersed, and will therefore swim upon the upper part of the fluid.

Of all the animals, however, thrown into the water, man is the most helpless; the brute creation receive the art of swimming from nature, while man can only acquire it by practice; the one escapes without danger, the other sinks to the bottom. Some have asserted, that this arises from the different sensibilities each have of the danger; the brute unterrified at his situation struggles, while his very fears sink the lord of the creation.

But much better reasons may be assigned for this impotence of man in water, when compared to other animals; and one is, that he has actually more specific gravity, or contains more matter within the same surface than any other animal. The trunk of the body in other animals is large, and their extremities proportionably small; in man it is the reverse, his extremities are very large in proportion to his trunk. The specific weight of the extremities is proportionably greater than that of the trunk in all animals, and therefore man must have the greatest weight in water, since his extremities are the largest.

Besides this, other animals to swim have only to walk, as it were, forwards upon the water; the motion they give their limbs in swimming is exactly the same they use upon land; but it is different with man, who makes use of those limbs to help him forwards upon water, which he employs to a very different purpose upon land.

LECTURE XXXIV.

ON THE METHODS OF ESTIMATING THE SPECIFIC GRAVITY OF BODIES; ON AIR-BALLOONS, &C.

FROM the principles explained to you in the preceding lecture, it will be easy to show you in what manner you may estimate the specific gravities of different bodies, whether solid or fluid. Now, the specific gravity of a body is the weight of that body, under a known and determinate magnitude; as a cubic inch, a foot, &c. To acquire this knowledge, the body is to be weighed hydrostatically; that is, 1. In air; 2. In water. You know, that a body immersed in water displaces a volume of water exactly equal to its own, and that it loses a portion of its weight exactly equal to the volume displaced; we therefore obtain by this mode, 1. The weight of the body; 2. The weight of a volume of water perfectly equal in bulk to that of the body. These two weights, compared together, give the relation between the specific gravity of water, which we suppose to be known, and that of the given body, by making the following proportion, in which 1000* represents the specific gravity of water. The weight of the volume of water displaced by the body, is to the weight of this body, as 1000 is to a fourth term, representing the specific gravity of this body: for the specific gravities are as the weights of equal bulks; therefore the specific gravity of the fluid is to that of the body, as the weight lost in the fluid is to the whole weight.

Now, let us suppose a piece of gold to weigh 38 grains in air, and only 36 grains when weighed in water; it has therefore lost two grains. Reasoning, therefore, from what has been already proved, we say, the gold has lost the weight of as much water as is equal in bulk to itself. But the gold itself weighs 38 grains; consequently, bulk for bulk, the weight of water is to that of gold, or the specific gravity of the fluid to that of the solid, as 2 to 38; that is, as the weight of the fluid is to the whole weight.

Thus the whole art of comparing the specific gravites of bodies, consists in finding out what a body weighs in air, and how much of that weight is lost in water; and then dividing the first weight by the difference between the first and second weight, and the quotient of this division shows how many times the body is heavier than water.

The definition of specific gravity implies comparison. Some kind of body must be fixed upon, whose gravity must be made a standard for the gravity of other bodies of equal bulk to be compared with. This standard-body should have two properties; first, it must be easy to be had, or come at, upon all occasions; and, secondly, it should be of as fixed and unalterable a nature as possible, that there may be no variation in its gravity in equal bulks, in different times or places.

Now, as the best way of discovering the specific gravities of bodies is by immersion, the body must be of the fluid kind; and, among fluids, water is that which possesses in the highest degree the requisites for a standard. Distilled water is the least objectionable; next to this pure rain-water; but common water, for many purposes, will answer exceedingly well.

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* In hydrostatic calculation, water, as the standard from which all the respective gravities are taken, is reckoned as unity, or 1, 10, 100, 1000, &c. as the case requires.

The specific gravity or weight of a given bulk of distilled water is nearly at all times the same; and by comparing this with other substances, the ratio of their specific gravities may be discovered; and denoting the specific gravity of water, by any number, taken at pleasure, the numbers expressing the specific gravities of other bodies are hence given.

As the weight of one cubical foot of pure distilled water is equal to 1000 ounces avoirdupois, if its specific gravity be denoted by 1, or 1000, the weight of one cubic foot, or other measure, of other substances, is hence found, and tables of the specific gravities of bodies are formed.

One ounce avoirdupois is equal to 437.5 grains, and an ounce troy to 480 grains; consequently, one avoirdupois pound is to one troy pound, as 437.5 × 16 to 480 × 12, or as 1750 to 1440.

A cubic foot of water is equal to 1000 ounces avoirdupois, or 62.5 lb. avoirdupois; whence we find it to be equal 75.95 lb. troy.

A cubic inch of water is equal to 253.18 grains, or .57869 parts of an avoirdupois ounce; and 253.18 grains=.5274 parts of an ounce troy.

THE USE OF THE HYDROSTATIC BALANCE, IN DETERMINING THE QUALITY OF GOLD, &c.

Being able to determine the specific gravities of bodies, you will thence be able, by weighing metals in water, to discover their adulterations or mixtures, with greater exactness than by any other method whatever. By this means the counterfeit coin, which may be offered you as gold, will be easily detected, and known to be a baser metal.

The principal and distinguishing qualities of pure gold, are the simplicity, minuteness, and close cohesion of its parts; whereby a greater number of those parts is contained in less space than any other body with which we are acquainted.*

As all bodies weigh in proportion to their quantity of gravitating matter, under the same bulk, the specific weight of gold must be superior to that of other metals.

It follows from hence, that if gold be adulterated with any other metal, its specific gravity, or comparative weight, must be less in proportion to the quantity of alloy. The weight, therefore, of gold, is a sure criterion of its quality.

In order to determine the precise quantity of alloy compounded with gold, gold must be compared with some other mass as a standard, and their relative gravities be computed. I have already shown you, that water is the most convenient standard.

It will therefore, from what has been said, be easy for you, by means of an accurate hydrostatic balance, to determine its quality and real value. Weigh a piece of gold first in air, weigh it then in water, subtract its weight in water from its weight in air, and the difference shows the loss it has sustained by being weighed in a denser medium. Divide the weight in air by the loss of weight in water; the quotient shows the specific gravity, or how many times gold is heavier than water.

On the contrary, the specific gravity of sterling gold being known, if the weight in air of any piece of gold coin be divided by the specific gravity of sterling gold, the quotient shows what ought to be its loss in water; and if it be found to lose more, the gold is bad, or has too much alloy.

Gold is about eighteen times as heavy as common water; the specific gravity of sterling gold being to the weight of water as 17.793 to 1. If, therefore, a guinea weighs in air 129 grains, when weighed in water it must lose 7.25, or 7 1-4 grains of its weight; because, as 7.250 is to 129, so is 1 to 17.793; so that a quantity of water equal in bulk to a sterling guinea, weighs 7 1-4 grains. I shall show you hereafter how to compute the proportion in which the quantity of alloy in counterfeit gold exceeds that which is allowed to a standard, and proceed to describe the hydrostatic balance, and the method of applying it to use.*

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* Platinum excepted.....A. EDIT.

OF THE HYDROSTATIC BALANCE.

The beams of these balances are in general made from eight to ten inches long; and with the perfections necessary to a good balance-beam, as pointed out in my lecture on Mechanics, it is made either to rest upon a simple stand or fulcrum, as at *plate* 2, *fig.* 3, or on a more complete and convenient stand, as at *plate* 2, *fig.* 4.

To this beam are adjusted a pair of scale-pans, which may be taken off at pleasure. There is also another small pan, A, *fig.* 3, of equal weight with one of the others, furnished with shorter strings, so as to admit a vessel of water to be placed under it. When the balance is used for hydrostatic purposes, this pan is to be suspended at one end of the beam, and one of the common scale-pans at the other.

The glass bucket, A, *fig.* 4, is to hold any solid body to be weighed in water, and is to be suspended by the horse-hair to the hook at the bottom of the small scale. There is a weight to be placed in the opposite scale, in order to balance the bucket exactly in water.

The brass tongs are for the same purpose, and to hold such substances as cannot conveniently be put into the bucket.

The small brass nippers are intended for the weighing of gold coin, by which it may be more accurately weighed than in the bucket; this being principally designed for such things as cannot be well placed in the nippers or tongs: the beam will turn much easier with either of these, than with the other. A scale-beam, loaded at each end with a considerable weight, is insensible to the addition of a small one: besides, the resisting medium of water, through which the whole surface of the bucket and its contents must pass, lessens the vibration of the beam, and renders the operation both tedious and uncertain.

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* See an excellent small tract on the use of the Hydrostatic Balance, by *Becket.*

The glass solid, B, *fig.* 3, is made use of to determine the specific gravity of fluids.

The beam of the best balance is elevated, while weighing, by turning the brass nut, B, of *fig.* 4.

These several appendages have, you see, respective weights, for a balance in water, which are distinguished by different marks. These weights are intended to balance them exactly; but their correctness may be injured by different circumstances, for water varies considerably in its density, according to the temperature of the air; in hot weather it is lighter, in cold it is heavier: in the former case the balance may appear rather too light, and in very cold weather rather too heavy. Whenever this happens, you must restore the equilibrium by a small weight, dropped into the scale that requires it, before any hydrostatic experiments can be performed with accuracy. From hence it is natural to conclude, that the specific gravity of the same substances will be different at different times: this variation is however so small, particularly in the weight of gold, as not to be regarded in common experiments.

For easier computation, I shall use tenths of grains for the subdivisions in these experiments.

TO FIND THE SPECIFIC GRAVITY OF SOLIDS.

Weigh the substancs first accurately in air, setting down with a pen the weight in grains and decimal parts; then hang on the small water-scale to one end of the beam; place under it the glass vessel, pouring water in till it be filled within three quarters of an inch of the brim: let the body to be weighed be then placed in the nippers, tongs, or bucket, as is most convenient; and, immerging it in the water, suspend it by a horse-hair to the hook at the bottom of the water-scale.

Take care that the same weights that balanced the body in air be in the opposite scale, and likewise the proper *balance water-weights*, and that no air-bubble adhere to any part of the substance in the water, which will render it apparently lighter.

The opposite scale to that which contains the substance will now greatly preponderate; weights should therefore be put into the water-scale till the equilibrium be restored.

The pen and ink will now finish the operation. Divide the weight in air by the loss of weight in water; that is, divide the number of grains in the large scale by those in the small one, and the quotient will show the specific gravity, or how many times heavier the substance that was weighed is than water.

If the weight in the small scale be subtracted from that in the other, it will show the respective gravity of the weighed substance, or the weight with which it will be evenly balanced in water.

This will be rendered plainer by an example or two. Weigh this new guinea in air; you have found its weight to be 129 grains. Now fix it in the nippers, suspend them from the water-scale, and immerge it in the water (the balance-weight of the nippers being in the opposite scale) it appears to be lighter; but put 7¼ grains in the water-scale, and the equilibrium will be restored, which shows that the guinea is good, or that sterling gold is 17.793 times as heavy as water.*

Example 2. Here is a guinea that I suspect to be bad, or counterfeit gold, though upon weighing it in air, it is found to be of full weight, or 129 grains. Now, let us weigh it in water: it has lost 8.12 grains; what is its specific gravity?

As the loss in water is to the weight in air, so is 1 *to the specific gravity;* therefore, as 8.12 to 129, so is 1 to

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* This is nearly the weight of a new guinea, but not strictly so; its real weight, valuing gold at £.3 17 10 1-2 per ounce, is 129.438, or nearly 5dwts. 9 1-2 grains: the loss in water will therefore be 7.27.

15.886; for 129.00, divided by 8.12, gives 15.886 for the specific gravity, showing that the gold is much worse than sterling.

Example 3. If a piece of silver weigh 636 grains in air, what is its specific gravity, supposing it to lose 60.7 when weighed in water? 60.7:636::1:10.478 the specific gravity; by which it appears to be good silver.

Example 4. How much heavier than water is flint-glass? A piece of white flint-glass weighed 169.05, and lost 50.60.

50.60 : 169.05 :: 1 : 3.341

so that it was somewhat more than $3\frac{1}{3}$ times heavier than its bulk of water.

Example 5. Mercury, though properly a fluid, is to be weighed as a solid. Thus, put one ounce, or 4800 grains of mercury into the glass bucket, and weighing it in water, it will be found to lose nearly about 351 grains. Mercury, therefore, is heavier than water as 13.67 to 1.

The method is the same for most other solid substances that are specifically heavier than water; for those that are lighter, the process is somewhat different.

When any thing floats on the surface of a fluid, it displaces just so much of the fluid as is equal in weight to the weight of the whole floating body. Suppose a hollow cube of tin, two inches square, to weigh one ounce; this being floated on water, may be supposed to sink in it half an inch, or one-fourth part of the whole: if so, a body of water two inches square, and half an inch thick, must weigh one ounce, equal to the weight of the cube, which would be four times specifically lighter than water. The cube would require an additional weight of three ounces to immerge it totally in the fluid; the respective gravity of the water to the cube is therefore as 3 to 1, and consequently a bulk of water equal to the cube would weigh four ounces.

From hence the method of weighing light substances specifically, is as easily conceived and put in practice as that of weighing the heavier ones. Though several different appendages have been made to the hydrostatic balance for this purpose, they appear in general to be unnecessary, as most substances that are not too large to go into the brass tongs, may very accurately be weighed in it.

Example 6. Suppose it were required to find the specific gravity of a piece of dry beech-wood, weighing 59.50 grains.

Having set down the weight of the wood in air, fix it in the tongs, and suspend the whole to the water-scale, placing the balance-weight of the tongs in the opposite scale. In the foregoing experiments, we let the weights with which the substance was weighed remain in the scale; we are now to take out all of them, except the balance-weight; and, upon immerging the tongs and wood into the water, they will appear to be lighter than the tongs alone. That adding a weight to one end of a scale-beam should make the other end preponderate, seems to be as paradoxical as the weighing of levity; but it ceases to be so when we consider, that no bodies ascend by means of their levity, but by reason of the greater density of the medium in which they are immersed. The wood pulls the tongs upward with a force equal to the relative gravity of the water; so that it requires as much weight to restore the equilibrium, as the wood is lighter than its bulk of water. Small weights are therefore to be put into the water-scale, till the balance be even, and the weight of the wood in air, added to these small weights, will be equal to the weight of a bulk of water of the same size as the wood; consequently, *the specific gravity will be as the weight of such a bulk of water is to the weight of the wood in air.*

When I weighed the wood, I put 16.70 grains into the water-scale, before the balance became even. 16.70 added to 59.50, the weight of the wood, make 76.20; therefore,

76.20 : 59.50 :: 1.000 : .781, the specific gravity of the beech.

Example 7. A piece of very dry cork, weighing in air 24.3 grains, required 149.2 to be put in the water-scale, to restore the balance. 149.2, added to 24.3, make 173.5, the weight of a bulk of water equal to the cork. As 173.5 : 24.3 :: 1.000 : .152, the specific gravity of the cork;—nearly 7 times lighter than water.

The specific gravity of another piece of cork was .241, or about 4 times lighter than water.

N. B. In weighing those bodies whose pores readily imbibe the water, the quicker the experiment is performed, the more likely it is to be correct.

METHOD OF FINDING THE SPECIFIC GRAVITY OF FLUIDS.

That which is meant by the term *specific gravity* of bodies, being nothing more than the difference, or comparative weight of those bodies to that of common water, we might easily find the specific gravity of any kind of fluid, by weighing a quantity of it against an equal bulk of water; but as a solid body, when immersed in a fluid, loses as much of its weight as a bulk of the fluid equal to the body weighs, a more convenient and accurate method is the immersion of a solid, of some determinate weight, in the fluid whose specific gravity we desire to know.

For this purpose is the conical piece of solid glass, belonging to the hydrostatic balance; whose weight, both in air and water, being known, shows immediately the weight of the fluid in which it is suspended; the solid being borne up by the fluid in a proportion equal to its respective gravity.

Suppose the glass solid to weigh in air 1464 grains, and that, when it is suspended from the water scale, and immersed in water, it loses of its weight 445 grains; this would be the weight of a bulk of water equal to the solid. The balance-weight for the solid must be made just equal to what it weighs in water, *i. e.* 1019 grains.

Whatever fluid is to be weighed, let it be put into the glass recipient; suspend the solid to the hook of the water-scale, and let it hang freely in the liquor, putting the balance-weight in the opposite scale. If the fluid be heavier than the water, the solid will rise in it; if lighter, it will sink to the bottom of the recipient. In either case, small weights are to be put into the light-scale, till the balance be made even.

When the fluid is lighter than water, the weight gained by the glass solid is to be subtracted from the weight of a bulk of water equal to the solid 445, and the remainder is the weight of an equal bulk of the fluid, or its specific gravity to water.

Example 1. When such a glass solid as the above was immersed in brandy, it balanced 38.2 grains more than in water. This, taken from 445.0, leaves 406.8; therefore, the specific weight of the brandy was to water as 406.8 to 445.

To reduce it to its proper terms, multiply the difference, 38.2, by 1000, the denominator of water, and divide the product by 445. As 445 : 38.2 :: 1000 : 86; subtract 86 from 1000, there remains .914, the specific gravity of the brandy.

From hence it appeas, that the brandy weighed 86 parts in 1000, or about $\frac{1}{12}$ less than water.

Example 2. In rum the solid balanced 40.3 grains more than in water; as 445 : 40.3 :: 1000 : 91, and 91 from 1000, remains .909. The specific gravity of the rum to water was therefore .909, or about $\frac{1}{11}$, less.

Example 3. When the solid was immersed in highly rectified spirit of wine, it balanced 73.6 more than in water; therefore, 445 : 73.6 :: 1000 : 165, and 165 from 1000, remains .835, or $\frac{1}{6}$ less.

It appears from these examples, that the hydrostatic balance is a certain and correct instrument for determining the strength of spirits, perhaps more so than the most accurate hydrometer that has yet been made for that purpose.

It is of considerable consequence to distillers, and dealers in spirituous liquors, to know the precise point of strength which is termed *proof*: though this, indeed, is rather arbitrary, than any fixed standard; but the degrees of strength which, I am informed, is now called *merchantable proof*, fixeth the specific gravity of the spirit to water at .930.

930 taken from 1000 leaves 70; therefore, 1000 : 70 :: 445 : 31.15. So that in proof spirit, a glass solid of the weight above-mentioned, must balance 31.15, or about 31$\frac{1}{4}$ grains more than in water.

It may easily be found in what proportion the spirit is above or below proof, by observing what quantity of water or alcohol is necessary to be mixed with it, in order to bring it to the above standard; and it might be immediately known, by comparing the weight of the spirit with that of water, if the specific gravity of both, when compounded, remained in the same ratio as when separate; but as it is found, that when water is mixed with spirit, the specific gravity of the compound is greater than that of the water and the spirit before they are compounded, the calculation must therefore turn out incorrect. For instance,

A quantity of the rum, before-mentioned, equal in bulk to the glass solid, weighed very nearly 405 grains, an equal bulk of water 445 grains, suppose then, that in order to reduce the rum to proof, one fifth part of water were to be mixed with it;

Water - - - - 1 = 445
Rum - - - - - 4 = 1620
5) 2065
Mean weight - - = 413

By this it appears, that a quantity of the compound, equal in bulk to the glass solid, should weigh 413 grains, and consequently, that the solid, when immersed in it, should balance 32 grains more than in water; in which case it would still be somewhat above proof.

But, upon trial, it will be found to balance not much more than 29½, and that there must be but little more than one-seventh part of water mixed with the rum to reduce it to the given standard.

Immediately after water is mixed with spirit, the compound appears lighter; but in a few hours afterwards, when the particles of each are more intimately united, its bulk diminishes, and consequently the specific gravity increases.

From a few experiments of this kind, the theory will appear sufficiently plain; and a table might easily be formed for showing by inspection what quantity of water is necessary to be put to any given quantity of spirit, to render it true proof.

2. When fluids are specifically heavier than water,

the glass solid, as before observed, will rise in such fluid, the water-balance weight being in the opposite scale; and appear to be lighter; small weights are therefore to be put in the water-scale, till the equilibrium be restored; and the loss which the solid sustains, by being weighed in the heavy fluid, is to be added to the weight of a bulk of water equal to the solid; the sum shows the specific gravity of the fluid to water.

Example 4. Suppose it were required to find the specific gravity of sea water, or how much heavier it is than rain water.

Let the solid be suspended, as usual, to the water scale, and immersed in the sea-water, putting the balance-weight in the opposite scale. It will require 11.6 grains to bring it to an even balance.

As 445 : 11.6 :: 1000 : 26.

The specific gravity is therefore 1026; which shows that sea-water is 26 parts in 1000, or $\frac{1}{38}$ heavier than rain-water; or that there must be 1026 measures of rain-water to weigh as much as 1000 measures of sea-water. The method is the same for every other fluid specifically heavier than water.

The specific gravity of salt and water, in equal quantities, in measure, is 1205; or about one-fifth heavier than common water.

METHOD OF DISCOVERING THE PROPORTION OF ALLOY MIXED WITH GOLD.

The specific gravity of sterling or money-gold, as before observed, is 17.79, which is the standard made use of in the following calculations, as it comes the nearest to the average weight of the current British coin. The specific gravity of pure, or very fine gold, is indeed considerably more; in some tables it is made 19,64, but it is seldom met with so heavy as this. The specific gravity of good silver is 10.37; of copper 8.83.

If, then, a mass be compounded of gold and silver, the specific gravity of the compound may be thus found, without the use of the balance.

Example 1. Suppose a mass of metal, half gold and half silver, in weight, to weigh in air 258.8 grains, what must be the specific gravity of this compound, to an equal bulk of water?

Rule. Compute what must be the loss in water of the proportional part of each metal, by dividing its weight in air by its specific gravity; add the two sums together, and by this product divide the weight of the whole mass; the quotient will show the specific gravity.

	Weight in air.	Loss in water.
Ratio of the gold	129.4	7.27
- - - - - - - silver	129.4	12.47
	258.8	19.74

258.8, divided by 19.74, gives 13.11 the specific gravity of the compound mass.

Example 2. If a guinea be adulterated with copper, in the proportion of 4.39 gold and 1 copper, what is its specific gravity?

Say, if a mass, weighing 5.39 grains, contains 1 grain of copper, how much will be contained in a mass weighing 129.4?

As 5.39 : 1 :: 129.4 : 24.

The guniea must therefore contain 24 grains of copper and 105.4 of gold.

Loss in water of 105.4 grains of gold = 5.93
- - - - - - - - - - - 24.0 - - - - copper = 2.71
129.4 8.64

129.4 divided by 6,64, gives 15 for the specific gravity.

The same method may be used for determining the specific gravity of any other mass, though compounded of three or more different metals, provided the ratio of each metal be known.

The reverse of these examples must, therefore, be the rule for discovering the proportion of alloy mixed with gold, after its specific gravity is found by the balance.

Example 1. What is the proportion of gold and silver in a mass weighing 258.8 grains, whose specific gravity is found to be 13.11 ?

Rule. Compute what must be the loss in water of a mass of gold of equal weight with the compound, and likewise of a mass of silver of the same weight ; subtract the loss of gold from that of the compound, the remainder is the ratio of silver : subtract the loss of the compound from that of silver, the remainder is the ratio of gold.

Loss in water of 258.8 grains of gold = 14.54
- - - - - - - - - 258.8 - - - - silver = 24.94
- - - - - - - - - 258.8 the compound = 19.74

14.54 taken from 19.74, there remain 5.20 ; 19.74 from 24.94, remain likewise 5.20 ; by which it appears that the weight of the gold and silver is in equal proportion, agreeably to the first preceding example.

Exampte 2. If a guinea of its full weight, or 129.4 grains, be adulterated with copper, and its loss in water found to be 8.64, what quantity of copper is mixed with it ?

Loss of 129.4 grains of gold = 7.27
- - - - - 129.4 - - - - copper = 14.65
- - - - - 129.4 the compound = 8.64

7.27 from 8.64, remain 1.37 ; 8.64 from 1465, remain 6.01 : so that the proportion of the copper is to that of the gold as 1.37 to 601 ; or, agreeable to the second preceding example, as 1 to 4.39, *i. e.* 24 grains of copper and 105.4 of gold.

If it be required to find the respective bulk of each metal in the compound mass ; first find its specific gravity, and then subtract the specific gravity of the inferior metal from that of the compound, and the remainder shows the ratio of gold ; then if the specific gravity of the compound be subtracted from that of the gold, the remainder is the ratio of the inferior metal.

It must be observed, that in the preceding examples, the quality of the alloy with which the gold is compounded is supposed to be known, together with its specific gravity; but this is seldom the case: it is not often discoverable with what particular metal or metals a counterfeit guinea is adulterated, much less their exact specific gravity, which is necessary to be known: the foregoing experiments, then, may appear to be calculated more for amusement, than for any real utility, in determining the value of bad gold. This conclusion, however, would be rather too precipitate; such experiments and calculations have not only a tendency to familiarize the subject, but they lead directly to the desired point.

Gold may be made to retain a very specious appearance, and yet be rendered much worse than sterling, by being adulterated with inferior metals, as silver, copper, some kinds of brass, &c. and though the weight of these, after the mass is compounded, cannot be precisely ascertained, nor, consequently, the exact ratio of the gold; yet, by taking the mean specific gravity of the metals generally used for adulterating gold, and attending to the weight, colour, &c. of the compound, a pretty correct judgment might be formed of the nature of the alloy; and if this be nearly true, the hydrostatic examination cannot be very erroneous, but will generally prove sufficiently accurate.

As a sterling guinea, when weighed in water, is found to lose $7\frac{1}{4}$ grains of its weight; if it lose more than this, the usual method has been to reckon the gold so much worse than sterling, by allowing or deducting from its value a certain sum for every grain that it loses more than $7\frac{1}{4}$; and perhaps a more correct method cannot be taken, if that sum be adequate to the weight of the alloy.

A piece of good silver, of equal weight with a guinea, or 1.294 grains, when weighed in water, loses of its weight 12.47 grains, which is 5.2 more than the guinea loses:

pence. pence.
5.2 : 252 :: 1 : 48.4.

If, therefore, a guinea, or other piece of gold coin, be adulterated with silver, four shillings, at least, must be deducted from its value, for every grain that it loses more than it would lose were it sterling gold.

A piece of copper of equal weight with a guinea, loses in water 14.65, which is 7.38 more than a guinea loses.

pence. pence.
7.38 : 252 :: 1 : 34.

So that, if sterling gold be adulterated with copper, two shillings and ten-pence is to be deducted from its value for every deficient grain; and the same method will ascertain the proportion of every other species of alloy.

Counterfeit guineas are seldom found to be adulterated with silver alone, but more generally with copper, and a small quantity of silver; or with metals whose weights are nearly equal to that of copper.

From the mean weight of these metals compared with different assays that have been made of counterfeit gold, it appears, that if three shillings be allowed for every grain deficient in the specific gravity, it will come very near the truth, and may be generally relied on as sufficiently correct.

OF MEASURING THE SPECIFIC GRAVITIES OF BODIES BY THE HYDROMETER.

When there are several fluids to be compared, and a given body which is specifically lighter than any of them is made to float upon their surface, the parts of the body that sink below the surface in these different fluids will be inversely as their specific gravities.

A piece of cork will sink deeper in spirits of wine than in water; and the part of it which sinks below the surface of the spirits will be to the part which sinks below the surface of the water, as the specific gravity of the spirits is to that of the water when they are inverted; that is, as the specific gravity of the water is to that of the spirits. The part which sinks below the surface in the spirits, is the bulk of as much spirits as is equal in weight to the whole cork; and the part which sinks below the surface of the water, is the bulk of as much water as is equal in weight to the same cork. These parts are, therefore to each other as the bulks of equal weights of spirits and water; but these bulks, and consequently the parts of the cork that sink below the surface, are inversely as the specific gravities of spirits and water.

Hence as we can discover the specific gravity of different solids, by plunging them in the same fluid; so we can discover the specific gravity of different fluids, by plunging the same solid body into them; for, in proportion as the fluid is light, so much will it diminish the weight of the body weighed in it. Thus we know that spirits of wine has less specific gravity than water, because a solid that will swim in water will sink in spirits of wine. The heavier any fluid is, the greater will be its resistance to any solid immersed: wort and spirits of nitre have greater specific gravity than water; a solid that will sink in water, will swim in wort and spirit of nitre.

The method of comparing fluids with each other by means of the hydrometer or aerometer, depends on this principle. Before I enter into a minute explanation of the hydrometer and its uses, I shall show you one or two pleasing experiments with fluids of different weights. Here is a glass vessel which is divided into two parts, communicating with each other by a small opening; fill the lower part with red wine, and then pour water gently into the upper part. As the red wine is lighter than water, you will soon see it rising like a small thread up through the water, and diffusing itself upon the surface; it has continued doing so till the wine and water have changed their places; the water is in the lower half, and the wine in the upper half of the vessel.

Here is a small bottle, whose neck is long, and not more than the one-sixth of an inch in width; I fill the bottle with red wine, and then place it in this glass vessel full of water, which is about two or three inches higher than the bottle; the wine, you see, begins to come out of the bottle, and rises in the form of a small column to the surface of the water, while the water enters the bottle and supplies the place of the wine; the weight of the water carries it down, and forces up the wine. You may alter this experiment by filling the bottle with water and the vessel with wine, and then placing the bottle inverted in the vessel, the water will descend to the bottom of the vessel, and the wine will mount up into the bottle.

The *hydrometer*, *plate* 1, *fig.* 19, is one of the most useful philosophical instruments; for, though the hydrostatic balance is the most general instrument for finding the specific gravities of all sorts of substances, yet the hydrometer is better to discover with ease and expedition those of fluids. It consists of four parts, 1. A ball of metal, ivory, or glass. 2. A tail and weight to poise the instrument, that a certain part of the instrument may be always downmost in the liquor. 3. A long stem arising from the opposite and upper part of the instrument. 4. A shoulder on the upper part of this instrument, for occasionally placing weights to cause the instrument to sink so far that the surface of the fluid may always cut the stem at a particular point.

When this instrument is swimming in the fluid, the part of the fluid displaced by it, will be equal in bulk to the part of the instrument under water, and equal in weight to the whole instrument.

Suppose the weight of the whole to be 4000 grains, and it is evident you can by this instrument compare together the different bulks of 4000 grains of various fluids: for instance, if the weight at bottom be such as shall cause the hydrometer to sink in rain-water, till its surface came to the middle point of the stem; and if, after this, it be immersed in common spring-water, and the surface thereof be one-tenth of an inch below the middle point, it is evident that the same weight of these waters differs in bulk only by the magnitude of one-tenth of an inch in the stem.

Now suppose, for ease in calculation, the stem were ten inches long, and weighed 100 grains, then every tenth of an inch would be one grain weight; and since the stem is of brass, and brass is about eight times heavier than water, the same bulk of water will be equal to one-eighth of a grain, and consequently to one-eighth of $\frac{1}{4000}$ part of the whole, that is, a 320,000 part of the whole bulk.

A variety of hydrometers have been constructed of late, for the purpose of ascertaining the strength of spirits and wort; but as government, to avoid disputes, has been under the necessity of passing an act to constitute *Clarke's* hydrometer (for a short time) the only legal one, it will be unnecessary to describe to you the various constructions that have been made, as they are by this means rendered of little use;* and, as the subject has been taken up by men of the first abilities, no doubt some instrument will be contrived that will answer the purposes of government for regulating the revenue, and at the same time to accommodate the public. Mr. *Ramsden*, in an excellent paper on this subject, has shown, that to answer these valuable purposes, four subjects must be well ascertained.

1. A method of proportioning and expressing in measures the quantities of spirits in compounds, and of determining their specific gravities.
2. A means of ascertaining the increase or diminution in the bulk of a given compound arising from different degrees of temperature.
3. The application of the experiments under the two preceding heads to the construction of a hydrometer, which shall give the specific gravity of any compound in thousandth parts of that of distilled water; and at the same time the quantity of spirits of a given strength in the compound in hundredth parts of the volume.
4. A method of determining the proportion of spirits to water in the compound, now called *proof*, which the commissioners of the customs have stated to weigh seven pounds twelve ounces per gallon, at the temperature of 55 degrees; for which purpose it is necessary to have an exact gallon-measure.†

From hence you may see, that the determining of the specific gravities of fluids, in order thereby to obtain the strength of spirituous liquors, is a very complicated problem. M. *de Luc* has shown, that when a hydrometer is employed, there are three physical effects, the degrees of which are not proportionate to their apparent causes, and which are united in one effect, namely, the different sinking of the hydrometer.

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* *Quin's* construction of the hydrometer, with only three weights and a sliding rule, is now very generally used by distillers, brewers, &c.

F. EDIT.

† *Ramsden's* Account of Experiments to determine the Specific Gravities of Fluids.

De Luc on Pyrometry, Aerometry, &c. PHILOS. TRANS.

1. It will not always sink in liquors of different densities proportionally to these densities, on account of the changes of its own bulk by heat, and the possible irregularities of its branch. 2. It will not sink in proportion to the changes of temperature of the fluid, because the changes of density in the latter do not follow the same law as the changes of temperature. 3. It will not sink exactly in the inverse ratio of the quantities of phlegm, because the specific gravity of the fluid does not follow the proportion of these quantities. It has an increasing progression, and here the intermediate cause of this disproportion, which is evident, may give you an idea of what takes place in nature, and hinders physical effects from appearing proportional to their causes.

The spirit and the phlegm penetrate each other; that is to say, *the bulk of the mixture is somewhat less than the sum of the two bulks before the mixture*; and thus the specific gravity, which is the weight under a certain bulk, increases but little in the mixture comparatively with the mean specific gravity of the component parts.

In order, therefore, to have equal degrees in the hydrometer, without sensible error in the spirituosity that it is intended to measure, you must fix these degrees by the comparison of effects observed within the limits of the common observations.

For as to physical joint effects, if you cannot fix all their relations, degree by degree, by immediate and sure observations, you must avoid deducing general rules from relations taken in the extremes. The action of causes, as well moral as physical, whether from the variety of subjects on which they act, or from secondary causes which escape our attention, is too complicated for the observable modifications to increase in the exact proportion of the evident causes, and consequently for the joint effects to be proportionate between themselves. If the joint effects be proportionate, there will be little lost in not taking distant points of comparison, provided they be taken exactly. If these joint effects be proportionate, there will be much gain; and the less proportionate they are, the greater will be the gain.

We are obliged to take up with probability in nature in so many respects, that it is perhaps of more importance to us to investigate the physical rules of probability, than to attend to its mathematical rules upon hypotheses.

OF A PHENOMENON NOT TO BE ACCOUNTED FOR ON THE GENERAL DOCTRINE OF SPECIFIC GRAVITY.*

Bodies of the greatest known specific gravity, when divided into very minute parts by the menstrua in which they are dissolved, will remain suspended therein for any length of time; thus aqua regia, or even ether, holds suspended the parts of the most ponderous of all metals, gold. Some mathematicians endeavour to account for this part of the phenomenon; but there is another difficulty, which will not bend to theory. It does not follow from any established principles, how a body divided into parts, however minute, can possibly ascend in a fluid specifically lighter than itself; whereas it is well known, that in some solutions, when the solid to be dissolved is placed at the bottom of a vessel into which the dissolving fluid is poured, the parts of the solid during the solution, without any motion whatever being communicated to the vessel, will be diffused throughout the substance of the dissolving fluid, appearing to overcome the natural tendency of bodies towards the centre of the earth, and to have some new power of ascent impressed upon its particles.

DIFFERENT METHODS OF ASCERTAINING THE SPECIFIC GRAVITY OF FLUIDS.

The specific gravities of water and any other fluid, may be compared together by the following process: weigh very accurately an ounce or other weight of distilled water in a cylindrical glass phial, and mark precisely the space occupied by it; then pour in any other fluid till it fill exactly the same space with the water, and weighing it you will know the weights of equal magnitudes of the water and the other fluid, and their specific gravities.

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* *Atwood's* Treatise on Rectilinear Motion, p. 155, 162, &c.

The magnitude of a body, however irregular, may be found by immersing it in a cylindrical vessel of water, and marking how far the fluid rises; for the space contained between the surfaces of the water before and after the immersion of the body, is equal to its magnitude; and this, together with its weight, being known, its specific gravity is also known.

The capacity of any irregular vessel may be known by filling it with water; for the water being weighed, its magnitude, or the number of cubical inches contained in it, will be found.

Let the vessel be filled with water, and let the weight of the water be A ounces; then make the following proportion, as 52746 to A, so is 1 to the capacity of the vessel expressed in cubic inches; this will be facilitated by the following table:

A TABLE.

oz.	*cubic inches.*
1	1.8959
2	3.7918
3	5.6877
4	7.5835
5	9.4794
6	11.3753
7	13.2712
18	15.1671
19	17.0630

To exemplify the use of this table, suppose the water contained in a receiver of an air-pump or other vessel to weigh 235.18 oz. then referring to the table,

oz.		*cubic inches.*
200	=	379.18
30	=	56.88
5	=	9.48
.1	=	.19
.08	=	.15

445.88 the number of cubic inches contained in the vessel.

If avoirdupois ounces be used in weighing the water, the numbers may be taken from the above table, but the resulting number must be multiplied into 91145, to give the true number of cubic inches contained in the vessel.

TABLE OF SPECIFIC GRAVITIES.

Refined gold	19.640
English guinea, by some 17.79, by others	18.888
Mercury	14.019
Lead	11.344
Refined silver	11.090
Bismuth	9.700
Copper from Japan	9.000
Copper from Sweden	8.843
Hammered brass	8.349
Cast brass	8.100
Turbeth mineral	8.235
Cinnabar, factitious	8.200
Cinnabar natural	7.300
Elastic steel	7.820
Soft steel	7.738
Iron	7.645
Pure tin	7.471
Glass of antimony	5.280
A pseudo topaz	4.270
A diamond	3.400
Crystal glass	3.150
Island crystal	2.790
Rock crystal	2.650
Common glass	2.620
Fine marble	2.704
Stone of a mean gravity	2.500

Selenites	2.252
Sal gemmæ	2.143
Brick	2.000
Nitre	1.900
Alabaster	1.875
Dry ivory	1.825
Brimstone	1.800
Dantzick vitriol	1.715
Alum	1.714
Borax	1.714
Calculus humanus	1.700
Oil of vitriol	1.700
Oil of tartar	1.550
Bezoar	1.500
Honey	1.450
Gum Arabic	1.375
Spirits of nitre	1.315
Aqua fortis	1.300
Pitch	1.150
Spirit of salt	1.130
Crassamen of the human blood	1.126
Spirit of urine	1.120
Human blood	1.054
Amber	1.040
Serum of the human blood	1.030
Milk	1.030
Urine	1.030
Dry box wood	1.030
Sea water	1.030
Common water	1.000
Camphire	0.996
Bees wax	0.955
Linseed oil	0.932
Dry oak	0.925
Oil olive	0.913
Spirit of turpentine	0.874
Rectified spirit of wine	0.866
Dry ash	0.800
Dry maple	0.755
Dry elm	0.600
Dry fir	0.550
Cork	0.240
Air	$0.001\frac{1}{3}$

This table exhibits the specific weights of the various substances contained in it, discovered by some of the methods already described; and the absolute weight of a cubic foot of each body is ascertained in avoirdupois ounces by multiplying the number opposite to it into 1000; as for example, S. g. of water : S. g. of mercury :: 1 : 14.019 :: 1000 oz. : wt. of a cubical foot of mercury, which is therefore equal to 1000×14.019 avoirdupois ounces. There are some uncertainties on this subject, for substances of the same kind, though denominated by the same name, may not be precisely similar, and some small errors may perhaps be inevitable in physical experiments; but they will be inconsiderable if the scales be nicely adjusted, and the experiments cautiously conducted, so that the body weighed do not touch the bottom or sides of the vessel, nor rise above the surface of the fluid, nor bubbles of air adhere to its surface. There is another cause of uncertainty; for as all substances are dilated by heat and contracted by cold, the dimensions of the same body, and consequently its specific gravity, will be different according to the different temperatures of the ambient air; hence the altitude of the thermometer ought to be considered in constructing a table of specific gravities. The different expansion of bodies in summer and winter, and consequently their different specific gravities, appear from the experiments of *Homberg*, and *Eisenschmedites*, in his Disquisitio nova de Ponderibus, &c. from the latter of whom the following table, exhibiting the weight of a cubical inch, Paris measure, of different substances is taken.

A TABLE OF THE SPECIFIC GRAVITIES OF SEVERAL FLUIDS.

A cubic inch.	In summer.			In winter.		
Paris measure.	*oz.*	*dr.*	*gr.*	*oz.*	*dr.*	*gr.*
Of mercury	7	1	66	7	2	14
Oil of vitriol	0	7	59	0	7	71
Spirit of vitriol	0	5	33	0	5	38
Spirit of nitre	0	6	24	0	6	24
Spirit of salt	0	5	49	0	5	55
Aqua fortis	0	6	23	0	6	35
Vinegar	0	5	15	0	5	21
Distilled vinegar	0	5	11	0	5	15

A cubic inch.	In summer.			In winter		
Paris measure.	*oz.*	*dr.*	*gr.*	*oz.*	*dr.*	*gr*
Burgundy wine	0	4	67	0	4	75
Spirit of wine	0	4	32	0	4	42
Pale ale	0	5	1	0	5	9
Brown ale	0	5	2	0	5	7
Cow's milk	0	5	20	0	5	25
Goat's milk	0	5	24	0	5	28
Urine	0	5	14	0	5	19
Spirit of urine	0	5	45	0	5	53
Spirit of tartar	0	7	27	0	7	43
Oil of olives	0	4	53	frozen		
Oil of turpentine	0	4	39	0	4	46
Sea-water	0	6	12	0	6	18
River-water	0	5	10	0	5	13
Spring water	0	5	11	0	5	14
Distilled water	0	5	8	0	5	11

It appears from this table, that the expansions of different fluids are different in the same changes of the temperature of the air; and it appears from observation, that substances, not fluid, are also in similar circumstances differently dilated; but the weight of given magnitudes, both of fluid and firm bodies, being diminished by heat, and increased by cold, the variation of their specific gravities is less than if the dimensions of one of them only had been variable.

OF AIR BALLOONS.

The romances of almost every nation have recorded instances of persons being carried through the air, both by the agency of spirits, and by mechanical inventions; but till the time of the celebrated Lord *Bacon*, no rational principle appears ever to have been thought of by which this might be accomplished. Before that time, indeed, Friar *Bacon* had written upon the subject, and many had been of opinion, that, by means of artificial wings fixed to the arms or legs, a man might fly as well as a bird: but these opinions were thoroughly refuted by *Borelli*, in his treatise *De Motu Animalium*, where, from a comparison between the powers of the muscles which move the wings of a bird, and those which move the arms of a man, he demonstrates, that the latter are utterley insufficient to strike the air with such force as to raise him from the ground. It cannot be denied, however, that wings of this kind, if properly constructed and dextrously managed, might be sufficient to break the fall of a human body from a high place, so that some adventurers in this way might possibly come off with safety; though by far the greater number of those who have rashly adopted such schemes, have either lost their lives or limbs in the attempt.

In the year 1672, Bishop *Wilkins* published a treatise, entitled, The Discovery of the New World, in which he mentions, though in a very indistinct and confused manner, the true principle on which the air is navigable; quoting from *Albertus de Saxonia* and *Francis Mendoca*, "that the air is in some part of it navigable; and, upon this static principle, any brass or iron vessel, suppose a kettle, whose substance is much heavier than that of water, yet being filled with the lighter air, it will swim upon it and not sink. So, suppose a cup or wooden vessel upon the outward borders of this elementary air, the capacity of it being filled with fire, or rather etherial air, it must necessarily, upon the same ground, remain swimming there, and of itself can no more fall than an empty ship can sink." This idea, however, he did not by any means pursue, but rested his hopes entirely upon mechanical motions, to be accomplished by the mere strength of a man, or by springs, &c. and which have been demonstrated incapable of answering any useful purpose.

The only person who brought this scheme of flying to any rational principle, was the jesuit *Francis Lang*, cotemporary with Bishop *Wilkins;* he being acquainted with the real weight of the atmosphere, justly concluded, that if a globular vessel were exhausted of air, it would weigh less than before; and considered, that the solid contents of vessels increase in much greater proportion than their surfaces: he supposed, that a metalline vessel might be made so large, that when emptied of its air, it would be able, not only to raise itself in the atmosphere, but to carry up passengers along with it; and he made a number of calculations necessary for putting the project in execution. But though the theory was here unexceptionable, the means proposed were certainly insufficient to accomplish the end; for a vessel of copper, made so thin as was necessary to make it float in the atmosphere, would be utterly unable to resist the external pressure, which being demonstrated by those skilled in mechanics, no attempt was made on that principle.

In the year 1766, Mr. *Henry Cavendish* ascertained the weight and other properties of inflammable air, determining it to be at least seven times lighter than common air. Soon after which it occurred to Dr. *Black*, that perhaps a thin bag, filled with inflammable air, might be buoyed up by the common atmosphere, and he thought of having the allantois of a calf prepared for this purpose; but his other avocations prevented him from prosecuting the experiment. The same thought occurred some years afterwards to Mr. *Cavallo;* and he has the honour of being the first who made experiments on the subject. He first tried bladders; but the thinnest of these, however well scraped and prepared, were found too heavy. He then tried Chinese paper; but that proved so permeable, that the vapour passed through it like water through a sieve. His experiments, therefore, made in the year 1782, proceeded no farther than blowing up soap-bubbles with inflammable air, which ascended rapidly to the ceiling, and broke against it.

But while the discovery of the art of aerostation seemed thus on the point of being made in Britain, it was all at once announced in France, and that from a quarter whence nothing of the kind was to have been expected. Two brothers, *Stephen* and *John Montgolfier*, natives of Annonay, and masters of a considerable paper manufactory there, had turned their thoughts towards this project as early as the middle of the year 1782. The idea was first suggested by the natural ascent of the smoke and clouds in the atmosphere; and their design was to form an artificial cloud, by inclosing the smoke in a bag, and making it carry up the covering along with it.

Towards the middle of November, that year, the experiment was made at Avignon, with a fine silk bag, of a parallelopiped shape. By applying burning paper to the lower aperture, the air was rarefied, and the bag ascended in the atmosphere, and struck rapidly against the ceiling. On repeating the experiment in the open air, it rose to the height of about seventy feet.

Soon after this one of the brothers arrived at Paris, where he was invited by the Academy of Sciences to repeat his experiments at their expense. In consequence of this invitation, he constructed, in a garden in the Fauxbourg of St. Germain, a large balloon of an elliptical form. In a preliminary experiment, this machine lifted up from the ground eight persons who held it; and would have carried them all off, if more had not quickly come to their assistance. Next day the experiment was repeated in the presence of the members of the Academy: the machine was filled by the combustion of fifty pounds of straw, made up in small bundles, upon which twelve pounds of chopped wool were thrown at intervals. The usual success attended this exhibition; the machine soon swelled, endeavoured to ascend, and immediately after sustained itself in the air, together with the charge of between four and five hundred weight. It was evident, that it would have ascended to a very great hight; but as it was designed to repeat the experiment before the king and royal family at Versailles, the cords by which it was tied down were not cut: but, in consequence of a violent rain and wind, which happened at this time, the machine was so far damaged, that it became necessary to prepare a new one for the time that it had been determined to honour the experiment with the royal presence; and such expedition was used, that this vast machine of near sixty feet in height, and forty-three in diameter, was made, painted with water-colours both within and without, and finely decorated, in no more than four days and four nights. Along with this machine was sent a wicker cage, containing a sheep, a cock, and a duck; which were the first animals ever sent through the atmosphere. The full success of the experiment was prevented by a violent gust of wind, which tore the cloth in two places, near the top, before it ascended; however, it rose to the height of 1440 feet; and, after remaining in the air about eight minutes, fell to the ground at the distance of 10,200 feet from the place of its setting out. The animals were not in the least hurt. The great power of these aerostatic machines, and their very gradual descent in falling to the ground, had originally showed, that they were capable of transporting people through the air with all imaginable safety; and this was further confirmed by the experiment already mentioned. As Mr. *Montgolfier*, therefore, proposed to make a new aerostatic machine, of a firmer and better construction than the former, Mr. *Pilatre de Rozier* offered himself to be the first aerial adventurer.

On the 21st of November, 1783, therefore, Mr. *Pilatre* determined to undertake an aerial voyage, in which the machine should be fully set at liberty. Every thing being got in readiness, the balloon was filled in a few minutes; and Mr. *Pilatre* placed himself in the gallery, counterpoised by the Marquis *d'Arlandes*, who occupied the other side. It was intended to make some preliminary experiments on the ascending power of the machine: but the violence of the wind prevented this from being done, and even damaged the balloon essentially; so that it would have been entirely destroyed, had not timely assistance been given. The extraordinary exertions of the workmen, however, repaired it again in two hours, and the adventurers set out. They met with no inconvenience during the voyage, which lasted about twenty-five minutes; during which time they had passed over a space of about five miles.

From the account given by the Marquis *d'Arlandes*, it appears, that they met with several different currents of air; the effect of which was, to give a very sensible shock to the machine, and the direction of the motion seemed to be from the upper part downwards. It appears also, that they were in some danger of having the balloon burnt altogether, as the Marquis observed several round holes made by the fire in the lower part of it, which alarmed him considerably, and indeed not without reason. However, the progress of the fire was easily stopped by the application of a wet spunge, and all appearance of danger ceased in a very short time.

This voyage of Mr. *Pilatre* and the Marquis *d'Arlandes* may be said to conclude the history of those aerostatic machines which are elevated by means of fire; for though many other attempts have been made upon the same principle, most of them have either proved unsuccessful, or were of little consequence. They have therefore given place to the other kind, filled with inflammable air; which, by reason of its smaller specific gravity, is both more manageable, and capable of performing voyages of greater length, as it does not require to be supplied with fuel, like the others.

The success of this experiment, and aerial voyage, made by Messrs. *Rozier* and *d'Arlandes*, naturally suggested the idea of undertaking something of the same kind, with a balloon filled with inflammable air. Thus there are two kinds of air balloons; one containing heated air, and the other inflammable air. Hot air occupies a much greater space than cold-air; inflammable air, at a given temperature, is much lighter than the common air of the atmosphere. The machine used on this occasion was formed of gores of silk, covered over with a varnish made of caouthouc, of a sperical figure, and measuring twenty-seven and a half feet in diameter. A net was spread over the upper hemisphere, and was fastened to a hoop, which passed round the middle of the balloon. To this a sort of carr, or rather boat, was suspended by ropes, in such a manner as to hang a few feet below the lower part of the balloon; and, in order to prevent the bursting of the machine, a valve was placed in it; by the opening of which some of the inflammable air might be occasionally let out. A long silken pipe communicated with the balloon, by means of which it was filled. The boat was made of basket-work, covered with painted linen, and beautifully ornamented; being eight feet long, four broad, and three and a half deep; its weight a hundred and thirty pounds. At this time, however, as at the former, they met with great difficulties in filling the machine with inflammable air, owing to their ignorance of the most proper apparatus; but at last, all obstacles being removed, the two adventurers took their seats at three quarters of an hour after one in the afternoon of the 1st of December, 1783. Persons skilled in mathematics were conveniently stationed, with proper instruments, to calculate the height, velocity, &c. of the balloon. The weight of the whole apparatus, including that of the two adventurers, was found to be 604 $\frac{1}{2}$ pounds; and the power of ascent, when they set out, was twenty pounds; so that the whole difference betwixt the weight of this balloon, and an equal bulk of common air, was 624 pounds: but the weight of common atmosphere, displaced by the inflammable gas, was calculated to be 771 pounds, so that there remains 147 for the weight of the latter; and this calculation makes it only five times lighter than common air.

At the time the balloon left the ground the thermometer stood at 9° *Fahrenheit's* scale, and the quicksilver in the barometer at 30.18 inches; and by means of the power of ascent with which they left the ground, the balloon rose till the mercury fell to 27 inches, from which they calculated their height to be 600 yards. By throwing out ballast occasionally, as they found the machine descending by the escape of some of the inflammable air, they found it practicable to keep it pretty near the same distance from the earth during the rest of their voyage; the quicksilver fluctuating between 27 and 27.65 inches, and the thermometer between 53° and 27°, the whole time. They continued in the air for the space of an hour and three quarters, when they alighted at the distance of twenty-seven miles from Paris; having suffered no inconvenience during their voyage, nor experienced any contrary currents of air, as had been felt by Messrs. *Pilatre* and *d'Arlandes*. As the balloon still retained a great quantity of inflammable gas, Mr. *Charles* determined to take another voyage by himself. Mr. *Roberts* accordingly got out of the boat, which was thus lightened by 130 pounds, and of consequence the aerostatic machine had now nearly as much power of ascent. Thus he was carried up with such velocity, that in twenty minutes he was almost 9000 feet high, and entirely out of sight of terrestial objects. At the moment of his parting with the ground, the globe had been rather flaccid; but it soon began to swell, and the inflammable air escaped from it in great quantity through the silken tubes. He also frequently drew the valve, that it might be more freely emitted, and the balloon more effectually prevented from bursting. The inflammable gas being considerably warmer than the external air, diffused itself all round, and was felt like a warm atmosphere; but in ten minutes the thermometer indicated a variation of temparature as great as that between the warmth of spring and the ordinary cold. His fingers were benumed by the cold, and he felt a violent pain in his right ear and jaw, which he ascribed to the dilatation of the air in those organs, as well as the external cold. The beauty of the prospect which he now enjoyed, however, made amends for these inconveniences. At his departure the sun was set on the vallies; but the height to which Mr. *Charles* was got in the atmosphere rendered it again visible, though only for a short time. He saw, for a few seconds, vapours rising from the vallies and rivers. The clouds seemed to ascend from the earth, and collect one upon the other, still preserving their usual form; only their colour was grey and uniform for want of sufficient light in the atmosphere. By the light of the moon, he perceived that the machine was turning round with him in the air; and he observed, that there were contrary currents, which brought him back again. He observed, also, with surprise, the effects of the wind, and that the streamers of his banners pointed upward; which he says, could not be the effect either of his ascent or descent, as he was moving horizontally at the time. At last, recollecting his promise of returning to his friends in half an hour, he pulled the valve, and accelerated his descent. When within two hundred feet of the earth, he threw out two or three pounds of ballast, which rendered the balloon again stationary; but in a little time afterwards he gently alighted in a field, about three miles distant from the place whence he set out; though by making allowance for all the turnings and windings of the voyage, he supposes that he had gone through nine miles at least. By the calculations of Mr. *de Maunier*, he rose at this time not less than 10,500 feet high; a height somewhat greater than that of Mount Ætna. A small balloon, which had been sent off before the two brothers set out on their voyage, took a direction opposite to that of the large one, having met with an opposite current of air, probably at a much greater height.

It would be tedious, as well as unnecessary, to recount all the other aerial voyages that have been performed in our own or other countries: it appears sufficient for the purpose of this article, to notice those which were most remarkable and interesting; and therefore an account of the ingenious Mr. *Baldwin's* excursion from Chester, alluded to above, shall now close our enumeration.

On the 8th September, 1785, at forty minutes past one, P. M. Mr. *Baldwin* ascended from Chester, in Mr. *Lunardi's* air balloon. After traversing in a variety of different directions, he first alighted, at twenty-eight minutes after three, about twelve miles from Chester, in the neighbourhood of Frodsham; then re-ascening, and pursuing his excursion, he finally landed at Rixton Moss, five miles N.N.E. of Warrington, and twenty-five miles from Chester. Mr. *Baldwin* has published his observations and remarks made during his voyage, and taken from minutes. Our limits will not admit of relating many of his observations; but the few following are the most important and curious. " The sensation of ascending is compared to that of a strong pressure, from the bottom of the car upwards, against the soles of his feet. At the distance of what appeared to him seven miles from the earth, though by the barometer scarcely a mile and a half, he had a grand and most enchanting view of the city of Chester, and the adjacent places below. The river Dee appeared of a red colour, the city very diminutive, and the town entirely blue. The whole appeared a perfect plane; the highest building having no apparent height, but reduced all to the same level; and the whole terrestial prospect appeared like a coloured map. Just after his first ascent, being in a well-watered and maritime part of the country, he observed a remarkable and regular tendancy of the balloon towards the sea; but shortly after rising into another current of air he escaped the danger. This upper current he says, was visible to him at the time of his ascent, by a lofty, sound stratum of clouds, flying in a safe direction. The perspective appearence of things to him was very remarkable. The lowest bed of vapour, that first appeared as cloud, was pure white, in detached fleeces, increased as they rose: they presently coalesced, and formed as he expressed it, a sea of cotton, tufting here and there by the action of the air in the undisturbed parts of the clouds. The whole became an extended white floor of cloud, the upper surface being smooth and even. Above this white floor he observed, at great and unequal distances, a vast assemblage of thunder-clouds, each parcel consisting of whole acres in the densest form: he compares their form and appearance to the smoke of pieces of ordnance, which had consolidated, as it were, into masses of snow, and penetrated through the upper surface, or white floor of common clouds, there remaining visible, and at rest. Some clouds had motions in slow and various directions, forming an appearance truly stupendous.

Mr. *Baldwin* also gives a curious description of his tracing the shadow of the balloon over the tops of volumes of clouds. At first it was small, in size and shape like an egg, but soon increased to the magnitude of the sun's disc, still growing larger, and attended with the most captivating appearance of an iris encircling the whole shadow, at some distance round it, the colours of which were remarkably brilliant. The region did not feel colder, but rather warmer, than below: the sun was hottest to him when the balloon was stationary. The discharge of a cannon, when the balloon was at a considerable height, was distinctly heard by the aeronaut; and a discharge from the same piece, when at the height of thirty yards, so disturbed him, as to oblige him for safety to lay hold firmly of the cords of the balloon. At a considerable height he poured down a pint bottle full of water; and as the air did not oppose a resistance sufficient to break the stream into small drops, it mostly fell down in large drops. In the course of the balloon's tract, it was found much affected by the water; a circumstance observed in former aerial voyages. At one time the direction of the balloon kept continually over the water, going directly towards the sea, so much as to endanger the aeronaut. The mouth of the balloon was opened, and he in two minutes descended into an under current blowing from the sea: he kept descending, and landed at Bellair Farm, in Rinsley, twelve miles from Chester. Here he lightened his car by thirty pounds; and instantly reascending, was carried into the interior part of the country, performing a number of different manœuvres.

At his greatest altitude he found his respiration free and easy. Several bladders which he had along with him cracked, and expanded considerably. Clouds and land, as before, appeared on the same level. By way of experiment, he tried the upper valve two or three times, the neck of the ballon being close; and remarked, that the escape of the gas was attended with a growling noise, like mill-stones, but not near so loud. Again, round the shadow of the balloon on the clouds, he observed the iris; and at fifty-three minutes past three he finally landed.

The general principles of aerostation are so little different from those of hydrostatics, that it may seem superfluous to insist much upon them. It is a fact universally known, that when a body is immersed in any fluid, if its weight be less than an equal bulk of that fluid, it will rise to the surface; but, if heavier, it will sink; and if equal, it will remain in the place where it is left. For this reason, smoke ascends into the atmosphere, and heated air into that which is colder. The ascent of the latter is shown in a very easy and satisfactory manner, by bringing a red-hot iron under one of the scales of a balance, by which the scale is instantly made to ascend; for, as soon as the red-hot iron is brought under the scale, the hot air being lighter than that which is colder, ascends, and strikes the bottom, which is thus impelled upwards; and the opposite scale descends, as if a weight had been put into it.

Upon this simple principle depends the whole theory of aerostation; for it is the same thing whether we render the air lighter by introducing a quantity of heat into it, or inclosing a quantity of gas specifically lighter than the common atmosphere in a certain space; both will ascend, and for the same reason. A cubic foot of air, by the most accurate experiments, has been found to weigh about 554 grains, and to be expanded by every degree of heat marked on *Fahrenheit*'s thermometer, about $\frac{1}{500}$ of the whole. By heating a quantity of air, therefore, to 200 degrees of *Fahrenheit*, you will just double its bulk when the thermometer stands at 54 in the open air, and in the same proportion you will diminish its weight; and if such a quantity of this hot air be inclosed in a bag, that the excess of the weight of an equal bulk of common air weighs more than the bag with the air contained in it, both the bag and air will rise into the atmosphere, and continue to do so till they arrive at a place where the external air is naturally so much rarefied that the weight becomes equal; and here the whole will float.

The power of hot air in raising weights, or rather that by which it is itself impelled upwards, may be shown in the following manner. Roll up a sheet of paper in a conical form, and, by thrusting a pin into it near the apex, prevent it from unrolling. Fasten it then, by its apex, under one of the scales of a balance, by means of a thread; and having properly counterpoised it by weights put into the opposite scale, apply the flame of a candle underneath; you will instantly perceive the cone to rise; and it will not be brought into equilibrum with the other, but by a much greater weight than those who have never seen the experiment would believe. If you try this experiment with more accuracy, by getting proper recepticles made, which contain determinate quantities of air, you will find, that the power of the heat depends much more on the capacity of the bag which contains it, than could well be supposed. Thus, let a cubical receptacle be made of a small wooden frame, covered with paper, capable of containing one foot of air; and let the power of a candle be tried with this, as above directed for the paper cone. It will then be found, that a certain weight may be raised; but a much greater one will be raised by having a receptacle of the same kind, which contains two cubic feet, a still greater by one of three feet, a yet greater by one of four feet, &c. and this even though the very same candle be made use of; nor is it known to what extent even the power of this small flame might be carried.

From these experiments it appears, that in the aerostatic machines, constructed on *Montgolfier*'s plan, it must be an advantage to have them as large as possible, because a smaller quantity of fire will then have a greater effect in raising them, and the danger from that element, which in this kind of machines is chiefly to be dreaded, will be in a great measure avoided. On this subject it may be remarked, that as the cubical contents of a globe, or any other figure of which balloons are made, increases much more rapidly than their surfaces, there must ultimately be a degree of magnitude at which the smallest imaginable heat would raise any weight whatever. Thus, supposing any aerostatic machine, capable of containing 500 cubic feet, and the air within it to be only one degree hotter than the external atmosphere, the tendency of this machine to rise, even without the application of artificial heat, would be near an ounce. Let its capacity be increased sixteen times, and the tendency to rise will be equivalent to a pound; though this may be done without making the machine sixteen times heavier than before. It is certain, however, that all aerostatic machines have a tendency to produce, or preserve, heat within them; which would by no means be imagined by those who have not made the experiment. When Messrs. *Charles* and *Roberts* made their longest aerial voyage, of 150 miles, they had the curiosity to try the temperature of the air within their balloon, in comparison with that of the external atmosphere; and at this time they found, that when the external atmosphere was 63°, the thermometer within the balloon stood at 104°. Such a difference of temperature must have given a machine of the magnitude which carried them a considerable ascending power, independent of any other cause, as it amounted to forty-one grains on every cubic foot; and therefore, in a machine containing 50,000 such feet, would have been almost two hundred pounds. Hence we may easily account for what happened at Dijon, and is recorded by Mr. *Morveau.* "A balloon, intended to be filled with inflammable air, being completed, was, by way of trial, filled with common air, and in that state exposed to the atmosphere. Now it was observed, and indeed a similar observation had been made before, that the air within the balloon was much hotter than the circumambient air: the thermometer in the former stood at 120; whereas in the latter, even when the sun shone upon it, the thermometer stood at 84. This showed a considerable rarefaction within the balloon; and, consequently, it was suspected, that by means of this rarefaction alone, especially if it were to increase a little, the balloon might ascend. On the 30th of May, about noon, the wind being rather strong, agitated the balloon so, that two men were employed to take care of it; but, notwithstanding all their endeavours, it escaped from its confinement, and, lifting up about sixty-five pounds weight of cords, equatoreal circle, &c. rose many feet high, and passing over some houses, went to the distance of 250 yards, where at length it was properly secured."

This difference between the external and internal heat being so very considerable, must have a great influence upon aerostatic machines, and will undoubtedly influence those filled with inflammable air, as well as the other kind; nor is it unlikely, that the short time which many aerial voyagers have been able to continue in the atmosphere, may have been owing to the want of a method of preserving this internal heat. It may naturally be supposed, and indeed it has always been found, that balloons, in passing through the higher regions of the atmosphere, acquire a very considerable quantity of moisture; not only from the rain or snow they sometimes meet with, but even from the dew and vapour which condenses upon them. On this an evaporation will instantly take place; and as it is the property of this operation to produce a very violent cold, the internal heat of the balloon must be soon exhausted in such a manner as to make it become specifically heavier than the common atmosphere, and consequently descend in a much shorter time than it would have done by the mere loss of air. To this, in all probability, we are to ascribe the descent of the balloon, which carried Messrs. *Blanchard* and *Jefferies*, on the 7th of January, 1785, actually across the British channel, from Dover to Calais; and which seemed so extraordinary to many people, that they were obliged to have recourse to an imaginary attraction in the waters of the ocean, in order to solve the phenomenon. This supposition is rejected by Mr. *Cavallo*;* who explains the matter by remarking, that in two former voyages made with the same machine, it could not long support two men in the atmosphere; so that we had no cause to wonder at its weakness on this occasion.

Many ineffectual attempts have been made to give a progressive motion to air balloons. The difficulty arises from the large surface which the balloons expose to the resisting medium in which they move; the quantity of air to be displaced by the travellers is so great, that they cannot move it with any degree of velocity; for, supposing them to give the balloon a small degree of velocity, the resistance of the air is so great, that their strength will be employed in overcoming this, instead of adding to the velocity of the machine. The great point is, therefore, to form a balloon of such a figure as will move through the air, without displacing any considerable quantity thereof.†

Having introduced these observations on air balloons, by observing that the art of penetrating into and being carried through the air had been the subject of the romances of almost every nation, I shall conclude this subject with an extract from *Johnson's* Rasselas, where we find the artist endeavouring to persuade the prince, that the fields of the air are open to knowledge, and that only ignorance and idleness need crawl upon the ground.

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* See the History and Practice of Aerostation, by *T. Callvallo*, 1785.

† *Nicholson's* Introduction to Philosophy, vol. ii. p. 96.

"The prince having seen what the mechanist had already performed, was willing to fancy he could do more, yet resolved to inquire further before he suffered hope to afflict him by disappointment. 'I am afraid,' said he to the artist, 'that your imagination prevails over your skill, and that you now tell me rather what you wish, than what you know; every animal has his element assigned him; the birds have the air, and man and beasts the earth.' 'So,' replied the mechanist, 'fishes have the water, yet beasts can swim by nature, and men by art. He that can swim need not despair to fly; to swim is to fly in a grosser fluid, and to fly is to swim in a subtiler. We are only to proportion our power of resistance to the different density of matter through which we are to pass. You will be necessarily upborne by the air, if you can renew any impulse upon it faster than the air can recede from the pressure.'

'But the exercise of swimming,' said the prince, 'is very laborious; the strongest limbs are soon wearied; I am afraid the act of flying will be yet more violent, and wings will be of no great use unless we can fly further than we can swim.'

'The labour of rising from the ground,' said the artist, 'will be great; but as we mount higher, the earth's attraction and the body's gravity will be gradually diminished, till we shall arrive at a region where a man will float in the air without any tendency to fall: no care will then be necessary, but to move forwards with the gentlest impulse. You, Sir, whose curiosity is so extensive, will easily conceive with what pleasure a philosopher furnished with wings, and hovering in the sky, would see the earth and all its inhabitants rolling beneath him, and presenting to him successively, by its diurnal motion, all the countries within the same parallel. How must it amuse the pendent spectator to see the moving scene of land, ocean, cities, and desarts! to survey, with equal security, the marts of trade, and the fields of battle; mountains infested by barbarians, and fruitful regions gladdened by plenty and lulled by peace! pass over distant regions, and examine the face of nature from one extremity to the other!'

'All this,' said the prince, 'is much to be desired; but I am afraid that no man will be able to breathe in those regions of speculation. I have been told, that respiration is difficult upon lofty mountains; yet from these precipices, though so high as to produce great tenuity of air, it is very easy to fall; therefore I suspect, that from any height where life can be supported, there may be danger of too quick descent.'

'Nothing,' replied the artist, 'will ever be attempted, if all possible objections must first be overcome; if you will favour my project, I will try the first flight at my own hazard; but I will work only on this condition, that the art shall not be divulged, and that you shall not desire me to make wings for any but ourselves.'

'Why,' said *Rasselas*, 'should you envy others so great an advantage? All skill ought to be exerted for universal good; every man has owed much to others, and ought to repay the kindness that he has received.'

'If men were all virtuous,' returned the artist, 'I should with great alacrity teach them all to fly; but what would be the security of the good, if the bad could at pleasure invade them from the sky? Against an army sailing through the clouds, neither walls, nor mountains, nor seas, could afford any security; a flight of northern savages might hover in the wind, and light at once with irresistible violence upon the capital of a fruitful region that was rolling under them.'

"The prince promised secrecy, and waited for the performance, not wholly hopeless of success; he visited the work from time to time, observed its progress, and remarked many ingenious contrivances to facilitate motion, and unite levity with strength. The artist was every day more certain that he should leave vultures and eagles behind him, and the contagion of his confidence seized upon the prince.

"In a year the wings were finished, and on a morning appointed, the maker appeared furnished for flight on a promontory; he waved his pinions awhile to gather the air, then leaped from his stand, and in an instant dropped into the lake: his wings, which were of no use in the air, sustained him in the water, and the prince drew him to land half dead with terror and vexation."

OF THE DIVING-BELL.

In order to facilitate our power of remaining on the surface of water, or of breathing when at the bottom, different methods have been contrived. As to the first, the cork waistcoat answers the purpose tolerably well; for the latter, the diving-bell is a well-known security. Dr. *Halley*, in a diving-bell of his own contrivance, remained fifty-two feet deep at the bottom of the sea, for the space of an hour and a half.

The diving-bell is an instrument long known and in use. That made by Dr. *Halley* was in the form of a great bell, and was coated with lead, so as to make it sink in water: it was three feet wide at top, five feet wide at bottom, and eight feet high. Into this great bell the diver entered, and sat upon a small seat within-side prepared for that purpose, and received light from a strong glass at top. Thus prepared, by means of a rope, the bell, the man, and all, were let down to the bottom, in order to search for goods, or fix cords to wrecks of ships, and such like purposes.

Though the machine is open at the bottom, and goes down many fathoms, only an inconsiderable quantity of water enters into, or rises in it, so that the diver has air enough above such water to breathe and live in for some time. This you may illustrate by an easy experiment; take a glass tumbler, and plunge it into water, with the mouth downwards and the sides perpendicular to the surface, and you will find very little water rise in the tumbler. It is the air contained therein which resists and prevents the water rising.

It is in this topmost part, which is empty, or only filled with air, that the diver keeps his head, and breathes that air which thus resists the ascending water; here he can remain for some time, living upon the condensed air, and at the same time performing what he descended for.

But, to be more particular in the description of Dr. *Halley's* bell. In the top was fixed, as before mentioned, a strong clear glass to let in the light from above, and likewise a cock to let out the hot air that had been polluted by repeated inspiration below. It was suspended to the mast of a ship, and so hoisted over the ship's side as to be let down without danger. In this, two or more divers were let down to the bottom, and two barrels of air were let down to them, to supply them with fresh air, which alternately rose and fell like two buckets. As the air from the barrels was let into the space in the bell free from water, it entered cold, and expelled the hot air, which had been spoiled, out through the cock at the top. By this method, air was communicated in such plenty, that, the Doctor informs us, that he was one of five who were together at the bottom in ten fathom of water, for above an hour and a half at a time, without any sort of ill consequence; and he might have continued there as long as he pleased, for any thing that appeared to the contrary. By the glass at the top of the bell, so much light was transmitted when the sun shone, and the sea was undisturbed, that he could see perfectly well to read and write, or to find any thing that lay at the bottom; but, in dark weather, and when the sea was rough, he found it as dark as night at the bottom. But then this inconvenience might be remedied, by keeping a candle burning in the bell as long as he pleased; for he found by experience, that a candle polluted the air by burning, just as a man would by respiring, both requiring about the same quantity of fresh air for their support, to the amount of nearly a gallon in a minute.

This machine was so far improved, that one of the divers might be detached to the distance of eighty or a hundred yards, by a close cap being put upon his head, with a glass in the fore part for him to see through, and a pipe to supply him with air, communicating with the great bell; this pipe was flexible, coiled round his arm, and served him as a clue to find his way back to the bell again. The only inconvenience that *Halley* complained of was, that upon their first descending, he and his companions found a small pain in their ears, as if the end of a quill were thrust forcibly through into the aperture of the ear. One of the divers, however, willing to remedy this inconvenience, stuffed his ears with chewed paper, which, as the bell descended, was so forcibly pressed into the cavities of that organ, that the surgeon could not extract the stuffing without great difficulty.

Triedwald, a Swedish engineer, has made some improvements on this machine since *Halley's* time. That contrived by him is less than *Halley's*, and consequently more easily managed; it is illuminated with three convex glasses instead of one. It has been found, that the nearer the diver's head is to the surface of the water in the bell, the better he breathes, for the air at that place is most comfortable and cool. In *Triedwald's* bell, the diver's head is therefore nearer the water, and when there is a necessity for his lifting up his head to the top of the bell, he has a flexible pipe in his mouth, with which he breathes only the air at the surface of the water at the bottom of the bell.

OF MR. SMEATON'S DIVING-CHEST, *fig.* 6, *plate* 2.

This machine was used with great success by Mr. *Smeaton*, at Ramsgate. Instead of the usual form of a bell, or of a conical tub of wood sunk by weights externally applied, Mr. *Smeaton's* was a square chest of cast iron, which being 50 cwt. was heavy enough to sink itself; and being 4½ feet in height, 4½ feet in length, and 3 feet wide, afforded room sufficient for two men at a time to work under it. It was peculiar to this machine, that the men therein were supplied with a constant influx of fresh air without any attention of theirs; that necessary article being amply supplied by a forcing air-pump, in a boat upon the water's surface. The figure will give you an idea of this machine; only in the model from which the drawing was made, the sides were of glass, for the purpose of rendering the effects visible. In Mr. *Smeaton's* chest, light was admitted through four strong pieces of glass fitted to the upper part of the chest.

Thus you find, that scarce any part of nature is wholly secluded from human visitation, since means have been found to ascend into the aerial regions, and to descend without danger to the bottom of the ocean.

LECTURE XXXV.

OF HYDRAULICS, OR THE MOTION OF FLUIDS.

HYDROSTATICS, as you have seen, determines the weight or pressure of fluids upon solids, or upon each other in vessels where the water is not suffered to escape, but remains at rest, *hydraulics* has for its object the motion of fluids. Upon the principles of this science many machines are constructed; several engines used in the mechanic arts, and various kinds of mills, pumps, and fountains, are the result of hydraulics judiciously applied.

If we knew with certainty the mass, the figure, and the number of particles of a fluid in motion, the laws of its motion might be determined by the resolution of a mathematical problem, namely, by finding the motion of a system of small free bodies acting one on the other in obedience to some exterior force, as that of gravity. We are, however, very far from being in possession of the data requisite for the solution of this problem; even if we were in possession of them, I do not know that we should be much farther advanced, as it would be difficult to deduce any satisfactory results from the intricate calculations in which the question would be involved. Some great mathematicians have endeavoured to deduce the laws of motion in fluids from the equilibrium of their particles, but unfortunately they are so complicated as to be of no practical use.

Accurate physical principles are always necessary before any utility can be drawn from mathematical abilities. Men may enter deeply into abstract speculations, and rise from assumed data to the most sublime efforts of the human mind; but if no physical existences correspond with those data, no advantage can arise to the general state of knowledge from exercises of this kind, and they can only be considered as mere amusements of the understanding.

It will be therefore necessary for those who wish to investigate this subject, to endeavour to establish their physical principles on experimental facts and accurate observation. In what I lay before you, I shall chiefly follow the *Abbe Boussut.*

OF THE SPOUTING OF FLUIDS THROUGH SMALL ORIFICES.*

When water spouts from a small hole in the bottom of a vessel, 1. The water descends nearly in a vertical direction, and the surface deviates very little from a horizontal plane; but at about three or four inches from the bottom, the particles turn from the vertical direction, and come from all parts with a motion more or less oblique towards the aperture. The same thing takes place when the water escapes from a small hole in the side of the vessel. The tendency of the particles towards the orifice is a necessary consequence of their perfect mobility, for they are hereby naturally directed towards that part where they meet with the least resistance, which part is the aperture.

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* Those who wish to study this subject fully, should consult *Boussut*, Traite Elementaire d'Hydrodynamique. *Buat*, Principes d'Hydraulique. *Prony*, Architecture Hydraulique.

2. At a small distance from the bottom of the vessel the water forms itself into a kind of funnel, whose point or summit corresponds with the centre of the hole. When the water runs out of a hole in the side of the vessel, it forms only a kind of half funnel, beginning when the surface nearly touches the upper edge of the hole. It is probable that the funnel begins to be formed as soon as the water begins to run out; but it does not become very sensible, except when the surface is at a small distance from the bottom.*

The funnel commences at a greater height from the bottom of the vessel, in proportion as the bottom is larger; the size, thereof is, however, varied by a number of circumstances.

The velocity of the water spouting from a small hole in the bottom of the vessel, is equal to that which a heavy body would acquire in falling vertically from a height equal to that of the surface of the fluid above the aperture.

The same law takes place when the hole is in the side of the vessel, for the pressure of the fluid is equal, at the same depth, in all directions, and will consequently produce the same velocity.

The fluid in issuing out of the hole gives a velocity sufficient to make it rise vertically to a height equal to that of the surface of the fluid above the aperture; in the same manner as a body falling from a certain height acquires a velocity sufficient to make it ascend to the height from which it fell.

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* The writers on this subject seem to have neglected a revolving motion in the water, but which, on making further experiments, they will find worthy their attention. If at the bottom of a vessel of water an aperture be made for the fluid to escape, it will revolve about the aperture, at some distance from it, and escape with the revolving motion; the water rushes from all sides in concentrating streams to supply the continual waste.

From the theory of falling bodies, it is plain, that if the fluid continued to move uniformly with the velocity it had acquired at coming out of the hole, it would move through a space equal to double the height of the fluid above the aperture, while a heavy body was descending through the same space.

The height being the same, the velocity of the fluid at the aperture will be always the same, and this though the fluid varies in density; for, though with a denser fluid the pressure is greater, the mass escaping is also greater, and the velocities are equal when the moving forces are proportioned to the masses they put in motion.

The quantities of a fluid proceeding in the same time through different apertures, each acted upon by a constant height or load, supposing of course that the vessels are kept equally full during the whole experiment, are to each other as the product of the areas of the apertures by the square-root of the heights; for example, it has been proved by experiment that a circular aperture of one inch diameter, in a thin vessel, gives in one minute of time, the water being four feet high, 5436 cubic inches of water. To know what will be furnished in the same time by an aperture two inches in diameter, the altitude of the water nine feet, use the following proportion, observing that the aperture of two inches is four times as large as that of one, because the areas of circles are as the squares of their diameters; as $1\times\sqrt{4}$ is to $4\times\sqrt{9}$, so is 5436 to x; or as 2 is to 12, so is 5436 to 32616 cubic inches of water, the quantity that will be furnished by an aperture of two inches diameter from a reservoir whose surface is always kept at nine feet from the aperture.*

If you fill with water a prismatic vessel, and let the water run out by an aperture in the bottom, observing the time employed by the water in running out, and then fill the vessel again, keeping the surface of the water at the same height, you will find in this last case, that in the same interval of time in which the vessel was emptying itself in the first instance, nearly double the quantity of water has been expended in the second.

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* The measures spoken of here are French.

In practice, the water often issues from lateral openings, which, although but small in comparison with the size of the reservoirs, cannot be considered as having all their points at an equal distance from the surface of the fluid. In these cases, the usual method of determining the quantity of water flowing through the aperture depends on the following principles: imagine the hole to be stopped by a plate, and this plate to be pierced with a great number of holes, through which the water escapes; now, considering each of these holes as a single insulated aperture, the velocity for each will be according to the correspondent height of the fluid. If the number of these holes be infinitely augmented, or, what comes to the same thing, if the plate be taken away, the velocity at each point of the given aperture will be as the height corresponding thereto; and in determining the quantity of effluent water, regard must be had to this inequality of velocity.

I must not, however, conceal from you, that this mode of reasoning is not very conclusive; for, though it may be just as far as it relates to the number of insulated holes, it does not appear clearly that the water will flow exactly in the same manner when the threads thereof are united, as when they proceed from small separate apertures: as the results of theory, however, upon this plan do not differ much from experiments, it may be useful to preserve it till some better method be discovered.

The quantity of water flowing through holes in a given time is not so great as might have been naturally expected, because the water does not flow in a compact parallel stream, but contracts in diameter on coming out of the aperture, and this contraction extends to a distance nearly equal to half the diameter of the aperture. The diameter of the contracted stream is to the diameter of the aperture as 3 to 4, or as $3\frac{1}{6}$ to 4, or as 19 to 24, so that its area is to that of the aperture as 10 to 16: it is nearly the same thing when the water flows from lateral apertures.

This contracted stream is a proof that withinside the vessel the lateral particles are directed towards the hole with different degrees of obliquity, which obliquity may be decomposed into two forces, one parallel to the plane of the hole, which contracts the fluid, the other perpendicular to the same plane, which occasions the efflux.

This contraction takes place also when water passes through tubes, and the contraction is at the entrance of the water into the tube, not at its going out, where it preserves its cylindric form. This contraction diminishes sensibly the quantity of water that should be furnished by the tubes.

To ascertain these facts, M. *Bossut* made a great number of experiments, the results of which I shall now lay before you. The apertures for the efflux of the water were all pierced perpendicularly in plates about half a line thick, and the time of each experiment is reduced to one minute.

Exp.	The water was kept constantly at eleven feet eight inches ten lines from the centre of each aperture.	Number of cubic inches furnished in one minute.
1.	With a horizontal circular aperture, six lines diameter	2311
2.	With a circular horizontal aperture, one inch diameter	9281
3.	With a circular horizontal aperture, two inches diameter	37203
4.	With a rectangular horizontal aperture, one inch by three lines	2933
5.	With a square horizontal aperture, the side one inch	11817
6.	With a square horizontal aperture, the sides two inches	47361
	Constant height nine feet.	
7.	Lateral circular aperture, six lines diameter	2018
8.	Lateral circular aperture, one inch diameter	8135

Constant height four feet.

9. Lateral circular aperture, six lines diameter . .	1353
10. Lateral circular aperture, one inch diameter . .	5436

Constant height seven lines.

11. By a lateral and circular orifice, one inch diameter	628

DEDUCTIONS FROM THE SRECEDING EXPERIMENTS.

1. *The quantities of fluid discharged in equal times from different-sized apertures, the altitude of the fluids being the same, are nearly to each other as the areas of the apertures.*

Thus in the second and third experiments the areas of the apertures are as one to four, and the water discharged 9281 cubic inches; which to 37203 is nearly in the same ratio.

2. *The quantities of water discharged, in equal times, by the same aperture, with different altitudes of the reservoir, are nearly as the square-roots of the corresponding altitude of the water in the reservoir above the centre of the aperture.*

Compare together the eighth and tenth experiments, in which the respective altitudes of the reservoir were of 9 and 4 feet, of which the square-roots are 3 and 2, and you will find the water discharged by the first was 8135 cubic inches, the second 5436 cubic inches nearly in the proportion of 3 to 2.

4. *That on account of the friction, small apertures discharge less water than those that are larger and of a similar figure, the water in the respective reservoirs being at the same height.*

5. *That of several apertures, whose areas are equal that which has the smallest circumference will discharge more water than the others, the water in the reservoirs being at the same altitudes, and this because there is less friction.* Hence, circular apertures are most advantageous, as they have less rubbing surface under the same area.

It is easy to perceive, that the quantities of water expended in the foregoing experiments, are not nearly so much as they ought to be, considering the size of the apertures, and the altitude of the reservoirs. The quantity discharged is diminished considerably by the friction, and by the contraction of the stream;* for the velocity which depends on the altitude of the reservoir is not sensibly altered. The difference in the discharge of water, supposing, 1. That the area of the stream is the same with that of the aperture; 2. That this stream is contracted; is as 16 to 10: in other words, by supposing the area of the orifice to be diminished in the proportion of 16 to 10, you may determine with sufficient exactness the efflux of fluids from vessels, where the surfaces are maintained at the same height.

OF THE DISCHARGE OF FLUIDS THROUGH ADDITIONAL TUBES.

If the water, instead of flowing through an aperture pierced in a thin substance, pass through the end of a vertical tube of the same diameter as the aperture, there is a much greater discharge of water, because the contracted stream is greater in the first instance than in the second.

In the following experiments, the constant height of the water in the reservoir above the upper aperture of the tube was 11 feet, 8 inches, 10 lines, the diameter of the tube 1 inch.

	Different lengths of the tube expressed in lines.		Number of cubic inches of water discharged in one minute.
	Lines		
Exp. 1	48	The stream filling the tube	12274
2	24		12188
3	18		12168
4	18	The water not filling the tube	9282

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* And probably on account also of the circular motion of the fluid.

You see, by comparing the three first experiments, that the longer the vertical tube is, the greater is the discharge of the water, because the contraction of the stream is less; it is, however, always somewhat contracted, even when it appears to fill the tube.

By comparing the quantities of water discharged in the third and fourth experiments, you will find the two discharges 12168, 9282, are to each other nearly in the proportion of 13 to 10; but you have seen, that the water discharged through a thin aperture without any contraction in the stream, would be to the same aperture with a contracted stream as 16 to 10. From hence we may conclude, that *the altitude in the reservoir and the apertures being the same, the discharge through a thin aperture without any contraction in the stream, the discharge through an additional tube, and the discharge through a similar aperture with a contracted stream, are to each other nearly as the numbers* 16, 13, 10; these proportions are sufficiently exact for practice. Hence you see that an additional tube only destroys in part the contraction of the stream, which contraction is greatest when the water passes through a thin aperture from a large reservoir.

If the additional tube, instead of being vertical, or placed at the bottom of the reservoir, were horizontal or placed in the side, it would furnish the same quantity of water, provided it were of the same length, and that the exterior aperture were at the same distance from the surface of the water in the reservoir.

If the additional tube, instead of being cylindrical, were conical, having its larger base nearest the reservoir, it would discharge a greater quantity of water. The most advantageous form that can be given in order to obtain the greatest quantity of water in a given time by a given aperture, is that which the stream assumes in coming out of the aperture; *i. e.* you must give the tube the form of a truncated cone, whose smaller base should be of the same diameter as the aperture, the area of the smaller base should be to that of the larger as 10 to 16, and the distance from one base to the other should be the semidiameter of the larger base; the efflux of water will then be as abundant as it would be through a tin aperture equal to the smaller base, and where the stream was not contracted. This form may be applied where it is necessary to obtain a certain quantity of water from a river, an aqueduct, &c. by a canal or lateral tube.

On comparing the efflux of water through additional tubes of different diameters, and with different altitudes of the water in the reservoirs, the following results were obtained; the additional tubes were two inches long, and placed vertically at the bottom of the reservoir.

Constant altitude of the water above the tubes.		Diameters of the tubes expressed in lines.		Number of cubic inch. in 1 min.
Ex. 1	3 feet 10 in.	6	Water filling the tube	1689
2		10		4703
3		6	The water not following the sides	1293
4		10		3598
5	2 feet	6	The water filling the tube	1222
6		10		3402
7		6	The water not filling the tube	935
8		10		2603

It follows from these experiments, 1. *That the discharge by different additional tubes, with the same altitude of the reservoir, are nearly in proportion to the area of the apertures, or to the squares of the diameters.*

2. *That the discharge of water by additional tubes of the same diameter, with different altitudes of water in the reservoir, are nearly proportional to the square-root of the altitude of the reservoir.*

3. *That in general the discharge of water in the same time, through different additional tubes, with different altitudes of water in the same reservoir, are to each other nearly as the product of the square of the diameters of the tubes by the square-root of the altitude of the reservoirs.*

So that the effluxes of water by additional tubes, follow amongst themselves, the same laws as through the thin orifice.

The following table was formed from the foregoing experiments.

Constant altitude of the water in the reservoir above the aperture expressed in feet.	Water discharged in one minute through a hole one inch diameter, the stream not contracted, in cubic inches.	Water discharged in one minute through an additional tube of 1 inch diameter, two inches long, in cubic inches.	Water discharged during 1 min. through a hole one inch diameter, with a contracted stream, in cubic inches.
1	4381	3539	2722
2	6169	5002	3846
3	7589	6126	4710
4	8763	7070	5436
5	9797	7900	6075
6	10732	8654	6654
7	11592	9340	7183
8	12392	9975	7672
9	13144	10579	8135
10	13855	11151	8574
11	14530	11693	8990
12	15180	12205	9384
13	15797	12699	9764
14	16393	13197	10130
15	16968	13620	10472

OF JETS D'EAU.

We are now going to consider the nature of fountains. There are few things that give more pleasure to the eye than a diversity in the play of water from a fountain: but these machines give additional pleasure in sultry climates, where they contribute to cool the air, as well as to enliven the prospect. With us they are chiefly made for the purposes of embellishment, for here the air is seldom disagreeable from too much warmth; and if there were fountains of fire, they would often make the most grateful ornament.

Whatever be the direction of the jet, the discharge of water is always the same; provided that the ajutage, and the altitude of the reservoir above the ajutage, be the same. This is a necessary consequence of the equal pressure of fluids in all directions.

Water, spouting from a small ajutage, has sufficient velocity to carry it to the same height as the water in the reservoir; but it never attains entirely this height, being prevented by various concurring causes. 1st. The friction in the tubes between the reservoir and the ajutage. 2dly. The friction against the circumference of the aperture. 3dly. The resistance of the air to the weight of the water at the top of the spout; for this, having lost its motion, rests on the part below, and by its weight obstructs the motion of the column. The resistance from this cause is so great, that the jet is frequently destroyed, the rising water being by fits and starts pressed down to the very orifice from which it spouts: but this inconvenience is remedied, if you give the jet a little inclination; for then the particles which have lost their motion upwards do not fall back as before, but fall off from the rest, and thus do not incumber the rising fluid; hence such jets as are a little inclined will rise higher than those that are vertical.

When the ajutage is inclined to the horizon, the projectile force and the gravity of the water cause the stream to describe a parabola, whose amplitude is greater in proportion to the height of the reservoir.

When the ajutage is in a horizontal direction, the jet describes a semi-parabola.

Jets of water rise higher in proportion as the aperture of the ajutage is larger; because, 1. Of two jets proceeding from the same reservoir with equal velocities, the larger undergoes less friction: 2. It has more mass, and consequently more force to overcome obstacles. But though a large jet will rise higher than a small one, it does not discharge more water; for the discharge is as the product of the aperture by the velocity at the moment of efflux; and this velocity is the same in each, friction not being considered.

In order to make large jets rise higher than small ones, the conducit-pipe must be large enough to furnish a sufficient quantity of water; for experience has shown, that if these be narrow, small jets will rise higher than those that are larger. The diameter of the conducit pipe should therefore bear a certain proportion to that of the ajutage, in order to make a jet rise to the greatest possible height. If you compare two different jets, and be desirous that each should attain its greatest altitudes, *the squares of the diameters of the conducit-pipes must be to each other in the compound ratio of the squares of the diameters of the ajutages, and the square-root of the altitude of the reservoir.* Thus, if you know by experience the diameter that ought to be given to a conduit-pipe, to furnish water for the discharge of a given ajutage, with a reservoir of a given altitude, you may determine the diameter of another tube, to feed a given ajutage with a reservoir of a given altitude.

Experience has shown, that, for an ajutage six lines diameter, with a reservoir of fifty-two feet, the conducit-pipe should be about thirty-nine lines; for an ajutage six lines diameter, and a reservoir sixteen feet, the conducit-pipe twenty-eight and a half lines. There is no inconvenience in giving a conducit-pipe a greater diameter than is required by the above rule, but there would be a considerable one in giving it a smaller diameter.

From the comparison of several experiments made on jets d'eau, it appears, that the difference between the altitudes of vertical jets, and the altitudes of the reservoirs is to each other as the squares of the jet's altitude. If you know, therefore, by experiment how far any jet falls short of the altitude of its reservoir, you may find, by the rule of three, how much any other jet falls short of its reservoir. If you wish to know the altitude of the reservoir, you have only to add to the altitude of the jet the quantity found by the proportion. It is to be observed, that the conducit-pipes should never be fixed at right angles to each other.

The following table will facilitate the application of the foregoing principles. In the two first columns are the altitudes of the jets and the corresponding altitudes of the reservoir. The third column contains (in Paris pints, thirty-six of which make a cubic foot) the discharge during one minute, through an ajutage six lines diameter, relatively to the altitudes of the second column. Knowing the discharge with an ajutage of six lines, by the rule of three, you will discover the discharge by any other ajutage with a reservoir of the same height; since it has been proved, that the discharges are as the areas of the ajutages, or as the squares of the diameters of these ajutages. In the fourth column you find the diameters for the conducit-pipes of an adjutage six lines diameter, relatively to the altitude of the second column.

Altitude of the jet in feet	Altitude of the reservoir in feet and inches.		Discharge in 1 minute by an adjutage of 6 lines diameter in Paris pints.	Diameter of the conduit-pipes relative to the 2d and 3d cols.
	Feet.	*In.*	*Pints.*	*Lines.*
5	5	1	32	21
10	10	4	45	26
15	15	9	56	28
20	21	4	65	31
25	27	1	73	33
30	33	0	81	34
35	39	1	88	36
40	45	4	96	37
45	51	9	101	38
50	58	4	108	39
55	65	1	114	40
60	72	0	120	41
65	79	1	125	42
70	86	4	131	43
75	91	9	136	44
80	101	4	142	45
85	109	1	147	46
90	117	0	152	47
95	125	1	158	48
100	133	4	163	49

OF PUMPS.

What I have said to you concerning conduit-pipes, the efflux of water from different apertures, &c. naturally leads us to consider the nature and action of pumps: their general principles were explained to you in my lectures on air. I there, however, only noticed two kinds: I shall now bring you acquainted with some further varieties in their construction, and notice some circumstances that I passed over in the former lecture.

OF THE CHAIN-PUMP.

Among these is what is called the chain-pump. This is generally made from twelve to twenty-four feet in length; consists of two collateral square barrels, and a chain of pistons of the same form, fixed at proper distances thereon. The chain is moved round a coarse kind of wheel-work, fixed at one end of the machine. The teeth of this are so contrived as to receive one half of the flat pistons, and let them fold in, and they take hold of the links as they rise. A whole row of the pistons, which go free of the sides of the barrel by near a quarter of an inch, are always lifting when the pump is at work; and as this machine is generally worked with briskness, they bring up a full bore of water in the pump. It is wrought either by one or two handles, according to the labour required. The chain-pump is so contrived, that, by the continual folding in of the pistons, stones, dirt, and whatever may happen to come in the way, may also be cleared: it is therefore used to drain ponds, sewers, and remove foul water, in which no other pump could be employed.

OF PUNPS WHERE THE PRESSURE OF THE ATMOSPHERE IS EMPLOYED.

These are pneumatic as well as hydraulic engines, and are usually divided into two kinds: 1. The common, or sucking-pump; 2. The forcing-pump. The nature of these, their action and operation, I shall explain by these glass models; in which you will see the motion of the pistons, and the play of the valves.

OF THE COMMON PUMP, *plate* 3, *fig*. 1.

I have poured water into the trough, which trough is to represent the well from which the water is to be raised. There are two valves in this pump: one, b, at the upper end of the small tube; the other, a, on the moveable piston. When the pump is not worked, their weight makes them lie close upon the holes over which they are placed. I raise or depress the piston by means of the piston-rod.

Let us now commence our operation. The piston being placed at the bottom of the barrel, before I begin to work, I raise the piston from the bottom to the top of the barrel; which makes room for the air in the pump, below the piston, to expand itself. The air in the pipe being thus dilated, presses less on the surface of the water within the pipe, than the atmosphere does without on that in the trough; and consequently the water rises in the tube till the pressures are equal; that is, till the air within is just as dense as that without; and it will there remain at rest between the two equal pressures. The valve at the bottom, which rose a little to let some of the rarefied air into the barrel, falls down again, and closes the hole at the top of the pipe.

I now depress the piston; and as the air in the barrel cannot get back again through the valve b, at the top of the pipe, it will raise the valve in the piston, and so make its way through the upper part of the barrel into the open air. I raise the piston again, and the air between it and the water in the lower pipe will again be left at liberty to fill a larger space; and so its spring being again weakened, the pressure of the atmosphere will force more water into the pipe; and when the piston is at the top of the barrel, the lower valve falls, and stops the hole at the top of the pipe, as before.

The same effect is produced by every stroke, till at last the water in the pipe reaches the bottom of the barrel. Now, upon depressing the piston, as the water cannot be forced back again through the lower valve, it will raise the upper valve as the piston descends, and will be lifted up by the piston when raised again.

The whole space below the piston is now full of water; and as the water cannot escape by the lower valve, it will, on my depressing the piston, raise the valve, in order to let the piston down. When this is quite at the bottom, the valve will fall by its weight, and stop the hole in the piston. When the piston is next raised, all the water above it will be lifted up, and run out of the spout; and thus, by alternately raising and depressing the piston, there is still more water raised; which, getting above the pipe into the wide part at top, will supply the spout, and make it run with a continual stream.

Thus you have seen, that every time I elevate the piston, the lower valve rises, and the upper valve falls; but that every time I depress the piston, the lower valve falls, and the upper one rises.

Now, as it is the pressure of the air of the atmosphere which causes the water to rise, and follow the piston, as it is drawn up; and as a column of water, of thirty-three feet high, is of equal weight with as thick a column of air, from the earth to the very top of the atmosphere; therefore the perpendicular height of the piston, from the surface of the water in the well, must always be less than thirty-three feet, otherwise the water will never rise above the piston. But when the height is less, the pressure of the atmosphere will be greater than the weight of the water in the pump, and will therefore raise it above the piston; and when the water has once got above the piston, it may be thereby lifted to any height, if the rod be made long enough, and a sufficient degree of strength be employed to raise it with the weight of water above the piston.

The force required to work a pump, is as the altitude of the water to be raised, and as the square of the diameter in that part where the piston works. Hence, if two pumps be of equal height, and one of them be twice the bore of the other, the larger will raise four times as much water as the narrower, and will therefore require to be worked with four times as much strength.

The wideness or narrowness of the pump, in any other part, besides that where the piston works, does not render the pump either more or less difficult to work, except what difference may arise from the friction of the water in the bore, which is always greater in a narrow bore than a wide one.

The pump-rod is generally raised by means of a lever, whose longer arm, where the power is applied, is generally five or six times the length of the shorter arm; by which means it gives five or six times as much advantage to the power.

Mr. *Ferguson* gives the following table for finding the dimensions of a pump that shall work with a given force, and draw water from a given depth, the handle being supposed to increase the power five times. It is also supposed, that one man can work a pump four inches diameter, and thirty feet high, and discharge 27½ gallons of water in a minute; the measure being that called English wine-measure.

Height of the pumps above the surface of the well	Diameter of the bore where the piston works.	Water discharged in a minute	
Feet.	*Inches.*	*Gals.*	*Pints.*
10	6.93	81	6
15	5.66	54	4
20	4.90	40	7
25	4.38	32	6
30	4.00	27	2
35	3.70	23	3
40	3.46	20	3
45	3.27	18	1
50	3.10	16	3
55	2.95	14	7
60	2.84	13	5
65	2.72	12	4
70	2 62	11	5
75	2.53	10	7
80	2.45	10	2
85	2.38	9	5
90	2.31	9	1
95	2.25	8	5
100	2.19	8	1

Now, to find the diameter of a pump that shall raise water with the same ease as a man can work a pump thirty feet high, with a four inch bore, look for the height in the first column, and over against it, in the second, you have the diameter or width of the pump, and in the third you find the quantity of water which a man of ordinary strength can discharge in a minute.

OF THE FORCING-PUMP, *plate* 3, *fig*. 2.

It is so called, because it not only raises the water into the barrel, like the foregoing, but it afterwards forces it up into the reservoir, in a lofty situation. The operation and nature of this pump will be evident by attending to the working of this model, *plate* 3, *fig*. 2. The pipe and barrel are the same as in the other pump, but the piston, G, is solid, having no valve, so that no water can get above it. At the bottom of the barrel, B, a pipe, M M, is fixed, and at right angles to this pipe, a cistern, or air-vessel, K K; at the bottom of the air-vessel there is a valve, b; from the top, a small pipe, O H I, is inserted so as nearly to reach the bottom of the air-vessel, and at the same time be air-tight at top.

I shall now begin to work the model, that you may see and comprehend the effects. The pipe-valve, a, rises when I draw the piston up; but falls down, and stops the hole, the moment the piston is at its greatest height. Now, as the water which has been raised above this valve cannot get back again into the pipe, but has a free passage by the pipe, M M, that opens into the air-vessel, it is forced into this vessel by depressing the piston, and retained therein by its valve b, which shuts the moment the piston begins to be raised, because the pressure of the water against the under side thereof then ceases.

The water, being thus forced into the air-vessel by repeated strokes of the piston, has now got above the lower end, I, of the pipe, and begins to condense the air in the air-vessel; for the air has no way to get out of this vessel, but through the tube, O H I, of the pipe, and is prevented from escaping this way when the mouth of the tube is covered with water. It is also gradually more and more condensed as the water rises in this vessel; till at last, as you see, it presses so strongly upon the water as to force it up through the pipe O H I; from whence it spouts, at F, in a jet to a great height, and is supplied by alternately raising and depressing the piston.

The higher the surface of the water is raised in the air-vessel, the smaller is the space into which the air is condensed; and, consequently, its spring will be stronger, and the pressure greater upon the water, which will be thereby driven with greater force through the pipe; and as the spring of the air continues to act even while the piston is rising, the stream will be uniform as long as the piston is worked.

The valve of the pipe opens to let the water follow the piston in rising. Whilst this valve is open, that of the air-vessel is closed, to prevent the water, which is forced into the air-vessel, from running back by its pipe into the air-vessel.

The effect of this kind of pump is not limited to the raising of water to any particular altitude; since the air's condensation may be increased to any degree.

If the air's condensation be double to that of the atmosphere, its elastic force will raise the water to about the height of thirty-four feet. If the condensation be increased three-fold, the altitude to which the water may be raised by it will be about twice the former height, or sixty-eight feet; the altitude of the raised water being increased thirty-four feet for each addition of unity to the number which expresses the air's condensation.

The engines used for extinguishing fire are upon this construction; consisting of two barrels, by which water is alternately driven into a close air-vessel. The forcing of the water therein condenses the air, which presses the water so strongly, that it rushes out with great impetuosity and force through a pipe that comes down into it, and makes a continued uniform stream by the condensation of air upon its surface.

OF DE LA HIRE'S PUMP, *plate 3, fig.* 3.

The model before you is that of a pump invented by M. *de la Hire*, which is so constructed, as to raise water as fast by the descent as by the ascent of the piston. As before, the trough in which the two pipes are placed represents the well: one of the pipes, B, is fitted to the lower end of the barrel in which the piston works; the top of the other pipe, C, is so connected with the smaller one, as to communicate with the upper part of the barrel. There is a valve on the top of the pipes B, C, and also on the two pipes E, F, which proceed from the pump-barrel into the air-vessel P. The piston is solid, or without any valve or opening.

As the piston rises, the air, pressing on the surface of the water in the trough, forces it up the pipe B, at the bottom of the barrel, and fills it with water up to the piston. The valves, e and s, lie close and air-tight at the top of their respective pipes E and F.

When the piston stops at its greatest height, the valve at the bottom of the barrel closes, and prevents the water from being forced back. Hence, as the piston is depressed, it forces all the water in the barrel up through the lower crooked pipe F, and through its valve, into the air-vessel.

The piston-rod moves through what is called a collar of leather, which makes it air-tight.

During the descent of the piston, the valve upon the upper crooked pipe falls down, and the pressure of the air on the water in the trough raises the water through this pipe, C, and the valve at the top of it, which is opened upwards by the power of the ascending water; and this water runs into the barrel of the pump, and fills all the space therein above the piston.

As soon as the piston is as low as it can go, the valve at the end of the upper pipe, D, falls down and closes it, so that no water can be forced back through it. As the piston is raised, all this water is forced through the upper pipe E, and, after opening its valve e, into the air-vessel P.

Thus, as the piston descends, it forces all the water below it up the pipe F; and, as it rises, it forces all the water above it up the pipe E; so that there is as much water forced up into the air-vessel by the ascent, as by the descent of the piston. The air is compressed in the air-vessel as in the preceding case; and the water being equally forced in, rushes out with a constant and very nearly equal stream. It is evident, from what we have already told you, that the top of the pipe that opens into the upper part of the barrel should never exceed thirty-two feet.

OF A NEW HAND-PUMP, INVENTED BY MR. WALTER TAYLOR, OF SOUTHAMPTON, AND USED BY THE ROYAL NAVY OF GREAT BRITAIN.

Every friend of mankind must rejoice on being informed, that the accidents to which ships that spring a a leak at sea were liable from the imperfections of the chain-pump, are happily removed by the ingenious contrivances of Mr. *Walter Taylor*, of Southampton, well known for his mechanical abilities, which, in other instances as well as this, have proved equally beneficial to himself and country. It seems rather surprizing, that the common pump, whose effects are so well known, should have remained for centuries inadequate to the purposes of the navy. The mechanism adapted by Mr. *Taylor* is so important, and, in various particulars, so different from what is in general applied to the common pump, that it may with great propriety be considered as a new invention.

Mr. *Taylor's* pumps have been in general use in the royal navy for some years: they have answered every expectation he first formed, though he has made many improvements on them. Here are three figures, which will give you a general idea of these pumps; they were copied from drawings which were kindly communicated to me for this purpose by Mr. *Taylor*. *Fig.* 4, *plate* 3, is a section of one of Mr. *Taylor's* pumps, of a simple construction. The piston is represented as descending in a chamber properly adapted thereto. At a and b you have a view of Mr. *Taylor's* pendulum valves; which, from their form, disengage themselves from chips, gravel, sand, &c. The piston is also so contrived, that no chips, gravel, or sand, can get between the leather and lower part of the piston; to both which defects the former constructions were liable. *Fig.* 7 is a separate view of the pendulum valve.

Fig. 4 represents a pump working with one piston-rod, and *fig.* 5 a pump working with two piston-rods; the one rising as the other falls: in *fig.* 4 and 5, the rods are supposed to be worked by levers. By a judicious application of ropes, to be carried on either deck, see *fig.* 6, Mr. *Taylor* is enabled, where men are plenty, as in a man of war, to raise any quantity of water. The drawing is taken from a pump with a seven-inch bore, and heaves one ton per minute twenty-four feet high, with ten men, five only working at a time. One is now constructing by Mr. *Taylor* to heave five tons per minute twenty-four feet high. The pumps are also so constructed, that a copper pump may be taken out of the wooden case, in order, when necessity requires, to make two pumps for separate work.

OF THE HESSIAN PUMP, *plate* 2, *fig.* 7 *and* 8.

ABC, DE, two tin vessels, soldered together, but communicating with each other by a hole at bottom. The larger vessel is furnished with a rim, to receive the water thrown up by the circulating tubes, and convey it into the vessel D E; m, n, o, p, represent four tubes of metal or glass, open at both ends, but bent at top, and fixed in an angular position to the axis K L. When in their place, the extremity, L, of the axis rests upon a point at the bottom of the large vessel, while the upper part is steadied, and kept in a vertical position, by passing through a hole in a bar going over the large vessel A B C.

Fill the vessels about two thirds with water, and then make the tubes circulate rapidly by turning the handle S, and the rotatory centrifugal motion will raise the water, and discharge it into the small vessel D E, by the pipe h.

OF VERA'S PUMP; OR AN ENGINE TO RAISE WATER BY MEANS OF HAIR-ROPES.

A and B, *plate* 3, *fig.* 8, are three hair-ropes passing over the pullies b and d, each of which has three grooves. The lower pulley, b, is immersed in water, and is kept therein by a weight suspended from it. These pullies are turned round with great rapidity by means of two multiplying wheels, one of which is seen at G. By turning the pullies, the cords revolve also with great rapidity, and the ascending sides carry up a considerable quantity of water, which they discharge with violence into the reservoir H, from whence it is conveyed into any convenient place by the pipe KL. The ropes should not be more than an inch asunder.

There are two of these machines at Winds. The depth of the well where one of them is fixed is ninety-five feet, and the quantity of water raised by the utmost efforts of a man is about nine gallons per minute.

In the beginning of the motion, the column adhering to the rope is always less than when it has been worked for some time, and continues to increase till the surrounding air partakes of its motion.

OF A SUPPOSED IMPROVEMENT OF THE COMMON PUMP.

In the year 1766, it was announced in the public papers, that at Seville, in Spain, a simple sucking pump had been constructed, which raised water sixty feet; and they concluded from thence, that those were strangely deceived who had asserted, that the pressure of the atmosphere would not support a higher column than thirty-two feet. On examination is was found, that an ignorant tinman at Seville had made a common sucking pipe with its lower valve sixty feet from the surface of the water; but, finding that he could raise no water by it, either through impatience or passion, with a stroke of a hatchet he made a small opening about ten feet above the surface of the water, and which forced a small quantity of water above the lower valve; the reason of which I shall explain by a diagram.

Let P F, *plate* 3, *fig*. 1, be the sucking tube, d the surface of the water, from d to F sixty feet; and that after a certain number of strokes of the piston the water was raised thirty-two feet in the tube, or to *e;* and that a small hole was made at ten feet from the surface of the water. The air which enters this, pressing equally every way, makes the water which is below *b*, fall down into the well; while the pressure upwards forces the water up thirty-two feet, through the valve, into the body of the pump. But this is not all, for it would have carried it to a much greater height; for the air near the earth is above eight hundred times rarer, or less dense than water; and supposing the density of a column thereof to be uniform, which is not the case, ten feet of water taken away would be equivalent to a column of eight thousand feet of air; so that the remaining twenty-two feet would be in equilibrium with the air, after being raised eight thousand feet. To have a second portion of water, the hole, *b*, must be stopped up, and the piston worked till the water rises to *e*, and then re-open the hole. In the first place you see, that this pretended discovery is so far from invalidating the principle of the pressure of the air, that it is a direct consequence thereof; secondly, that even to make it answer at all, it is necessary that the pipe be very small, or the column of water would be broken to pieces, the air would pass through, and very little would rise.

OF THE MOTION OF WATER IN CONDUIT-PIPES.

In conducting water from one place to another, the conduit-pipes must be longer in proportion as the places to which it is to be conveyed are more distant from each other. In the additional tubes heretofore spoken of, I took no notice of fricton, as in the cases then under consideration it was scarcely sensible. In long tubes it is different; the friction of these lessens considerably the velocity of the water.

On this head I can only relate to you the result of various experiments that have been made; which it is impossible for us to repeat, as they require not only a very large apparatus, but conveniencies that are only to be procured by those whose interest is concerned in the experiments. The experiments of which I shall give you the results were made by Mr. *Bossut;* the tubes were straight; one of them was sixteen lines inside diameter, the other two inches; and the tubes were successively lengthened from thirty to a hundred and eighty feet. The constant altitude of the water in the reservoir, above the axis of each tube, was in some cases one foot, in other cases two feet. This is a branch of hydrostatics, in which theory is necessarily imperfect, and the only means of arriving at truth must be from experiment.

Constant altitude of the water in the reservoir above the axis of the tube, expressed in ft.	Distance to which the water was conveyed, expressed in feet.	No. of cubic inches of water discharged by the tube of 16 lines diameter in a minute.	No. of cubic inches of water discharged by the tube of two inches diameter in a minute.
1	30	2778	7680
1	60	1957	5564
1	90	1587	4534
1	120	1351	3944
1	150	1178	3486
1	180	1052	3119
2	30	4066	11219
2	60	2888	8190
2	90	2352	6812
2	120	2011	5885
2	150	1762	5232
2	180	1583	4710

By comparing this with the table in page 429, you will find that the discharges of water there are much greater than the corresponding ones in the present table, and that the discharge is lessened as the tubes is lengthened, because there is a greater rubbing surface.

We may also notice that the diminution in the discharge is not in proportion to the length of the tube; for, the first thirty feet diminishes the discharge much more than the second thirty feet, and the third length of thirty feet diminishes still less in the discharge; and so on.

From these experiments it appears, that great accuracy is not necessary in practice; and perhaps we may adopt for a general rule, *that the discharge made in equal times by a horizontal tube, with the same altitude of reservoir, but at different distances from the first aperture of the tubes, are to each other nearly in the inverse ratio of the square-roots of the distances.*

The discharge is more in proportion from the larger tube than from the smaller one; because there is less rubbing surface in proportion in the larger tube.

If the tubes be curved, instead of being straight, the discharge will be somewhat diminished. This diminution in the discharge appears to rise from the impact of the water against the angular parts of the tube, whereby its velocity is diminished. This diminution will therefore vary with the degree of curvature.

When the plane of the carvature of the tube is in a vertical direction, there will be proportions of the tube where the air will fix itself, so as to lessen the velocity, or even stop the course of the water. Let ABCDE, *plate* 2, *fig.* 9, be a tube, whose upper end, A, is joined to the reservoir that furnishes the water; G, the end by which the fountain is supplied. When the communication at A is opened, the tube is filled with air; the water will fill the tube A B, drive out the air therefrom, and rise to C. Here experience has shown, that the water runs down the lower part of the curvature, and fills up the neck, D, leaving behind it the column of air CD; which will remain there, notwithstainding the pressure of the column of air, AB. The water continuing to flow, runs down the lower part of EF, and fills the neck F, leaving the second column of air at EE; so that the water will be only raised to I, and will not run out at G.

OF THE SYPHON.

Though I treated of the syphon in an early stage of the lectures,* as the account there given does not seem so correct as that given by a late author,† I shall here endeavour to state with great accuracy the principle on which it acts.

A syphon is an instrument used to decant fluids, or convey them from one place, over an obstacle that is higher than their surface, to another that is lower. Its form, as you have seen, is exceedingly simple, being nothing more than a crooked tube. Its effects are accounted for from the gravitation of fluids of different weights, one upon another.

If one leg of a syphon be immersed in a vessel of water, and the other leg hang out of it, in such manner that the lower end be below the surface of the water, on opening both the orifices at the same instant, the water will be found to flow out at the lower orifice, till its surface has sunk down to the orifice of the leg in water.

Now, on examining this experiment, you will find that the columns of air pressing on the two equal orifices, differ from each other in length only by the perpendicular distance between the surface of the water, and the horizontal plane of the lower orifice of the syphon; which space, compared with the whole height of the atmosphere, is too inconsiderable to be taken into the account; and we may therefore conclude, that the action of the atmosphere on both the orifices is equal.

Now, as we supposed the tubes full of water when the holes are first opened, these equal pressures of the atmosphere will be counteracted by the weight of two different columns of water; one in the shorter, the other in the longer leg of the syphon. The difference of the force of these counteracting pressures is equal to the weight of a column of water, whose base is equal to the diameter of the tube, and whose height is equal to the perpendicular height of the surface of the water from the orifice of the longer leg. Now, equal* pressures of the atmosphere will be counteracted by unequal forces of gravitating waters, which will make the opposite pressures of the vertex unequal: and as the superior weight of the longer column carries it downwards, there is less pressure on that side of the vertex; the water will be pressed forwards and continue to flow till the water be fallen to the bottom of the immersed leg, or, if it be the longer leg, as low as the end of the flowing one; for, the descent of the water in the longer leg, by its own gravity, would leave a vacuum in the tube, if not immediately succeeded by other water. This descent gives the atmosphere, which urges the water up the syphon, the same power to act as if it were not at all opposed at the issuing orifice.

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* Lecture iv. vol. i.

† Theory of the Syphon. London, 1781.

For the same reason that the atmosphere urges the water in the vessel after that which descends, it would fill the whole syphon, provided it were void of air; and by sucking the air out of the smaller kind of these instruments with the mouth, through a pipe placed for that purpose by the side of the issuing leg, the liquor will directly flow. In larger syphons, for the draining of pits, quarries, &c. the evacuation is effected by a pump placed in like manner at the issuing end.

Of the Distillers' Syphon. This is usually about an inch in diameter, and three feet in length, with a cock fixed into the issuing end. To use it, the cock is shut and the contrary end put into the bung-hole, till the liquor reach within about five or six inches of the bend. Then, on opening the cock pretty quickly, the contents flow out of the syphon in the usual manner. By the immersion of the drawing leg, the liquor is prevented by the pent-up air from rising as high within the syphon as it is on the outside thereof. On opening the cock, the ambient fluid obtains power to raise that within the tube to its own level; but, by a law already explained, the contained liquor, before it rises as high as that of the other, will have acquired a velocity nearly sufficient to carry it as much above that surface as it was before below it. Hence the fluid shoots over the bend; and there falling into a tube with a contracted orifice, the syphon is soon filled, and the liquor of course continues to flow, as in the common experiment.

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* For, if you suppose the atmosphere more than able to sustain the water in each leg, it will at the first instant of opening the orifices, press their respective contents against each other at the vertex.

s'Gravesande's Syphon. This is a syphon for raising water into a cistern by means of the expenditure of other water through the outer leg, and may be applied in many cases where water, &c. is to be raised ten or twenty feet, and where you have at the same time water sufficient to supply the lower reservoir. This syphon has been lately much improved by a very ingenious gentleman.*

OF THE VIBRATORY MOTION OF WATER IN A SYPHON.

I have proved to you in the Lectures on Mechanics, that the vibrations of a pendulum are isochrone or of the same duration, though the arcs it describes be unequal. I have also shown you, that the durations of the vibrations of two unequal pendulums are to each other as the square-roots of their respective lengths. The motion of water vibrating in a syphon follows the same laws.

To illustrate this, let us suppose *l n o m*, *plate* 3, *fig.* 9, to be a syphon consisting of three parts, or legs; two, *l n*, *m o*, vertical, and one, *n o*, horizontal; and that it is of an equal diameter throughout its whole extent. Let us further suppose that the fluid, while at rest, occupies the space *a n o d*, the two surfaces, *a b*, *c d*, will be level. Now, if by any means the fluid be forced to descend to *g h* in the leg *m o*, it will rise to *e f* in the leg *l n*; and as soon as this cause ceases to act, the fluid is left alone to the action of its gravity. The excess in length of the column, *e n*, over the column, *h o*, will force the fluid to descend even below the level of the other, on account of the acceleration it acquires in descending, which will raise the fluid in the other leg; and it will thus continue rising and falling alternately, forming oscillations similar to those of a pendulum; and the duration of each vibration will be precisely the same as the vibration of a pendulum, whose length is half the length of a column, *p q r*, of the fluid.

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* *s'Gravesande's* Elements of Philosophy, vol. i. p. 235.

As the oscillations of water follow the same laws as those of a pendulum, if the length of a column of water be increased or diminished, the duration of the oscillations will be also increased or diminished, and will be in a subduplicate ratio of this length.

OSCILLATOY MOTION OF WAVES.

Newton, in his Principia, compares the undulatory motion of waves to the vibratory one of water in a syphon. Let ABCDEF, *plate* 2, *fig*. 10, represent a stagnant water, whose surface is elevated or depressed by successive waves. Let A, C, E, be the convex, and B, D, F, the concave part of the waves. As waves are formed by the successive ascent and descent of the water, so that those parts which were the highest become the lowest alternately and successively; and as the moving force which makes the lowest part rise, and the highest sink, is the weight of the elevated water, this ascent and descent may be considered as analogous to the vibratory motion of water in a syphon, and observing the same laws.

If, therefore, you have a pendulum whose length is equal to half the transversal distance between the most convex point, A, and the most concave point, B, that is, equal to half of Ab, the highest part will become he lowest during the vibration of such a pendulum, and in another vibration it will again become the highest, going through its whole space while the pendulum performs two vibrations. And as a pendulum, whose length is quadruple the preceding one, that is, which is equal to the width, AC, of the waves, would perform but one vibration while the other performed two, we conclude that the waves perform their vibrations in the same time with a pendulum, whose length is equal to the breadth of the waves.

From hence it follows, that a wave, whose breadth is 3 feet $8\frac{17}{30}$ lines, by advancing its whole breadth in one second, would in a minute describe 183 feet 6 inches 10 lines; and in an hour, 11014 feet 2 inches.* If the breadth were quadruple, it would describe the breadth in double the time; consequently, the broader they are, the greater space they describe in a given time.

In the foregoing reasoning, I have assumed that the waves rise and fall in straight lines; but this is not exactly true, and consequently the deductions can only be considered as approximations towards the truth.

OF THE RESISTANCE OF FLUIDS.

One of the most important problems in hydrostatics, is to determine the resistance that a body in motion meets from a fluid at rest; and to know the effort necessary to keep a body at rest in a fluid in motion.

Water and air are two of the great inanimate agents in nature, and they are those which man renders most easily subservient to his purposes. Necessity first pointed out the use of these agents, and engaged him to investigate their properties. In this respect, however, much of his labour has been spent in vain; particularly that which has been employed in the resolution of the above-mentioned problems. These have hitherto evaded every research, though they have engaged the attention, and exercised the talents of the greatest mathematicians.

I have shown you, by many instances, that the philosophy of the ancients was neither so unreasonable, nor so limited as it has often been represented. It does not, however appear, that they were well versed in the science that is termed *mixed mathematics*, or mathematical philosophy; a science which consists in the application of calculation to the phenomena of nature. Among the branches of this science which they have the least studied, we may reckon that of the resistance of fluids; for we must confess, that they had obtained some knowledge thereof, as it was necessary for the construction of their ships, the principles of building which they had carried further than the moderns.

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* By the breadth of a wave, we mean the transverse space, A C, which is between the two greatest elevations, or that, B D, between the utmost depressions.

Modern mathematicians have imagined themselves able to discover the motions and penetrate into the elements of bodies, by the aid of geometry and calculation. By the assistance of these alone, they conceived it was possible to investigate the nature of fluids, discover the working of the parts, and the action of those innumerable particles which go to constitute a fluid; particles which, at the same time, are, as it were, united and separated, dependent and independent one of the other. Notwithstanding, however, the aid of geometry and the fluxionary process, they have made little or no progress in the knowledge of the resistance of fluids. The desire of using calculation has determined their principles; whereas their first business should have been to examine these principles by experiment and observation, instead of bending and twisting experiments to make them subservient to the powers of calculation.

Newton, to whom philosophy and geometry are so much indebted, was the first who undertook to determine, on mechanical principles, the resistance a body meets with when moving in a fluid medium. Unfortunately for science, his labours were not successful. His first theory consists of ingenious researches, they may awaken curiosity, but which are not applicable to nature; his second, though more conformable to the nature of fluids, is too complicated, and subjected to too many difficulties, to be reduced to practice.*

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* For the proofs of this, see the works of *Bernouilli*, *Euler*, and *d'Alembert*, particularly those of the latter, who is justly considered as one of the first among modern mathematicians.

Since his time, many able geometricians have endeavoured to render this theory more perfect; among these, we are to reckon *Bernouilli*, *d'Alembert*, and *Euler*, who have made upon this subject the most profound researches, but which are too complicated for practice. New experiments were afterwards made by these gentlemen, which were so far from according with the theory, that they contradicted some of its most important rules. M. *Bossut* and *Borda* endeavoured in vain to solve these difficulties, and remove these contradictions.

In 1775, Messrs. *d'Alembert*, *Condorcet*, and *Bossut*, instituted, by order of government, a set of experiments on the impact of fluids, which they have published in a work on that subject: after a number of experiments, they were obliged to confess, that the generally-received theory was found to be essentially defective. The importance of this subject is so great, that there is little room to doubt, that the society lately established for promoting the branches of science relative to naval affairs, will find means to extricate this part from its present opprobrious state.

Though the theory delivered by Sir *Isaac Newton* is confessedly imperfect, yet as another and more perfect one has not been established, it will be necessary to give you a short account of its principles.

A body cannot move forward in water, or any other fluid, without removing the parts of the fluid which lie before it out of the way; but as these particles possess that general property of matter, which is called their inertia, this resistance will be made by the most perfect as well as the most imperfect fluid, by air as well as by liquid honey. For, if a body move in a fluid, it must give motion to a certain quantity of that fluid, and the re-action of that quantity will destroy part of the motion of that body.

But by displacing the fluid, and communicating motion, it loses an equal quantity of its own motion, from whence we obtain some idea of the resistance of the fluid; much here will, however, depend on the form, magnitude, &c. of the moving body, and the velocity of its motion; for a greater body will displace a greater quantity of the fluid than a smaller one, every thing else being the same;

and the greater the velocity wherewith a body moves in a fluid, the more motion will be communicated thereto, and consequently lost to the body.

There is another cause of resistance, which arises from the tenacity of the parts of a fluid; for, as a body cannot move forward in a fluid, till the parts that lie before it be removed out of the way, the adhesion or tenacity must necessarily resist its motion.

There is a third cause of resistance, that is, the friction of the body against the particles of a fluid; but this, from the nature of fluids, is doomed to be very inconsiderable.

The resistance will also depend on the fluid's density, every thing else being the same; for, it is manifest, that it will require more force to displace a given quantity of mercury than the same quantity of water, and a quantity of water than an equal quantity of air.

But the principal resistance which fluids give to bodies in motion, is supposed to arise from the inertia of their parts, and this depends on the velocity of the moving body; and that for two reasons. In the first place, the quantity of fluid moved out of its place, in any determinate space of time, must be greater in proportion as the body moves with greater velocity through the fluid. And, in the next place, the velocity with which each particle of the fluid is moved, will also be proportional to the velocity of the body; for, it communicates a greater or less quantity of motion to each particle in proportion to the velocity of its motion, and will therefore be resisted on this account also in the proportion of the velocity. Since the resistance which any body makes against being put in motion, is proportional both to the quantity of matter moved, and the velocity it is moved with; but as the resistance of a fluid is as the velocity of the body moving therein, it will be doubly increased: 1. Because the quantity of particles moved is as the velocity of the moving body. 2. Because the resistance arising from a given number of particles is also as the velocity of the moving body. Therefore the resistance is considered as being in a duplicate proportion of the velocity of a moving body, or as the square thereof.

A cylinder moving in a fluid, in the direction of its axis, is resisted by a force equal to the weight of a column of the fluid, the base of which is the base of the cylinder, and altitude equal to the space through which a body must fall freely from rest to acquire the velocity of the cylinder's motion.

A sphere moving in a fluid is opposed by a resistance, which is to the force which resists a cylinder moving in the direction of its axis with the same velocity, in the proportion of 1 to 2.

Two suppositions are generally taken for granted, in proving the propositions on the resistance of fluids: 1. That the fluid in which the body moves is so compressed, that its pressure on every part of the moving bodies shall be the same as when they are at rest. 2. That the hinder parts of the solids contribute nothing to the resistance, which will be the same as if the anterior part only were exposed to the fluid. This last supposition is not admissible, for the hinder part of most solids contribute to lessen the resistance by the power it receives from the returning curves of the fluid.

The theory of resistance opposed to bodies moving in perfect fluids, could not even be demonstrated by Sir *Isaac Newton*, but under certain conditions and restrictions.

1. The particles of fluid wherein the bodies move are supposed to be perfectly non-elastic.

2. The fluid is imagined to be infinitely compressed.

The second condition is allowed not to obtain in any fluid whatsoever, and it is doubted whether the first be strictly applicable even to the most perfect known fluids.

It is certain, that the resistance of fluids depends on the cohesion, tenacity, and friction, as well as the inertia of the matter moved; but the illustrious author of the theory, here slightly touched upon, considered the geometrical estimation of these circumstances as of no use in physical inquiries. He therefore chiefly noticed the properties of retardation, which bodies suffer when moving through fluids, the cohesion and friction among whose parts were in a physical sense evanescent.

The lectures on hydrostatics, &c. like every other part of philosophy, serve to show you the weakness and imperfection of human knowledge, and how ignorant we are on those subjects in which we are deeply interested, and with which we are continually engaged; they also show us how long human ingenuity may be exercised, without improving the science on which it is exercised. In every other part of natural science new discoveries are made, and new phenomena are brought to view, which enlarge the boundaries of knowledge, though they convince us of greater objects and numerous phenomena that remain concealed from our observation; but in hydrostatics and hydraulics, little that is new has been discovered, and a general shade of ignorance seems to be cast over the whole science.

The lectures on hydrostatics and hydraulics clearly show, that those modern philosophers err much, "who, in zeal for the honour and perfection of their favourite study, contend that almost every thing in physics is demonstrable; not considering how far a mathematical conclusion will extend, and how far not; and forgetting that hypothetical forces will afford them as much matter to work on, as those that are real.

"To make a satisfactory progress in natural philosophy, you should be very attentive to the nature of physical truth. Experiments, which are the basis and support of all sound philosophy, though chosen with propriety, and made with the utmost accuracy, are incapable of penetrating into the real essences of things. They can only inform the senses of some of the apparent qualities of bodies, and enable us to investigate some of the secondary or physical causes; and sometimes they do not reach so far, and we learn nothing but the bare phenomena or effects."

Man has before him all nature, the whole world with which he is surrounded for the object of his view, and the subject of his consideration; but his capacity is so circumscribed, his knowledge so straightened, and his powers so limited, that he can by no means conceive the mechanism of so vast and complicated a structure. No experiments can show him the internal frame of any one part; the component elements thereof escape his sight through their minuteness; the heavenly bodies, be they ever so large, are too remote for his introspection.

So that our views of nature are like the map of an inland country, where you see rivers without any sources continually discharging their waters without a sea to receive them; roads that you know not whence they come, nor whither they go; mountains, forests, and plains, cut off in the middle by the marginal lines of the paper: but, even of those things which we know well, there is much that surpasses the extent of our faculties. We know the sun stands still, and that the earth rolls round him with inconceivable rapidity; yet to think of the stately fabrics, the spacious cities, the seated mountains, the brimming ocean, and the universal quiescence of a still evening, rushing forward with such incessant impetuosity, is an idea too unwieldly for our imagination; we know fire only dissipates, but does not destroy, yet we currently talk of its drying up, consuming and destroying. We know matter to be inert and senseless, and yet we ascribe force to storms and inundations, activity to spirituous liquors and drugs, inclemency to seasons, kindliness to dews, benignity to vernal gleams; giving bodies these and other powers belonging to ourselves. Thus, wherever we turn our view, we find in science no reason for boasting, no foundation for pride.

This visible world in which we are placed, when carefully viewed and examined by the enlightened powers of the human understanding, proclaims, indeed, the power, wisdom, and mercy of the eternal Godhead. His invisible glories are somewhat represented to us by the things that are seen; the sun, beaming forth light and life to our system, proclaims, in language known to all, his Maker's sacred glory, whose power supplies the never-failing fountain with its endless heat and light. When we consider the great and wide ocean, as carried orderly round in its daily revolutions, how are we astonished at the power that guides the tremendous mass? How mighty is the word that says, "Thus far shalt thou go, and no farther?" How powerful the arm that keeps within bounds its tumultuous workings, and that directs it in the beautiful regularity of its course? Who can refrain from falling down with reverential silence, when he considers the magnificent structure of the universe? Yet, how small a part of the great things around us can be taken in by our limited faculties? We see enough to surprize us; the great visible effects proclaim the majesty of the Invinsible Cause; further than this they do not, they cannot go.

No other eye than the Divine is equal to the view of nature; and no mind, less than that of the Great Designer, is able to perceive with perfect clearness the uniformity of the design, amidst the vast variety of parts that are to outward appearance so dissimilar and heterogeneous. There is nothing in which the nature of man prides itself so much, as in superior knowledge and wisdom; and yet how low is the point to which human knowledge can ascend! In the objects that surround us, in subjects in which we are most nearly interested, how small is the part we can discover! The best directed and most successful researchers only inform us how little is known, and give us no cause to be satisfied with the discoveries they have made.

LECTURE XXXVI.

ON ASTRONOMY.

OF the various branches of science cultivated by mankind, *astronomy* is one of the most important and most useful; our faculties are enlarged by the grand ideas it conveys, and the mind is exalted above the contracted prejudices of the vulgar and illiterate. It explains to you the laws or rules that govern and direct the motions of the heavenly host, and by which the Almighty carries on and continues the wonderful harmony, order, and connexion, observeable throughout the planetary system; it enables us to discover the size of the planets, and determine their respective distances. By the knowledge derived from this science, we discover the bulk of the earth, and ascertain the situation and extent of the countries and kingdoms into which it is divided; by aiding the navigator, and facilitating his passage through the trackless ocean, trade and commerce are carried on to the remotest part of the world, and the various products of the several countries are distributed for the health, comfort, and convenience of its different inhabitants.

There can be little or no occasion to excite your attention to the wonders continually exhibited in the heavens; the magnificent objects you there discover, forcibly awaken the idea of their Author, by presenting the most striking instances of his power and his glory. It is towards heaven you turn your eyes, when you look up for assistance to that infinite goodness and power, from whom alone it can be received. No one, indeed, can view the immense expanse of heaven, without being struck with admiration at the variety of bodies, apparently in motion, with which that expanse is adorned; at the uniformity and regularity with which they seem to move from one part of the heavens to the other, always keeping within a certain limit, and pointing out by their different situations the different seasons of the year, and diffusing the various degrees of heat and light necessary for the preservation of animal and vegetable life.

GENERAL PRINCIPLES OF ASTRONOMY.

Of the various phenomena that offer themselves for your observation, the most obvious is the *diurnal motion*, or that which seems to be common to the whole heavens, and which is completed in twenty-four hours.

Of the moving bodies, the *sun* is the first that engages our attention, not only on account of his almost infinite superiority over the rest, but because of our own more immediate dependence on him. In common with the other celestial bodies, he has an apparent motion from east to west; for we see him every day rising in the east, topping the sky as it were archwise, then descending till he sets in the west.

If you consider the *stars*, you will find that the greater part rise and set every day; and after a few days observation you will conclude, that the stars have a motion common to all, by which they are carried round the earth in twenty-four hours.

If, in order to consider more attentively the circumstances of this diurnal motion, you place yourself in an elevated situation, you will perceive a circle terminating your view on all sides by the meeting, or apparent meeting of the heavens and the earth; your day and night depend upon it, it prescribes to every star in heaven the race that it has to run, and bounds your view so effectually that you can neither see whence they rise, nor whither they tend. This circle is called the *horizon*, it divides the heavens into two parts; but that part alone which is above the horizon is visible, and as this part appears to us under the form of a concave hemisphere, it is usually termed the *visible hemisphere.*

On considering with attention for one or more nights the motion of the stars, you will find each star describing a circle in about twenty-four hours; those stars that are most northward, describe smaller circles than those that are more to the south; you will find the circles described by different stars diminishing gradually, till they are lost and confounded in one point, that is called the *pole* of the world. To be more particular, if you look towards the south, you will observe some stars just appearing above the horizon, grazing this circle, but not rising above it, and then vanishing; observe a group a little farther from the south, and you will find them rise above the horizon, making a small arc, and then going down; take another group still more distant from the south, and you will find these describing a larger arc, and taking a longer time to go down. Turn now your face to the north, and you will find some that just skim the horizon, mount to the top of the heavens and then descend, and again touch the horizon and mount again without ever disappearing; others that are higher than these, describe complete circles in the sky, without touching the horizon, and these circles diminish till at last you arrive at a star that seems scarce to move from the point where it is stationed, the rest wheeling round it; and you will find this course to be performed in about twenty-four hours.

To form clear ideas in astronomy, it is necessary for you to learn how to discover or distinguish the pole of the world, or the place in the starry heavens corresponding thereto. The star, you have already discovered, round which the others wheel, is very near the pole, and from hence has been called the *polar star;* it never departs from its situation, and every hour of the day, and every season of the year, is to be found in the same place,* uniformly maintaining its position. Now, as there is one star motionless at one end of the axis, there must undoubtedly be some point motionless at the other end of the axis.

The general appearance, therefore, of the starry heavens, is that of a vast concave sphere turning round two fixed points (diametrically opposite to each other, the one in the north, and the other in the south), once in twenty-four hours.

This pole-star is more or less elevated, according to the part of the earth from which it is viewed; thus, more northward, as at Lapland, it is much more elevated above the horizon than with us. If you go southwards, it is lower than with us; we see it more elevated than the inhabitants of Spain, and these again see it more elevated than those of Barbary. By continually travelling southward, you would see the pole-star depressed to the horizon, and the other pole would appear in the south part of the horizon, round which the stars in that part would revolve. There is, however, no star in the southern hemisphere so near the pole as that in the northern. If you were still to travel onward to the south, the north pole would disappear, and the whole hemisphere would appear to turn round a single point in the south, as the northern hemisphere appears to turn round the pole-star.

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* This is spoken in conformity to the appearance of the object, for though this remarkable star revolves round the pole, its motion is so slow, and the circle it describes so small, as to render both the revolution and the change of situation hardly perceivable.

Here, however, it may be proper to observe, that what we call the *sky*, in which the heavenly bodies seem as it were to be fixed, is no real substance. If there were no atmosphere surrounding our earth, whose particles might reflect other rays of light to our eyes, than those which come directly from the sun, all parts of the heavens, even at mid-day, would be dark, and the stars be visible at noon. But as our atmosphere abounds with particles capable of reflecting light every way, some of it will fall upon our eyes whithersoever they are directed ; from the nature of this reflection, we receive the idea of colour, and the mind immediately imagines a substance wherein it may reside ; in the same manner, the regular reflection of light from an object in a looking-glass, is combined by the mind into an image of that object. Mr. *de Saussure*, when at the top of Mount Blanc, in Savoy, a mountain which is elevated 15673 feet perpendicularly above the sea, and where, consequently, the atmosphere must be much rarer than ours, says, that the moon shone with the brightest splendour in the midst of a sky as dark as ebony, while Jupiter, rayed like the sun, rose from behind the mountains in the east.

The polar star may be discovered by a continual attention to the motion of the stars, as already described; but there is a readier method, by attending to those stars that the country people call the waggon and horses, but which astronomers call the *great bear*. It consists of seven very conspicuous stars : four of these make a kind of square ; prolong a straight line through the two which are farthest from the three out of the square, and it will at last reach a solitary star, which, from its vicinity to the pole, is called the pole-star. In early ages mariners had no other guide for their wandering vessel; by this they formed their observations, and regulated their voyage. When this was for a length of time obscured by clouds, they were bewildered in the watry waste, and knew not where they advanced, nor whither they should steer; but when this auspicious star broke through the gloom, they re-assumed with alacrity the management of the helm, and shaped their course with tolerable satisfaction and certainty. It may be proper to observe to you, that the polar star is always on the same side of the square with the star marked α on the celestial globe, or, which is the same thing, on the convex side of the bear's tail.*

Knowing the pole-star, you will be able to distinguish the *cardinal points*, the *north*, *south*, *east*, and *west*. 1. The north, the side to which you are turned when you look at the pole. 2. The south, the opposite side, or that in which the sun appears in the middle of the day. 3. The east, the side on which the sun appears to rise. 4. The west, that side on which the sun seems to set. The east is on the right hand when you look at the pole, and the west on the left. The east and west points are placed exactly at equal distance, or at right angles, from the north and south.

The ancients, in *Homer's* time, divided the horizon into four cardinal points; afterwards they reckoned twelve; but in *Pliny's* time the common division was into eight points. The earliest division known to the ancients being the four cardinal points, east, west, north, and south, winds were said to blow from that point to which they were nearest. Thus, the east wind is said "to have brought the locusts in Egypt," though it was probably a north-east wind. The horizon is now subdivided into thirty-two points, which are usually termed *the points of the compass;* because the compass shows in what part of the horizon these points are situate.

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* Upon referring to a celestial globe, it will be seen that the two foremost stars in the Great Bear always point to the pole-star, and have therefore been called *pointers*.—E. EDIT.

In considering the heavens, astronomers find it necessary to speak of a point supposed to be directly over our heads, and this point they call the *zenith:* it is the point where the thread of a *plumb-line* would meet the heavens, were it sufficiently extended.

The zenith, being the most elevated point of the heavens, is always ninety degrees, or a quarter of a circle from every part of the horizon. If a star, therefore, be found sixty degrees above the horizon, it will be thirty degrees from the zenith; because sixty and thirty make the ninety, that is, the distance from the horizon to the zenith. In future, therefore, I may say, that the height or *altitude* of a star is the *complement* of its distance from the zenith, because the complement of an arc is what it wants of ninety degrees.

The *nadir* is the lowest point of the celestial sphere; that which is directly under our feet, or opposite to the zenith, and towards which the plumb-line directs itself by its natural gravity.

The zenith and nadir being directly opposed to each other, if you conceive a circle making the whole tour of the heavens, and passing through the zenith and nadir, there will be one hundred and eighty degrees, or a semicircle on each side. All circles passing through the zenith and nadir are called *vertical circles*, because the line through which a plumb-line passes is called a vertical line. The altitude of objects above the horizon is estimated by these vertical circles.

Whenever you consider the heavens from any elevated situation, you naturally conceive, that as there is a hemisphere above, there is also another beneath, though invisible; and that, of course, the horizon is a great circle of the sphere, dividing the concave heavens into two parts, the visible above, the invisible below; and, consequently, that this circle determines the rising and setting of every celestial body.

The earth being, as I shall hereafter prove to you, a spherical body, the horizon, or limit of our view, must change as we change our situation; every place upon the earth, every particular observer, has a different horizon. Thus, to an observer placed at *a*, *plate* 3, *fig.* 2, the horizon is GC; but if he remove to *b*, his horizon will be HD.

As one pole opposes another, the *south pole* is directly opposed to the *north pole*, and the one is as much below the horizon as the other is above it: the north pole at London is about 51° 30′ above the horizon. These two poles are the extremities of a line going from one to the other, through the centre of the earth. This line is called the *axis* of the world; it is about this line that the whole heavens seem to turn every day.

Knowing the two extremities of the axis, it is easy to conceive a wheel or circle in the middle, between the two poles or extremities, and this will be the *equator*. To understand this more clearly, I must refer you to a diagram. Let HPZER, *plate* 3, *fig.* 5, represent a vertical circle, passing through the two poles, P the north pole, R the south pole, PR the axis of the world; then the line, EQ, will represent the diameter of the equator, or of a circle at equal distances from the two poles, and whose plane is perpendicular to the axis, as a wheel is perpendicular to its axle-tree. You are therefore to conceive in the heavens a circle exactly between the two poles, and at right angles to the axis of the world. This was indeed the first circle that the ancient astronomers imagined, and to which the Egyptians and Chaldeans referred all the stars, even as far back as the time of *Herodotus*, 450 years before Christ.

The equator being at equal distances from the two poles, you may either say, that the sphere, with its equator, turns about the axis P R, or about the poles P and R of the equator.

As we call the points, P and R, poles of the equator, because they are every where equally distant from the equator; so we also call, in general, poles of any circle those two points of the sphere which are at the greatest distance from this circle, and which are situate in a line perpendicular to the plane of the circle, and passing through its centre. Thus the zenith is the pole of the horizon: it is the same with every other circle; the pole thereof is always ninety degrees therefrom in every derection.

The line which passes through the two poles of a circle, is called in general the axis of this circle; for instance, the vertical line is the axis of the horizon. You are not to confound the axis with the diameter of a circle, they are two things altogether different; the diameter is drawn in the plane of the circle, but the axis is elevated on both sides of the circle, and at right angles thereto, out of the plane of the circle, and having but one point common with the axis, that is, the centre through which the axis passes.

Let us for a moment now return to the sun, whose influence extends over every thing round about us. The rising and setting of the sun are two events which form, as it were, every day two remarkable and interesting epochas; they are the first and most obvious phenomena that we notice. He every day seems to describe a circle in the heavens moving from east to west. He rises in the east, and all the attendants of night gradually disappear, or sink from his presence. Nature wakes at his call, and his far-extended beams discover to our eyes all her variegated beauties. He mounts to the highest point of the circle, in which he moves, and then descends to the west, when the lamps of heaven again illumine the divine scene of night.

After having examined each day the points where the sun rises and sets, you would naturally call the highest point in the sky to which the sun rose, and from whence he began to descend again, the *mid-day point.* By extending this point over your head, you would trace in imagination another circle round the globe, and call it the *meridian of your place.*

If you notice in the same manner the stars which rise and set, you will find that they are at their greatest height in the middle of the interval between their rising and setting, though their altitudes may be different; they are then also said to be on the meridian.

A circle, as P Z R H, *plate* 3, *fig.* 5, passing through the zenith, the nadir, and the poles, is the meridian. It is thus named, as I have observed, because when the sun is upon this circle, it is mid-day. Every point of this circle is equally distant from the horizon; that is, as far from the east side as from the west side. The stars that rise and set are in the meridian twice in the course of their diurnal motion; once above the horizon, and once when they arrive at the lower half thereof, under the horizon. Their diurnal motion may therefore be divided into four parts; one from the time of their rising to their arrival on the meridian; from this to their setting; thence to their reaching the lower meridian; and, lastly, from thence to their rising again the next day.

The meridian of a country situate east or west of London, is a different one from that of London. If you travel east or west, you change your meridian by a quantity equal to that which you have advanced towards the east or west; for your meridian, at every change of situation, passes through a new zenith, and the poles of the world. Thus Rome is about 12° 34′ east of London, and consequently the meridian of Rome differs 12° 34′ from that of London. There is but one method of changing your situation without changing your meridian; which is, to go directly north or south, that is, towards either of the poles.

Every place is supposed to have a meridian passing over its zenith, and going through the poles of the world. All the meridians therefore meet, and intersect each other, at the poles. They are all at right angles to the equator; and the equator, because it is every where at an equal distance from the poles, divides every meridian into two equal parts.

You have now established three circles in the celestial sphere, to which you may refer all the stars you observe. These circles, though in themselves imaginary, will become, as it were, fixed points for your observations. The horizon is the first circle which you must use; not only as it distinguishes night from day, and the rising and setting of all the heavenly bodies, but as the higher and farther they are elevated above the horizon, the longer they stay with us; their elevation above it became one of the first objects of an astronomer's attention: this was one of the means by which he could refer them to the horizon.

To this end, suppose a circle rising perpendicularly out of the horizon, passing through the star, and going up to a point directly over your head, or to your zenith. Upon this circle you may, by the help of your quadrant, reckon how many degrees it is from the horizon; and this is called its *altitude* or height. The circle upon which I count the degrees of altitude is called a *vertical circle*, because it stands perpendicularly to the horizon, and passes through the zenith.

Thus, let B D, *plate* 14, *fig*. 3, represent a quadrant whose circumference is divided into 90 degrees. Place one of the sides vertically by means of a plumb-line; place the eye at the centre O; observe what point, C, on the circumference of the quadrant, corresponds with the star, and the number of degrees between D and C will be the altitude of the star above the horizon. For instance, if the arc, D C, be the half of B D on the small instrument, the celestial arc, A R, will be also the half of Z R, both one and the other will be forty-five degrees; but as the measure of angles by a quadrant, or some other instrument, is the basis of astronomy, it will be necessary here to enter somewhat more fully into the nature thereof, before we proceed further in our investigation of the heavens.

OF THE ASTRONOMICAL QUADRANT.

The small quadrant before you, *plate* 14, *fig*. 1, is a very simple and useful instrument. With it you may not only perform a great number of interesting and entertaining problems,* but may also attain a good idea of practical astronomy.

Every circle is supposed, by geometricians and astronomers, to be divided into 360 parts or degrees; ninety degrees, therefore, or one-fourth part of a circle, will be sufficient to measure all angles formed between a line perpendicular to the horizon, and other lines which are not directed to points below the horizon.

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* See my Essays on Astronomy, &c. third edition, 8vo. 1795.

An angle is the opening between two lines, which touch one another in a point. In describing the extent of an angle, regard is not to be had to the length of the lines, but to the width of their opening. This is easily illustrated by a pair of compasses; the legs of this instrument will represent the sides of the angle, and the joint the vertex of the angle. With the same pair of compasses it is possible to form a great variety of angles, by opening them to different widths; for the angle that is formed by the legs differs in proportion to that opening, and its quantity is greater in proportion as they are placed farther asunder, and smaller as they are brought nearer together.

Suppose an arc of a circle to be placed in such a manner as to be passed over by these points, then the opening or angle will be in proportion to the part of the arc that is passed over; and if the whole circle be divided into any number of equal parts, as, for example, 360, the number of these will denote the magnitude of the angle.

But here you are carefully to observe, that the angle is neither enlarged nor diminished by any change in the length of the legs, provided their position remain unaltered; because it is the inclination of their legs, or the space between them, not their length, which constitutes the angle; for, with the same opening, the imagination may extend the length into indefinite space.

So that if a pair of compasses, with very long legs, were opened to the same angle as another smaller pair, the intervals between their respective points would be very different; but the number of degrees on the circles supposed to be applied to each would be equal, because the degrees themselves, on the small circle, would be exactly proportioned to the shortness of the legs. It is owing to this principle, plain, easy, and simple, as it is, that we are able to measure out the whole expanse of the universe, and give its place to every point in the sky, and to measure the arcs of those vast circles that we have imagined in the heavens: for the arcs of the two circles, though very different in their absolute size, would be altogether the same in their relative dimensions; that is, the smaller would contain just as many of the 360 degrees as the larger; they would both be archs of the same quantity in proportion to the general division of a circle. Though you do not, by this means, obtain a real measure, it enables you to settle the comparative distance between all objects in the heavens.

For instance, that star we see in the meridian, rose in the east, and has now run one quarter of its circle. This I may express by saying, it has passed through ninety degrees, one fourth of 360. That other star, midway between the horizon and the zenith, is elevated forty-five degrees.

DESCRIPTION OF THE QUADRANT.

Every circle being supposed to be divided into 360 equal parts or degrees, it is evident that ninety degrees, or one fourth part of a circle, will be sufficient to measure all angles formed between a line perpendicular to the horizon, and other lines which are not directed to points below the level. We have here a very simple and useful instrument of this kind, *plate* 14, *fig.* 1. A B C is a quadrant mounted on an axis and pedestal: by means of the axis, it may be immediately placed in any vertical position, and the pedestal being moveable in the axis of the circle E F, at bottom, serves to place it in the direction of any azimuth, or towards any point of the compass. The limb, A B, is divided into degrees and halves, numbered from A; and upon the radius, B C, are fixed two sights, of which the one on B is perforated with a small hole, and is provided with a dark glass, to defend the eye from the sun's light; and the other sight, on C, has a large hole, furnished with cross wires, and also a smaller, which is of use to take the sun's altitude, by the projection of the bright image of that luminary upon the opposite sight. From the centre, C, hangs a plumb-line C P. The horizontal circle, F E, is divided into four quadrants of ninety degrees; and the arm E, connected with the pedestal, moves along the limb, and consequently shows the position of the place of the quadrant, as will hereafter be more minutely explained. Lastly, the screws, G, H, I, render it very easy to set the whole instrument steadily and accurately in its proper position, notwithstanding any irregularity in the table or stand it may be placed upon.

The rationale of this instrument is very clear and obvious. It is used to measure the angular distance of any body, or appearance, either from the zenith or point immediately above our heads, or from the horizon or level. The plumb-line C P, if continued upwards from C, would be directed to the zenith Z; and the line C L, supposed to be drawn from the centre of the quadrant to an object L, will form an angle L C Z, which is the zenith-distance, and is equal to the angle B C P, formed between the opposite parts of the same lines. We see, therefore, that the degrees on the arc, comprehended on the limb of the quadrant, between the plumb-line and extremities next the eye, measure the angle of the zenith-distance.

Again, the line C K, forming a right angle with the perpendicular C Z, is level, or horizontal; the angle, L C K, must therefore be the altitude or elevation of L above the horison; and this last angle must be equal to the angle measured between the plumb-line and the end A, farthest from the eye; because both these are equal to the quantity which would be left, after taking the zenith-distance from a right angle, or the whole quadrant.

The determination of the altitude or zenith-distance of an object is not sufficient to ascertain its place, because the object may be placed in any direction with respect to the azimuth, or the points of the compass, without increase or diminution of its altitude. Hence it is, that a horizontal graduated circle is a necessary addition to a quadrant, which is not intended to be always used in the same plane. The bearing or position of an object relative to the cardinal points, together with the altitude, is sufficient to ascertain the place of any object or phenomenon.

To adjust the quadrant for observation. The quadrant is adjusted for observation when its plane continues perpendicular to the horizon, in all positions of the line of sight. For this purpose, bring the index to 90° on the horizon, and turn one or both of the screws which are fixed opposite 60°, till the plumb-line lightly touches the plane of the quadrant. Then turn the index to 0°, and make the same adjustment, by means of the screw at 0°, and the quadrant is ready for observation.

Or otherwise, set the index at 0°, and observe the degree marked by the plumb-line on the limb: then turn the index to the other 0°, which is diametrically opposite, and observe the degree marked by the plumb-line: if it be the same as before, there will be no occasion to alter the screws at 60°; but, if otherwise, one or both of those screws must be turned, till the plumb-line intersect the middle degree, or part, between the two. After this operation, the degree marked by the plumb-line must be observed, as before, by setting the index at both the 90°, and the adjustment of the plumb-line to the middle distance must be made at the screw at O, taking care not to touch the other screws.

The latter method of adjustment being more accurate in practice, may be used after the former.

TO FIND THE ALTITUDE OF THE SUN, OR ANY OTHER CELESTIAL BODY.

The quadrant being adjusted, as it must be in all cases previous to its use, the celestial body must be viewed through the sights, and the plumb-line will show its altitude on the graduated limb of the instrument.

If the observation be made on the sun, the dark glass must be used to defend the eye; or the luminous spot formed by the smaller hole of the sight, which carries the cross-wires, must be made to fall on the mark on the other sight, which lies immediately beneath the eye-hole.*

CONTINUATION OF A VIEW OF THE GENERAL PRINCIPLES.

Let us now return to our observation on the celestial sphere: whilst this sphere turns upon its poles, the points situate in the equator describe a circle, which is of the same size with the sphere, its centre passing through the centre of the sphere; but the points which are nearer the pole describe smaller circles, these are called *parallels to the equator*, or simply *parallels*. Every point in the heavens, that is not situate in the equator, describes a parallel; these diminish in size, in proportion as they are more distant from the equator.

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* By the addition of a small telescope, with a reflecting eye-piece, vernier scales to the arc and circle, rack-work and pinion to the arc A B, and circle E F. &c. I have rendered this small instrument useful for observing angles up to the zenith, and with more ease and accuracy for angles in general.
E. EDIT.

All these parallels are divided into equal parts by the meridian; thus, any star supposed on the meridian, will describe by its diurnal motion a circle parallel to the equator, and will be as long on one side of the meridian as it is on the other; this circle dividing the time of the diurnal revolution into two equal parts.

If the parallel described by the star be entirely above the horizon, you may see it pass the meridian twice in a day; once in that part of the meridian above the pole, and once in that part of the meridian below the pole. But if part of the parallel described by the star be below the horizon, you will only see it pass the meridian once in 24 hours.

Mathematicians call all those circles on the surface of the sphere, whose centres coincide with the centre of the sphere, *great circles;* the plane of these circles divides the sphere into two equal parts. The *equator* the *meridian*, and the *horizon*, are great circles. *Small circles* are those which divide the sphere into two unequal segments; such are the parallels to the equator.

TO FIND THE HEIGHT OF THE POLE BY MEANS OF THE STARS.

The disposition of the three great circles of the sphere that we have fixed upon, namely, the equator, the horizon, and the meridian, will henceforth become the basis of all your observations; and to these you will refer the stars, in order to determine their situations and motions. The first thing, therefore, we ought to do, is to discover the reciprocal situation of these circles: 1. The situation of the equator with respect to our horizon. 2. The elevation of the pole. 3. The elevation of the equator at the meridian, or south point.

If the north or pole-star, of which we have already spoken had been precisely and accurately situate at the pole of the world, it wovld have been a sure and permament mark, and you would have had nothing more to do than to take the altitude of this star to obtain that of the pole; but this star is nearly two degrees distant from the pole. To the naked eye unassisted with instruments, this difference is scarcely perceptible; but, with instruments, and by attentive observation, it is found to describe a small circle about the pole; so that, though it does not mark directly the pole, yet the middle of the circle it describes being the pole, its place may be very readily discovered.

Thus the pole-star describing about the pole a circle about 2° distant from the pole, there will be nearly 4° difference between the altitude of this star, when it is in the upper point of its circle, and when it is in the lowest point. If therefore the altitudes be taken when it is in both situations, in order to have the height of the pole, you must subtract say 2° from the superior altitude, and add two to the lower; thus, if the altitude had been observed at London first to be 53° 32′, and then 49° 32′, the difference is 4°, the half of which added to 49° 32′, or subtracted from 53° 32′, gives 51° 32′, the altitude of the pole at London; or you may add the two heights together, and take the half. The elevation of the pole may be discovered in the same manner by any other of the circumpolar stars.

The elevation of the pole being discovered, you thereby obtain that of the equator. Thus, in the diagram before you, *plate* 3, *fig.* 5, P represents the pole, and E Q the equator, H O the horizon, P H the elevation of the pole, Z the zenith: H Z O, or the visible part the heavens, contains twice 90° or 180°, it being 90° from Z to H, and 90° from Z to O; but it is also 90° from the pole P, to E the equator. If you take away P E, there remain 90° for the other two arcs, or, in other words, the elevation of the pole and the equator are together equal to 90°; and the one being known, and subtracted from 90°, gives the other; in technical words, *the elevation of the pole at any place is the complement of the elevation of the equator.*

It follows from hence, that the elevation of the equator is equal to the distance from the pole to the zenith, for the elevation of the equator is the difference between that of the pole and 90°; the same elevation subtracted from 90°, gives its distance from the zenith.

For the same reason, the distance from the equator to the zenith is equal to the elevation of the pole, for H Z is equal to 90°, and P E is equal to 90°; if, therefore, you take away P Z, which is common to both, the remainders, P H, Z E, must be equal to each other.

OF THE PHENOMENA OF THE MOON.

From considering the diurnal motion which is common to all stars, and obvious even to those who have given very little attention to these subjects, I shall pass on to that motion of the planets which is performed in a direction contrary to the diurnal revolution of the stars, that is, from west to east.

Among these, there is none so remarkable as those of the moon; every month she changes her figure, and makes the tour of the heavens in a contrary direction to the general motion thereof; so that, though the moon appears to rise and set every day, like the other stars, by a motion from east to west, yet she seems also every day to be retarded in her motion, and to stay as it were behind the stars, or go backwards towards the east about 13°.

This motion of the moon towards the east, at the same time that it partakes of the general diurnal motion, is called her *periodical* motion, and is so considerable, that in 27 days, the moon, which appeared near any remarkable star, separates from it, moving in a contrary direction: the first day it will have removed rather more than 13 degrees from it, the second 26, the third 39, &c. so that in 27 days it completes the 360, and meets the star on the opposite side, having made the tour of the heavens.

" Her nightly changes in her circling orb," is a phenomenon still more striking than the preceding. At the new moon, as soon as she becomes visible, she is seen in the western part of the heavens, at no great distance from the sun, and just as it were emerged from his light. She increases every night in size, and removes farther and farther from the sun, till at last she appears in the eastern part of the horizon when the sun is disappearing in the western, rising at the same time he sets, and appearing with a full round face, equaling if not exceeding the sun in size. After this, she gradually removes farther and farther eastward, till at last she seems to approach the sun as nearly in the east, as she did before in the west, and rises only a little before him in the morning ; whereas in the first part of her course, she set in the west along with him. This is a prelude to her total disappearance ; and, though from her first appearance in the west she takes almost a month to pass to the eastern part of the heavens, she seems to pass from the eastern to the western in one or two days ; for, immediately on her vanishing in the east, she appears again in the west, and very near the sun. All these different appearances happen in the space of a month, after which they recommence in the same manner. " Sometimes she half-restores day with her waxing brightness ; sometimes waning into dimness, she scarcely disperses the nocturnal gloom."

This luminary, like the sun, seems to have a motion from north to south, but at opposite seasons ; for, in summer time, when full, she is low in the southern part of the heavens, but approaches considerably nearer to the northern regions in winter. By her phases she divides the times and seasons; and we find that in early ages men were much affected with this two-fold consideration of the services of the moon, enlightening the night, and regulating their time. I need not here mention the various sacrifices, offerings, and processions in honour of Diana, Luna, Hecate, that is, of the moon under different names ; her votaries were numerous and superstitious. Ignorant of philosophical truth, and uninstructed by revelation, they practised absurdities, and believed in lies.

OF THE SUN'S ANNUAL MOTION, AND OF THE ECLIPTIC.

Though the periodical motion of the moon is quick and remarkable, yet there is still one more important, that is, the apparent periodical or annual motion of the sun; on it depend the changes of season, the summer's heat and the winter's cold, the different lengths of day and night. This motion is, indeed, only apparent, arising from the annual motion of the earth; at present, however, we are only considering appearances, from these we shall afterwards ascend to the contemplation of the causes which occasion them.

A little attention will soon convince you, that the sun does not always rise exactly at the same point of the horizon.

Thus, if you commence your observations of the sun, for instance, in the beginning of March, you will find him every day, rising more to the northward, to continue longer above the horizon, to be more vertical or higher at mid-day; this continues till towards the end of June, when he moves backward in the same manner, and continues this retrograde motion till near the end of December, when he begins to move forward; and so on.

It is this change in the sun's place that occasions him to rise and set in different parts of the horizon, at different times of the year. It is from hence that his height is so much greater in summer than in winter. In a word, the change of the sun's place in the heavens is the cause of the different lengths in the days and nights, and the vicissitudes of the seasons. "Sometimes we perceive him very high, as if he would come over our heads; at those times his rays fall upon us with greater force, he continues longer visible, the earth and air are considerably heated, vegetables flourish and arrive at their perfect state, every animal seems cheerful, and this agreeable season we call *summer*. By and by, however, he removes farther south, his time of continuing visible becomes constantly shorter and shorter, until at last it is reduced to a few hours; the sun is then sunk almost in the southern part of the heavens, the earth is covered with frost and snow, the air is disturbed with violent winds and tempests, vegetables perish, animals are benumbed with cold, and we call this dreary season *winter*. The intermediate season, when the sun begins to return towards us, and by his genial influence to revive the animal and vegetable world, is called *spring*. The season when he again begins to withdraw his influence, when fruits ripen, and vegetable life begins to decay, is named *autumn*.*

As the knowledge of the sun's apparent motion is of great importance, and a proper conception of it absolutely necessary, in order to form a true idea of the phenomena of the heavens, you will excuse my dwelling something longer upon it. If on an evening you notice some fixed star near the place where the sun sets, and observe it for several successive evenings, you will find that it approaches the sun from day to day, till at last it will disappear, being effaced by his light, though but a few days before it was at a sufficient distance from him. That it is the sun which approaches the stars, and not the stars the sun, is plain, from this reason; the stars always rise and set every day at the same points of the horizon, opposite to the same terrestrial objects, and are always at the same distance from each other; whereas the sun is continually changing both the place of rising and setting, and its distance from the stars.

The sun advances nearly one degree every day, moving from west to east, so that in 365 days you will see the same star near the setting sun, that you observed to be near him on the same day in the preceding year. In other words, the sun has returned to the place from whence he set out, or made what we call his *annual revolution*.

We cannot, indeed, observe the sun's motion among the fixed stars, because he darkens the heavens by his splendour, and effaces the feeble light of those stars that are in his neighbourhood; but we can observe the instant of his coming to the meridian, and his meridional altitude; we can also compute what point of the starry heavens comes to the same meridian, at the same time, and with the same altitude. The sun must be at that point of the starry heavens thus discovered. Or we can observe that point in the heavens, which comes to the meridian at midnight, with a declination as far from the equator on one side as the sun's is on the other side; and it is evident, the sun must be in that part of the heavens which is diametrically opposite to this point. By either of these methods you may obtain a series of points in the heavens through which the sun passes, forming a circle called the *ecliptic.*

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* New and Concise System of Geography.....Edinburgh.

To combine more clearly the annual and diurnal motion of the sun, let us consider for a moment this globe; by turning it on its axis, you obtain an idea of the diurnal motion of the celestial sphere. If you place an insect at equal distances from the poles, it will turn with the globe, and describe the equator; if you place another nearer one pole, it will describe a circle parallel to the equator. But while the globe is moving in one direction, the insect may be moving very slowly in another, and will thereby imitate the annual motion of the sun, which advances by degrees towards the east, though it is carried round every day by the diurnal motion towards the west.

The *ecliptic*, or path of the apparent annual motion of the sun, differs in situation from the equator, whose situation we have already ascertained. This the Chaldeans first observed at Babylon, where the equator was elevated 54° above the horizon; now, if the annual path of the sun had been in the equator, every day at noon the sun's altitude would have been 54°; it was so far from this, that in summer they found it rise 24° above the equator, and in winter descend 24° below it; so that his meridian height in summer was 78°, and only 30° in winter; the ecliptic was therefore evidently differently situate from the equator, but cut it in two points diametrically opposite; for they observed twice in the year, in spring and autumn, that at noon the sun's altitude was exactly the same with that of the equator.

The ecliptic, or path of the sun, is then that circle of the sphere which cuts the equator in two points, but which is afterwards removed about 24° from the equator, being one half of the year on the north side, the other half on the south side; and, as the points where it cuts the equator divide it into two equal parts, you may conclude that the ecliptic is a great circle of the sphere, for it is one of their properties to bisect each other. The next business is to determine in the celestial concave and among the fixed stars, those by which the sun passes every day, so as to delineate his path on a globe.

For this purpose, remember there are two days, six months distant from each other, when the meridian altitude of the sun is the same with that of the equator; these are called the *equinoctial days*, because the sun on those days describing the equator is twelve hours above and twelve beneath the horizon, so that the day is equal to the night; one is called the *vernal*, the other the *autumnal equinox*.

Having remarked at midnight on the vernal equinox a star having the same altitude with the sun, that is, equal to the elevation of the equator, you are certain of having obtained the point diametrically opposite to the sun, that, where he will be at the autumnal equinox. In the same way you may discover at the autumnal, the sun's place at the vernal equinox, and thus ascertain two points, the knowledge of which is essential to the progress of astronomy.

The points of the ecliptic, where the sun is situate when he is most distant from the equator, are called *solstitial points;* because the sun at these points appears stationary for some days, that is, about the 21st of June and the 21st of December. These points being observed, there remains, for completing the ecliptic, only to fill up the intermediate parts of the path.

The distance between the equator and the ecliptic at the solstitial points, is called the *obliquity of the ecliptic;* to ascertain this, it is necessary to measure how much the sun rises above the equator in summer, and descends below it in winter; or how much higher he was at noon in summer than in winter. Astronomers have done this, and found the difference to be 47°, the half of which, 23½, is the greatest distance between the ecliptic and the equator.

About 2000 years ago, this obliquity was 24°, it is now about 23° 28′, and diminishes about one minute every 100 years.

To reckon and measure the motion of the sun and other celestial bodies, it is necessary to fix upon some point in the heavens from whence to proceed, and to which every thing may be referred. The return of the seasons being the most remarkable and interesting circumstance in all astronomy, the commencement of the reckoning was governed thereby.

The sun, by his annual course in the ecliptic, traverses the equator every year, bringing spring with him; and this renewal of nature served to mark the beginning of the year, and the astronomers used the point where this change took place, or the intersection of the equator and ecliptic for the beginning of their measures, and called *longitude* the distance of the sun from the vernal equinox reckoned along the ecliptic.

OF THE PLANETS.

The diurnal motion of the heavens, as being the most striking, was that which was first noticed by mankind: then the periodic motions of the sun and moon. In process of time, by repeated and more assiduous observations, they found among the stars six which had motions peculiar to themselves, and these they called *planets;* they are thus named, Mercury ☿, Venus ♀, Mars ♂, Jupiter ♃, Saturn ♄, and one lately discovered is named the Georgium Sidus ♅; they never depart far from the ecliptic, their motions are very irregular, sometimes they appear to go forwards, sometimes backwards, and sometimes to be stationary.

MERCURY emits a bright white light, but keeps so near the sun, that he is very seldom visible; and when he does make his appearance, his motion towards the sun is so swift, that he can only be discerned for a short time. He appears a little after sun-set, and again a little before sunrise.

VENUS is the most beautiful star in the heavens, known by the names of the *morning* and *evening star*. She also, like Mercury, keeps near the sun, though she recedes from him much farther, and like him, is never seen in the eastern quarter of the heavens when the sun is in the western; but always either attends him in the evening, or gives notice of his approach in the morning.

MARS is of a red fiery colour, giving a much duller light than Venus, though he sometimes appears almost equal to her in size. He is not subject to the same limitations in his motions as Venus and Mercury, but appears sometimes very near the sun, at others at a greater distance from him, and sometimes opposite, or rising when the sun sets, and setting when he rises.

JUPITER and SATURN likewise often appear at great distances from the sun. The former shines with a bright light, the latter with a pale faint one. The motion of Saturn among the fixed stars is so slow, that unless carefully observed, and that for some time, he will not be thought to move at all.

The GEORGIUM SIDUS is the planet discovered by Dr. *Herschel*. It is reckoned to be twice the distance of Saturn from the sun, but cannot be readily perceived without the assistance of a telescope.

From the preceding observations, any person may easily learn to distinguish all the planets. For, if after sun-set he sees a planet nearer the east than the west, he may conclude that it is neither Mercury nor Venus; and may determine whether it be Saturn, Jupiter, or Mars, by the colour and light; by which also he may distinguish between Venus and Mercury.

That the light of each planet has its peculiar tinge, and that there are certain fixed stars that have peculiar teints, was known to the Chaldeans. It is an observation best verified in those countries where the air is clearest.

Besides the motions which we observe in all the planets, their apparent magnitudes are very different at different times. Every one must have observed that Venus, though she constantly appears with great splendour, is not always of the same size: but this difference of magnitude is most conspicuous in Mars, it is remarkable in Jupiter, but less so in Saturn and Mercury.

The only phenomena visible to the unassisted sight, besides those already described, are those unexpected obscurations of the sun and moon, called *eclipses*, of which we shall hereafter speak more particularly.

OF THE ARMILLARY SPHERE, *fig.* 1, *plate* 13.

Hitherto I have contemplated the heavens according to nature, I shall now endeavour to render some of the particulars more easy, by referring you to one of the instruments that have been contrived to elucidate the elements of astronomy. Of these there are none which imprint so clearly on the mind as the armillary sphere, the nature and use of those circles which astronomers have supposed to be applied to the concave sphere of the heavens, for investigating with greater accuracy the motions of the celestial bodies, and facilitating the communication of science.

If the circumference of a circle be turned about its diameter as an axis, it will generate in its motion the surface of a globe or sphere.

And the centre of the circle will be the centre of the globe.

All straight lines reaching from the centre of a globe to its surface are equal.

Every straight line that goes through the centre of a sphere, and is terminated at both ends by the surface is a diameter.

The diameter about which any sphere turns is its axis.

The extremities of such diameter or axis are its poles.

On the surface of a globe or sphere several circles may be described; those circles, whose centre is the same with the centre of the globe, are called by way of distinction *great circles*, these divide the sphere into two equal parts. The angular distance of two points situate on the surface of the sphere, is measured by the arc of a great circle intercepted between them.

Lesser circles divide the sphere into two unequal parts.

The sphere before you, by its real circles, serves to represent, and will enable me to explain to you those imaginary circles by which astronomers divide the heavens into the same parts or portions as you see these circles divide the sphere. If your eye could be placed in the centre of this sphere, you would see its circles upon or against those very points of the heavens where the imaginary circles of the astronomers are supposed to be situate. It is called *armillary*, because it consists of a number of rings of brass, called by the Latins *armillæ*, from their resembling bracelets or rings for the arms.

There are six great circles of the sphere which require your particular attention: the horizon, the meridian, the equator, the ecliptic, the equinoctial colure, and the solstitial colure.

The sphere is sustained in a frame, on the top of which is a broad circle representing the horizon.

On the inside of the horizon are two notches for receiving a strong brass circle representing the meredian.

It is suspended on two pins, at two opposite points of the meridian; these pins are a continuation of the axis of the sphere both ways, and as the sphere turns round upon them, they are considered as poles, and are called one the *north*, the other the *south* pole.

The *equator* is that circle which goes round the sphere exactly in the middle between the two poles.

The *ecliptic* is that circle which crosses the equator obliquely; it is divided into twelve parts, each of which consists of thirty degrees.

The *equinoctial colure* is the great circle which passes through those two points of the equator that are intersected by the ecliptic.

The *solstitial colure* is the other great circle at right angles to the equator.

There are four lesser circles of the sphere, two tropics, and two polar circles; these four circles are all parallel to the equator.

The *tropic of cancer* is the parallel which is on the north side of the equator, and 23½ degrees distant therefrom.

The *tropic of capricorn* is situate on the south side of the equator, and also 23½ degrees distant therefrom.

The two *polar circles* are at the same distance from the two poles, that the tropics are from the equator, that is, twenty-three degrees and a half.

That towards the north pole is called the *arctic circle*, that towards the south pole, the *antarctic circle*.

OF THE HORIZON, AND ITS USES.

This broad circle, H O, *plate* 13, *fig*. 1, is termed the *horizon*, because it represents that imaginary circle which bounds or terminates the view of the spectator, dividing the sphere into two equal parts; that which is above the horizon is called the upper or visible hemisphere, that which is below, the lower or invisible hemisphere. When the sun, moon, or stars descend below this circle, we say, *they are set;* on the contrary, when they appear above it, we say, *they have risen.*

The poles of the horizon are the two points every where equally distant from it; 1. The zenith, or the point directly over the head of the spectator; and, 2. The nadir, or the point directly under his feet.

On the broad circle, representing the horizon, are marked the thirty-two points of the mariner's compass; the east, west, north, and south points are principally to be regarded; these are called cardinal points. Thus to refer the celestial bodies to the horizon, it is usual in astronomy to say, the sun, moon, or planets, rise on such a point, or so many points from the east or west, north or south. If you consider their distance from the east and west points of the horizon, at rising or setting, it is called their amplitude; the azimuth is their distance from the meridian, at any time, reckoned on the horizon.

OF THE DISTINCTION OF THE HORIZON INTO RATIONAL AND SENSIBLE.

In general terms, the horizon may be defined to be an imaginary circle, that separates the visible from the invisible part of the heavens.

If you suppose the floor or plane, on which you stand, to be extended every way, till it reach the starry heavens, this plane is your *sensible horizon.*

The *rational horizon* is a circle, whose plane is parallel to the former, but passing through the centre of the earth.

The rational horizon divides the concave sphere of the heavens into equal parts, or hemispheres; the objects that are in the upper hemisphere will be visible; such as are in the lower hemisphere will be invisible.

Though the globe of the earth appears so large to those who inhabit it, yet it is so minute a speck, when compared to the immense sphere of the heavens, that at that distance the planes of the rational and sensible horizons coincide; or, in other words, the distance between them in the sphere of the heavens is too small for admeasurement.

To illustrate this, let A B C D, *plate* 3, *fig.* 1, represent the earth; z h n o, the sphere of the starry heavens. If an inhabitant of the earth stand upon the point A, his sensible horizon is *s e*, his rational one h o; the distance between the planes of these two horizons is A F, the semidiameter of the earth, which is measured in a great circle upon the sphere of the heavens, by the angle e F o, or the arc e o; this arc, in so small a circle as z h n o, would amount to several degrees, and consequently the difference between the sensible and the rational horizon would be great enough to be measured by observation. If we represent the sphere of the heavens by a larger circle, the semidiameter of the earth, A F, measured in this circle, will amount to fewer degrees; for the arc E O is less than the arc e o; and the larger the sphere of the heavens is, in proportion to the globe of the earth, the less sensible is the difference between the two horizons. Now, as the sphere of the earth is but as a point, when compared to the starry heavens, the difference between the sensible and rational horizon will be insensible.

From what has been said it sppears, that the only distinction between the sensible and rational horizon, arises from the distance of the object we are looking at.

The sensible horizon is an imaginary circle, which terminates our view, when the objects we are looking at are upon the earth's surface.

The rational horizon is an imaginary circle which terminates our view, when the objects we are looking at are as remote as the heavenly bodies.

As the rational horizon divides the apparent celestial sphere into two equal hemispheres, and serves as a boundary, from which to measure the elevation or depression of celestial objects; those in the upper or visible hemisphere are said to be high or elevated above the horizon; and those in the other hemisphere to be low, or beneath the horizon.

The earth being a spherical body, the horizon, or limits of our view, must change as we change our place; and therefore every place upon the earth has a different horizon. Thus, if a man live at a, *plate* 3, *fig*. 2, his horizon is G C; if he live at b, his horizon is H D; if at c, it is A E. From hence we obtain a proof of the sphericity of the earth; for if it were flat, all the inhabitants thereof would have the same horizon.

The point in the heavens, which is directly over the head of the spectator, is called the zenith.

That point, which is directly under his feet, is called the nadir.

If a man live at a, *plate* 3, *fig*. 2, his zenith is A, his nadir E. If he live at b, his zenith is B, his nadir F: consequently, the zenith and horizon of an observer remain fixed in the heavens, so long as he continues in the same place; but he no sooner changes his position, than the horizon touches the earth in another point, and his zenith answers to a different point in the heavens.

OF THE GENERAL MERIDIAN, AND THE DEGREES MARKED THEREON.

The name *meridian*, which signifies mid-day, I have before observed to you, is given to this circle, because the centre of the sun coincides with this circle at noon; or, in other words, when the sun comes upon the meridian of any place, it is their noon or mid-day at that place.

This large brass circle, M R, *plate* 13, *fig.* 1, is called the *general meridian*, because it serves here to represent any particular meridian; for you know, that a particular meridian is an imaginary circle in the heavens passing through the poles of the world, and the zenith-point of every person wheresoever situate on the surface of the earth; consequently, there will be as many such meridians as you can imagine points from east to west on the earth's surface, for those places which lie north and south of each other have the same meridian, because the meridian circles lie north and south.

In the sphere, this strong circle or general meridian suffices for all the rest; for as it is fixed, and the sphere is moveable about its axis, you may, by turning the sphere, bring any part of its surface under this circle, which will then represent the meridian of that part.

One side of this circle is graduated or divided into four quadrants, each consisting of ninety degrees. The numbers on two of these quadrants commence from the equator, and proceed each way to the north and south poles; the numbers on the other two quadrants increase from the poles towards the equator.

The numbers which proceed from the equator are to point out the distance of the zenith of any place from the equator, which is called *latitude.*

The latitude is called *north* or *south*, according as the places are on the north or south side of the equator.

The degrees which are numbered the contrary way from the poles, are to show the height or elevation of the pole above the horizon. I have before shown you, that the elevation of the pole is equal to the latitude of the place.

By this circle we also measure the distance of the sun or of a star from the equator, which is called its *declination*; thus, if you bring the highest or most northern part of the ecliptic to the meridian, you see it cut the same in 23½ degrees, and so much is the declination of that point.

To rectify the sphere is to put it in that position in which it may represent exactly the apparent motion of the heavens.

For different places that position must be different, according to the elevation of the pole, or latitude of the place.

Therefore to rectify the sphere, elevate the pole till the same number of degrees is above the plane of the horizon, as the pole of the place you are considering is above the horizon, and then the circles on the sphere will, with respect to the globe within it, correspond with the imaginary circles of astronomers in the heavens.

OF THE EQUATOR. EQ.

You may remember, that in speaking of this circle, I told you, that this great circle surrounds the sphere exactly in the middle between the two poles of the world; you also observe that the ecliptic, which represents the sun's path, cuts this circle in two opposite points, and therefore, of course, the sun is twice a-year in this circle; and as one half of this circle is always above the horizon, and the other half below it, when the sun is in those two points the days and nights will be equal. This circle is called the *equator*, because it equates or divides the globe of the earth into two equal parts; it is often also called the *equinoctial*, because when the sun is in this circle the days are equal to the nights.

It is divided into 360 degrees, beginning and ending in the equinox-point, and numbered from west to east; these degrees on the sphere are called degrees of *right* *ascension*, but the degrees on the equator laid down on a terrestial globe, are called degrees of *longitude.*

Hence a meridian, passing through the sun, a planet, or star, shows on the equator the degree of its right ascension, or distance from the equinoctial point.

OF THE HOUR-CIRCLE.

This is a small circle fixed about the north pole, with a hand on the axis of the sphere, so that by turning the sphere on its axis, the index will be carried round the circle.*

This circle is used to convert the degrees of the equator into time; it is divided into twenty-four equal parts, answering to twenty-four hours, or the time of an entire revolution of the heavens. The index, by pointing successively to those hours, shows in what space of time any part of that revolution is performed.

The twelve hours are twice engraved on this circle; the first twelve hours show the time from noon to midnight, the other, from midnight to noon the next day.

Astronomers begin the day at noon, therefore the two hours of twelve stand directly upon the graduated edge of the meridian.

To find the time in which any motion is performed, I bring the beginning of the degrees on the equator to the meridian; holding the sphere in that position, I turn the index about to the hour of twelve upon the edge of the meridian next me; I now turn the sphere once round, till the same point come again to the meridian, and the index passes once round the hour-circle, which shows, that one revolution of the sphere is performed in twenty-four hours.

If one revolution, or three hundred and sixty degrees, be made in twenty-four hours, then half a revolution, or one hundred and eighty degrees, is made in twelve hours; a quarter, or ninety degrees, in six hours; and in that proportion fifteen degrees of the equator pass the meridian in one hour, and fifteen minutes, or one quarter of a degree, in one minute of time.

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* The horary circle *a*, *fig.* 1, is not completely represented by the figure; but, as it is exactly similar to that of a globe, the reader can easily conceive the nature of it.---E. EDIT.

OF THE QUADRANT OF ALTITUDE.

This is, you see, a long slip of brass with a nut at top to fix it to the meridian; it is divided into ninety degrees, and being fixed at the zenith it reaches the horizon; it may be carried to any part of the sphere or globe, and thus will show the height or altitude of any point above the horizon in any position of the sphere.

OF THE ECLIPTIC AND CELESTIAL LONGITUDE.

We are now come to the great circle called the *ecliptic*, C D, because there can be no eclipses of the sun and moon, but when the moon is in or near this circle; for this circle, represents the sun's apparent annual path, and therefore whenever the moon obscures the sun, or is obscured by the earth, she must be near this circle.

It is generally graduated into twelve equal parts, consisting of thirty degrees each, the beginning of the parts, being marked with characters representing the twelve signs. Astronomers divided the ecliptic in this manner in order to judge more easily, and express more clearly the places of the sun, moon, and planets; for it is easier to obtain an idea of their situation by referring them to some part of a known figure, than it could possibly be by merely considering degrees in the circumference and expanse of the heavens. The astronomers, therefore, to reduce the science to order and method, arranged the stars, which are distributed irregularly over the surface of the skies, into certain and determinate classes or constellations.

Thus the stars near the ecliptic are distributed into twelve constellations or signs, which are named, Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, and Pisces.

The sun appears always in this circle, advancing in it nearly a degree in a day, and going through it exactly in a year. The points where this circle cuts the equator are called the equinoctial points; the vernal equinoctial point being marked Aries, the autumnal one, Libra.

It is by the ecliptic that astronomers estimate the latitude and longitude of the heavenly bodies.

The longitude of the heavenly bodies is reckoned in signs and degrees of the ecliptic, beginning from the first point of Aries, or vernal equinoctial point. Thus, suppose the sun to be now in the fifth degree of Leo, we say, *his longitude is four signs, four degrees, and part of another;* for he has already passed the four signs of Aries, Taurus, Gemini, Cancer, and four degrees of Leo, and is in the fifth.

The latitude of the heavenly bodies is reckoned on an arc of a great circle passing through the poles of the ecliptic and a given star, and intercepted between the ecliptic and the star.

The *zodiac* is a broad circle in the heavens, and which is sometimes applied to an armillary sphere; it contains the twelve signs, and is called the zodiac, because, according to the Chaldean division, it consisted only of animal figures; what is now called Libra, being by them reckoned a part of Scorpio, and making the claws of that animal.

The division of the zodiac into twelve parts corresponds with the division of the year into twelve months, and therefore plainly answers to the apparent motion of the sun. This zone represents the space or limits to which the planets stray on each side the ecliptic, which is about five degrees, so that the breadth thereof is about ten degrees.

OF THE COLURES, TROPICS, AND POLAR CIRCLES.

The next circles of the sphere we are to consider are those two called the *colures*, both of which pass through the poles of the world, and cut the equator at right angles.

One of these passes through the equinoctial points, and is called the *equinoctial colure*, N; the other passes through the solstitial point P, and is called the *solstital colure.*

The colures are in a manner the boundaries of the year, for they point out the seasons by their two opposite parts of the ecliptic. The solstitial colure passes through the poles of the ecliptic.

The *two tropics*, S and T, are parallels, and distant 23½° from the equator on each side of it, at which distance they touch the ecliptic in the *solstitial points.*

While the sun is going from Aries to Cancer, he advances every day more northward, till he comes to Cancer, his most northward situation; after which he begins to descend to Libra, getting every day more southward than before. Now, when he is in the beginning of Cancer he changes his motion, and turns from going northward to go southward; this turning back is meant by the word *tropic*, from whence the parallel passing through Cancer is called *the tropic of Cancer;* and for the same reason the other is called *the tropic of Capricon*, because there the sun returns from the southerly course to the northern one.

On the sphere they represent the parallels in which the sun moves on the longest and shortest days in the year; in other words, when he has the greatest and least meridian altitude; that part of the tropic which is above the horizon represents the length of the day, and that part which is beneath the horizon represents the night. Now you must observe from the sphere, that the day is the longest when the sun is in the tropic of Cancer, and the night the shortest of any one in the year; in the same manner, the two parts of the tropic of Capricorn show the longest night, and shortest day of the year.

The *polar circles*, V and U, are two imaginary circles in the heavens, which correspond to those parts of the earth that were termed by the ancients the *frigid zones;*

they include 23½ degrees on each side their respective poles.

These circles have been by many authors called the *arctic and antarctic circles*, but improperly, as the ancients always meant by the arctic circle the largest parallel that was always above the horizon of any particular place, containing all those stars which never set at that place, but seem to be carried round above our horizon in circles parallel to the equator.

The large parallel, which is entirely hid below the horizon of any place, they call the antartic circle, or circle of perpetual occultation. This includes all the stars which never rise to an inhabitant of the northern hemisphere, but are perpetually below the horizon.

All arctic circles touch their horizons in the north point, and all antarctic circles touch their horizons in the south point.

Those who live under either tropic have one of the polar circles for their arctic, and the other for their antarctic circle; thus the names of *mutable circles* have been given to the immutable polar circles, which are only arctic and antarctic circles in one particular case.

OF A RIGHT, PARALLEL, AND OBLIQUE SPHERE.

There are three situations of the celestial sphere relative to the inhabitants of the earth, of which you may form a very clear conception by means of the armillary sphere.

Of a right sphere, or of those inhabitants who live under the equator. To show the position of the circles of the sphere with respect to these inhabitants, bring the equator into the zenith, and the poles of the world lie in the horizon. The equator, the tropics, and polar circles, all stand perpendicular to the horizon; the heavenly bodies always rise and set perpendicularly; and as the horizon cuts all the circles in half, the days and nights will be equal throughout the whole year. This position is called a *right sphere*.

Under the equator the stars must every day appear to rise out of the horizon, then to ascend perpendicularly for six hours, and descend perpendicularly for the six following hours, then set and remain below the horizon twelve hours successively. The planets do the same, there being no difference between them and the fixed stars, but that the latter always describe the same parallels; whereas the parallels described by the former differ every day, being greater or less according to their distance from the equator.

A parallel sphere. As this position is to show the situation of the heavens to an inhabitant who lives under the poles; bring the pole therefore to the zenith, as shown in *fig.* 1, the axis is now at right angles to the horizon, the equator and horizon coincide, and the tropic and polar circles are parallel thereto; here there will be but one day and night throughout the whole year: for the same reason, the moon, in half her monthly revolution, will never rise, and in the other half will never set. The fixed stars will every day describe circles parallel to the horizon, some never rising, others never sitting.

Hence, also, an inhabitant of the north pole has the sun above his horizon, and therefore perpetual all the time the sun is on the north side of the equator, that is, for six months together. But the sun is below his horizon, and it is night with him all the time the sun is on the south side of the equator, which is also for six months: or, in other words, the sun will be seen for half a year, and then it will be day; and it will be hidden for half a year, and then it will be night.

An oblique sphere. We are situate in an oblique sphere. This position agrees with the inhabitants who live neither under the poles, nor under the equator; here the equator, and all the circles parallel to it, make oblique angles with the horizon.

It is evident, that in this situation, all the parallels to the equator are divided by the horizon into two unequal parts, but the equator into two equal parts; consequently, the day and night are never equal to an inhabitant in an oblique sphere, but when the sun is in the equator, that is, twice a year, viz. on the 20th of March, and the

22d of September. All the rest of the year the days are either longer or shorter than the nights; and the sun, which always appears to move in the ecliptic, describes one of the parallels to the equator, which are all cut by the horizon into two unequal parts. On the northern side of the equator, the days are longer than the nights, as long as the sun is on the north side of the equator; but the nights are longer than the days, when the sun is south of the equator.

The portion of the parallels above the horizon is greater in proportion as they are nearer the elevated pole; but when the distance of the parallel from the pole becomes less than the elevation of the pole, then that parallel, and all those which are included within it, are wholly above the horizon, no part of them setting or passing under it. The contrary happens in the parallels that are situate towards the depressed pole, a smaller portion of these being above the horizon, and the greater part lying under it. Those parallels which are nearer the depressed pole, than the elevation of the pole, or latitude of the place, remain perpetually, together with the stars included within them, under the horizon, and are never visible to us.

The arc of the equator intercepted between the beginning of Aries, and a star, &c. when on the meridian, is called its *right ascension*. It is so called, because in a right sphere, that point of the equator, which rises with a star, &c. comes also to the meridian with it. Hence you find the right ascension of a star, &c. by bringing it to the meridian; for that point of the equator which comes to the meridian with it, is its right ascension. The right ascension of a fixed star is always the same, but that of a planet varies.

The arc of the equator intercepted between the beginning of Aries and the point of the equator, rising or setting along with any heavenly body, is called its *oblique ascension or descension*.

The difference between the right and oblique ascension is called the *ascensional difference*. Many problems solved by the globes may be performed with the sphere; it is unnecessary to speak of these here, as I have only introduced the sphere to give you clear and satisfactory ideas of the circles used by astronomers.

In the middle of this sphere I have placed a small globe of the earth, supported by an axis, the ends of which go through the sphere in the places of the celestial poles. It is so contrived, that you may either turn the globe round within the sphere, or turn the sphere round the globe; thus this instrument will exhibit the real motion of the earth round its axis within the sphere of the heavens, or the apparent motion of the heavens round the earth. It will, therefore, show you the correspondence between the terrestrial and celestial spheres, and that the appearances of the heavenly bodies would be the same to us, whether they moved round the earth, as they appear to do, and the earth stood still; or that they stood still, and the earth was carried round the contrary way.

THE APPEARANCES OF THE STARRY HEAVENS ILLUSTRATED BY AN ARMILLARY SPHERE.

By the sphere you will readily perceive the truth of the positions stated, when I was treating of the apparent motion of the starry heavens.

You here see that the poles of the earth, when extended, reach to the poles of the heavens. By placing a small patch on the different circles of the sphere to represent stars, you perceive, that those which are farthest from the poles will describe the greatest circles; hence those will describe the largest possible circles that are situate in the equator, which is equidistant from both poles. You here perceive, that a star has acquired its greatest elevation when it comes to the upper semicircle of the meridian, and its greatest depression when it is at the lower circle of the meridian; and that the arc of its apparition is bisected by the meridian.

The sphere also shows you plainly, that if the circle of revolution be between the equator and the pole which is above the horizon, the greatest portion thereof will be visible; but if it be on the other side of the equator, the smallest portion will be visible. One half of the equator is always visible; some circles of revolution are wholly above the horizon, and some wholly below it; a star in one of the first is always seen, and in one of the last is never seen. A phenomenon whose circle of diurnal revolution is on the same side of the equator with the elevated pole, is longer visible than it is invisible; the contrary obtains if it be on the other side of the equator.

OF THE FIXED STARS, CELESTIAL SIGNS, AND CONSTELLATIONS.

No part of the universe gives such enlarged ideas of the structure and magnificence of the heavens, as a consideration of the number, magnitude, and distance of the fixed stars. We admire, indeed with propriety, the vast bulk of our own globe; but when we consider how much it is surpassed by most of the heavenly bodies, what a point it degenerates into, and how little more even the vast orbit in which it revolves would appear when seen from some of these stars; we conceive more just ideas of the extent of the universe, and of the boundaries of creation.

Among the many distinctions that characterize the fixed stars from other luminaries of heaven, that which is afforded by their light, or peculiar lustre, is the most obvious.

The light of the planets is steady, because it is reflected; that of the stars is bright and lively, and accompanied with a kind of vibration of light, which we call twinkling. This is supposed to arise from the nature of their light as intrinsic, and not received and reflected, and principally from the smallness of their apparent diameters. Our atmosphere is full of innumerable little particles, which are continually floating in it; many of these are large enough, on coming between the eye and such a point as a fixed star, to hide that point, or take that star out of our view, by intercepting its rays. But these atoms are in perpetual motion, so that the star is no sooner hid by one of them, than it appears again, because the atom has changed its place; then another comes, and again intercepts the view: the swift succession of these moving particles may assists in causing that appearance which we term twinkling. Or, it may probably arise from something in the eye itself, for we are continually shifting and closing them, even when we are not conscious of doing it, and a star being but a point, the smallest motion of the eye is sufficient to make us lose it; perhaps the impressions are so weak, as not to be able to keep the visive faculty awake, except as it were by fits.

As the stars do not change their relative situations one to the other, and as they are used to compare the relations and motions of the planets, it became necessary to find out some method of distinguishing them from each other; and as they do not change their relative positions, astronomers endeavoured to make an exact description of them, and by repeated observations determined the position and order which subsist among them.

To avoid confusion in description, and be able to point out any particular star, without being obliged to give a name to each, they are divided into several parcels; to each of these parcels is assigned a figure at pleasure; these assemblages, or groups of stars, are called *constellations*. Thus a number of stars near the north pole is called the *Bear*, because the stars which compose it are at such distances from each other, that they may fall within the figure of a bear. Another constellation is called the *Ship*, because that collection of stars, which compose it, is represented upon a celestial globe as comprised within some part of the figure of a ship.

The division of the stars into constellations is of the highest antiquity; there are no books so ancient in which the heavens are at all considered, but we find them treated of as distinguished into constellations; *Hesiod* and *Homer* mention several by names that are now known, and we find *Job* speaking of Orion and the Pleiades.

As the fixed stars appear of different degrees of magnitude and splendor, they are divided into six different classes. Those which seems the largest and brightest, are called *stars of the first magnitude ;* the smallest that we can see with the naked eye, are called *stars of the sixth magnitude ;* and the intermediate ones, according to their different apparent sizes, are called *of the second, third, fourth,* or *fifth magnitudes.* Those stars, which cannot be seen without the assistance of a telescope, are not reckoned in any of these classes, and are called *telescopic stars.*

By a knowledge of the fixed stars and their positions, we obtain so many fixed points, by which we may observe the motions of the planets; and the relation of these motions to each other, which become so many land-marks, if the word may be allowed, to ascertain the situations of other celestial bodies, and the varieties to which they are subject. For, from the same place, the motions of the heavenly bodies can only be estimated by the angle formed at the spectator's eye, by the space which the moving body passes over.

To measure these spaces the stars must be used, and they are considered as so many luminous points fixed in the concavity of a sphere, whose radius is indefinite, and of which the observer's eye is the centre. We learn from hence the necessity of forming an exact *catalogue* of the stars, and of determining their positions with accuracy and care. With such a catalogue, the science of astronomy begins.

Although to those who are unacquainted with the nature of celestial observations, it might at first sight appear almost impossible to number the stars; yet their relative situations have been so carefully observed by astronomers, that they have not only been numbered, but even their places in the heavens have been ascertained with greater accuracy than the relative situation of most places on the surface of the earth.

The greatest number of stars that are visible to the naked eye, are to be seen on a winter's night, when the air is clear, and no moon appears. But even then a good eye can scarce distinguish more than one thousand at a time in the visible hemisphere: for though on such a night they appear to be almost innumerable, this appearance is a deception that arises from our viewing them in a transient and confused manner; whereas, if you view them distinctly, and only consider a small portion of the heavens at a time, and after some attention to the situation of the remarkable stars contained in that portion, begin to count, yon will be surprized at the smallness of their number, and the ease with which they may be enumerated.

Hipparckus, the Rhodian, about a hundred and twenty years before the birth of Christ, was the first among the Greeks who reduced the stars into a catalogue; daring, according to *Pliny*, " to undertake a thing, which seemed to surpass the power of a divinity; that is, to number the stars for posterity, and to reduce them into order; having contrived instruments, by which he marked the place and magnitude of each star. So that by these means we can easily discover, not only whether any of the stars perish, and others grow up, but also whether they move, and, if so, the direction of their motion, whether they increase or diminish; thus putting posterity in possession, as it were, of the heavens." His catalogue, first adopted by *Ptolemy*, contained only 1026 stars. Since that time the list has been considerably augmented, and is daily receiving fresh increase, by the improvement of telescopes.

Several astronomers have followed *Hipparckus* in the same arduous undertaking. In 1603, *J. Bayer* published his *Uranometria*, or celestial charts of all the known constellations, and of the visible stars of which they are composed. In these charts every star is distinguished by a letter. The largest star in the constellation is marked with the first letter of the Greek alphabet, the next in apparent size is marked with the second letter; and so on. When there are more stars in the constellation than letters in the Greek alphabet, he marks the remainder with the letters of the Roman alphabet. When a star is mentioned with the letters of the Greek or Roman alphabet. it is always with the additional name of the constellation to which it belongs; and thus to those, who are acquainted with the figures of the constellations, and with the catalogue of fixed stars, it becomes as determinate a denomination as if the star were called by a proper name, and the same purpose is answered in a more familiar manner, and with less burden upon the memory. Among the various catalogues, the most copious, and hitherto esteemed the best, is that called Historia Cœlestis of our countryman, *Flamsteed*. A new one has lately been published by the Rev. Mr. *Wollaston*.

The number of the ancient constellations was forty eight; in these were included 1022 stars. Many constellations have been added by modern astronomers; so that the catalogues of *Flamsteed* and *de la Caille*, when added together, are found to contain near 5000 stars. The names of the constellations, their situation in the heavens, with other particulars, are best learned by studying the artificial representation of the heavens, a celestial globe.*

The *galaxy* or *milky way* must not be neglected; it is one of the most remarkable appearances in the heavens: it is a broad circle of a whitish hue, in some places it is double, but for the most part consists of a single path surrounding the whole celestial concave. The great *Galileo* discovered by the telescope, that the portion of the heavens which this circle passes through, was every where filled with an infinite multitude of exceedingly small stars, too small to be discovered by the naked eye; but, by the combination of their light, diffusing a shining whiteness through the heavens. Mr. *Brydone* says, that when he was at the top of Mount Ætna, the milky way had the most beautiful effect, appearing like a pure flame that shot across the heavens.

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* A set of new engravings for a pair of eighteen-inch globes has just been completed, under the title of the NEW BRITISH GLOBES. They are the only globes of the size in English, and it is upwards of forty years since any plates, for globes of about this dimension, have been engraved. The *terrestrial* contains all the latest discoveries and communications, from authentic observations and surveys, to the year 1798, from an accurate drawing by Mr. *Arrowsmith*. The *celestial* contains the position of nearly 6000 stars, clusters, nebulæ, planetary nebulæ, &c. as computed by me for the year 1800, from the latest observations, &c. by Dr. *Maskelyne*, Dr. *Herschel*, the Rev. Mr. *Wollaston*, &c. For particulars, I must refer the reader to a Treatise on their Uses, which I am preparing for the press.

E. EDIT.

This idea of the milky way, that it is formed of an innumerable cluster of small stars, is not, however, new; for, among the various conjectures of *manilius*, we find the following:

> "Or is the spacious band serenely bright
> From little stars, which there their beams unite,
> And form one solid and continued light?"

The stars appear of a sensible magnitude to the naked eye, because the retina is not only affected by the rays of light which are emitted directly from them, but by many thousands more, which falling upon our eyelashes, and upon the visible aerial particles about us, are reflected into our eyes so strongly, as to excite vibrations, not only in those points of the retina, where the real images of the stars are formed, but also in other parts round about it. This makes us imagine the stars to be much bigger, than they would appear if we saw them only by the few rays which come directly from them to our eyes, without being intermixed with others. You will be sensible of this, by looking at a star of the first magnitude through a long narrow tube; which, though it takes in as much of the sky as would hold a thousand of such stars, scarce renders that one visible.

The number of the stars almost infinitely exceeds what we have yet been speaking of. An ordinary telescope will discover, in several parts of the heavens, ten times as many stars as are visible to the naked eye. *Hooke*, in his Micrographia, says, that with a telescope of twelve feet he discovered seventy-eight stars among the Pleiades, and with a more perfect telescope, many more. *Galileo* reckoned eighty in the space between the belt and the sword of Orion, and above five hundred more in another part of the same constellation, within the compass of one or two degrees square. *Antonia Maria de Rheita* counted in the same constellation above two thousand stars. Future improvements in telescopes may enable us to discover numberless stars that are now invisible; and many more there may be, which are too remote to be seen through telescopes, even when they have received their ultimate improvement. Dr. *Herschel*, to whose ingenuity and assiduity the astronomical world is so much indebted, and whose enthusiastic ardour has revived the spirit of discoveries, of which we shall speak more largely in another part of this essay, has evinced what great discoveries may be made by improvements in the instruments of observation. In speaking here of his discoveries, I shall use the words of M. *de la Lande*.* "In passing rapidly over the heavens with his new telescope, the universe increased under his eye; 44000 stars, seen in the space of a few degrees, seemed to indicate that there were seventy-five millions in the heavens." He has also shown, that many stars, which to the eye, or through ordinary glasses, appear single, do in fact consist of two or more stars. The galaxy or milky way owes its light entirely to the multitude of small stars, placed so close as not to be discoverable even by an ordinary telescope. The nebulæ, or small whitish specks, discerned by means of telescopes, owe their origin to the same cause; former astronomers could only reckon 103, Dr. *Herschel* had discovered upwards of 1250 of these clusters, besides a species which he calls planetary nebulæ. But what are all these, when compared to those which fill the whole expanse, the boundless fields of ether! Indeed, the immensity of the universe must contain such numbers, as exceed the utmost stretch of the human imagination. For who can say, how far the universe extends, or where are the limits of it? where the Creator stayed "his rapid wheels," or where he "fixed the golden compasses?"

TO OBTAIN A KNOWLEDGE OF THE CONSTELLATIONS.

Let us suppose ourselves, in a frosty and bright evening of the month of January, seated about nine or ten o'clock on a commanding eminence, beneath an open sky; when every planet comes forward in stateliness, when the host of stars crowd the heavens with glory.

The first view of the heavens creates a confusion of pleasure and astonishment. The general irradiation elevates, but the seemingly irregular assemblage confounds the mind. Willing to form an intimacy with each portion of these glorious masses, fix on some of the most distinguishable groups, and make use of those as marks to conduct you through the rest. To every clump of stars, if we may use the expression, we will affix the name that antiquity first gave it.

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* Memoires de l'Academie de Dijon....1785.

The collection we proceed from is that large irregular square, within which you may observe three bright stars, placed very near each other, on a straight oblique line; beneath which are three others of inferior lustre, hanging from them in a manner. This the ancients called *Orion.* The two highest stars in this square form his arms; the two lowest, his legs; the three bright stars in the middle are his belt; and the three smaller, which hang from them, are understood to design the garment that covers his thighs, or more properly, his faulchion.

These three bright stars, sometimes called *Orion's Belt*, sometimes the *Three Kings*, by their direction point out to us, on one side *Sirius*, or the *Dog-Star;* on the other, the *Pleiades.* Sirius is that remarkable star, distinguishable from any other by its scintillation and lustre. You will see it on the south-east side of Orion.

The Pleiades, or the Seven Stars, lie on the north-west of Orion: they are very easily known, by being so closely massed together. Besides, they are almost in a direct line from Sirius, through Crion's Belt; they are on the back of another constellation, called the *Bull.*

That very large star, on a right line, half way between the Pleiades and the star that forms the western shoulder of Orion, is called the *Bull's Eye*, or *Aldebaran.*

That large star, which, you may perceive, forms a triangle of equal sides with Sirius and Orion's Belt, is called *Procyon*, or the *Lesser Dog.* It lies to the north of Sirius, and to the east of Orion.

Imagine a straight line, extending towards the north from Procyon or the Lesser Dog; and the first bright star it meets with is *Castor*, or one of the constellations called *Twins:* very near which is another bright star, called *Pollux*, the second of the Twins. This last lies south-east with regard to Castor.

Those four stars that lie in a right line, at equal distances, and about half way between the east shoulder of Orion and the Twins, are the *four feet* of the Twins.

A line drawn from the bright star that forms the west foot af Orion, through the star that forms his western shoulder, carries you on to the next star, called the *southern horn of the Bull.* This star, and the west foot of Orion, are equally distant from his west shoulder.

The *northern horn of the Bull* is brighter than the southern; it lies on a straight line with the east shoulder of Orion and the southern horn of the Bull.

The ecliptic, or the annual path which the sun seems to make in the heavens, passes betwixt the two horns of the Bull.

A line prolonged from the west foot of Orion, through Procyon, or the Lesser Dog, brings you to that very bright star called *Regulus*, or the *Lion's Heart*, which lies east by north-east from Procyon, and at some distance from it.

If you fancy a line drawn from the star in the middle of the Twins, through Regulus, it will there pass beneath a square of bright stars near Regulus, which form the constellation called the *Lion.* The first bright star, lying east by north-east from Regulus, and almost as large as itself, is called the *Lion's Tail.*

Extend, in imagination, a line from a bright star, that lies half way between the feet of the Twins and their heads, towards a bright star that lies east, and it will pass through the *Crab*, which lies exactly half way between the Twins and the Lion, remarkable by one bright star, and a cloudy clump of small stars adjoining to it. This constellation, you may observe, is difficult to be remarked.

If you suppose a line drawn through the Twins, north-west by north, it will touch a bright star in the helmet of the constellation called *Auriga*, which is at a considerable distance.

A very conspicuous star lies south-west by south from this one. This beautiful star is called the *Goat;* and due east, opposite to it, lies another, which, with three or four more near them, situate to the south, forms the whole constellation called *Auriga.*

A line drawn from Procyon, by Aldebaran, westward, leads you to the constellation called the *Ram.* This is reckoned the first constellation in the heavenly order, since the mass of stars that form its head, lies nearest that point where the sun equally divides the year, making the nights equal to the days. The first star in the Ram's Horn, which is the star astronomers reckon from, lies thirty-six degrees more to the west than Aldebaran.

Draw a line in fancy from the Seven Stars, of the Pleiades, north-north-west, and the first very bright star it meets is the first star in, or the *Breast of, Perseus ;* the star to the north-west of this is his right shoulder ; the star to the west is the left ; and the very brilliant star, south by south-west of the Breast of Perseus, is the first star in the constellation called *Medusa's Head :* next to which, there are three others very near, that form the whole head, something in the form of a square.

The very bright and beautiful star, lying east by north-east from the Lion's Tail, is *Arcturus*, the largest in the constellation called *Bootes*, situate between his legs.

The mass of stars west by north-west of Arcturus, between it and the Lion's Tail, is called *Berenice's Lock.*

A line drawn from Arcturus, north by north-west falls in with the last star of the tail of the capital constellation called the *Great Bear.* This last is formed by seven stars, in the form of a plough : it is the most conspicuous constellation in the heavens, and therefore may serve to point out others. Of this constellation, the four stars that lie towards the north form an irregular square ; of which, the two stars that are the most northern point northward to a bright star not very distant, which is called the *Polar Star*, as it lies only two degrees, or thereabouts, from the pole of the world.

The *Swan* is a very remarkable constellation, in the form of a great cross. A line drawn from the Twins, through the polar star, meets the Swan on the other side, at much about the same distance.

A line drawn from the square of the Great Bear, through the pole, passes through the middle of the constellation of *Pegasus.* This too is a square, formed by four bright stars, the most northern of which forms the head of *Andromeda*. A line carried in fancy from the Pleiades to the Ram, falls upon *Algenib*, the beautiful star in the wing of Pegasus. The most northern of the southern stars of Pegasus are called *Scheat*, and *Markab* : Scheat lies to the north, and Markab to the south.

Cassiopeia is a constellation directly opposite to the Great Bear, through the polar star, in such a manner, that the line that passes through the middle of the Great Bear, by the polar star, passes also through Cassiopeia on the other side of the pole. This constellation is formed of six or seven stars, in the shape of a chair turned upside down.

Cepheus is that constellation you see contained between the polar star, Cassiopeia, and the Swan. A line drawn from the polar star to the Swan's Tail, passes through the two first great stars in the constellation of Cepheus.

The *Lesser Bear* has almost the same shape as the Greater, and is parallel to it, but the situation is inverted. The polar star is the last in its tail. The two largest stars in this constellation are on a line drawn through the centre of the square of the Great Bear, perpendicular to both its greater sides.

The *Dragon's Tail* lies between the polar star and the square of the Great Bear. The four stars in its head lie south by south-east with regard to the Lesser Bear, and almost form an exact square.

Such are the constellations that most conspicuously adorn the face of heaven, in a winter night. Those which show themselves during the nights of summer, are not easily noted; but by the help of what I have pointed out to you, I think I shall be able to render them distinguishable.

When the star in the middle of the tail of the Great Bear is in the meridian above the polar star, and in the highest point of the heavens, which happens about nine o'clock on one of the evenings in the end of May, you will then observe towards the south, in the meridian, that very beautiful star called the *Virgin's Spike*.

You see this star forms a triangle of equal sides with Arcturus and the Lion's Tail.

A little more to the right, but lower than the Virgin's Spike, you may remark a kind of square, formed by four principal stars: this constellation is called the *Crow*.

A line drawn from the two most eastern parts of the square of the Great Bear, through the Lion's Heart, Regulus, meets towards the south the *Heart of Hydra.* Its head is to the south of the Crab, between Procyon and Regulus. This constellation of Hyra extends from the Lesser Dog to the lower part of the Virgin's Spike.

Between Hydra and Crow, but to the west of this last, is situate the *Crater*, or the *Cup*. The kind of small square formed by it is remarkable enough, so that you will not be at a loss to fix upon it.

A line drawn east-north-east from the brightest star in the Swan, reaches the bright star in the *Lyre*, which is one of the most splendid in the heavens. It almost makes a right-angled triangle with Arcturus and the polar star.

The *Crown* is that small constellation, situate between Arcturus and the Lyre: it sometimes shows itself in the form of an oval, with eight pretty visible stars: sometimes only as three bright stars lying very close to each other, bearing west by south-west from the Lyre. A line carried from the two first stars in the tail of the Great Bear points out the Crown, generally called *Corona Borealis* or the *Northern Crown.*

That very resplendent star which you see to the south of the Lyre and the Swan, is called the *Eagle*. It is very preceivable, by being placed in a straight line between two other bright stars that lie very near it.

The clump of stars that lie to the south of, and next to the Eagle, is called *Antinous*.

The line of the arc of a great circle, which passes through Regulus and the Virgin's Spike, leads you on, east south-east, to a very bright star called *Antares*, or the first of the *Scórpion.* This constellation is very remarkable; for, to the south of this very bright star there is an arc of stars, with the concave towards the north, and the convex to the south, which forms the tail or sting of the Scorpion. Antares seems likewise to be a centre to a circle of bright stars around it.

That bright star that lies half way between the Virgin's Spike and Antares, is the southern scale of the constellation called the *Balance*. The next bright star to this, north-east by north, is the northern scale of the same constellation.

That other constellation which follows the Scorpion, that is, somewhat more to the east, is called *Sagittarius*: the Virgin's Spike and Antares point to it east south-east. It lies upon a line drawn from the middle of the Swan through the middle of the Eagle.

An arc drawn from Antares to the polar star, first passes through the constellation called *Ophiucus*, or *Serpentarius*, and a little higher meets the constellation *Hercules*. A line from Antares to the Lyre, passes between the heads of Hercules and Ophiucus, which lie very near to one another. The most eastern and southern of the two is the head of Ophiucus. The stars that lie immediately to the north of the head of Hercules, form the constellation of Hercules; and those that you see to the south of the head of Ophiucus, constitute the constellation of that name, together with a part of the *Serpent*, which lies east and west of Ophiucus.

A line drawn through the Lyre and the Eagle, towards the south-east, leads you to those two stars near each other which form the head of the constellation called *Capricorn*. Those two other stars, much about the same distance, lying beneath the Water-bearer towards the south, form the tail of Capricorn.

A line extended from the Eagle through Capricorn's tail, almost south-east, leads you to that effulgent star called *Fomalhout*, or the *Southern Fish*.

The small group of stars in the form of a bright cross you see next to the Eagle, lying north-east by east, is called the *Dolphin*.

Draw a line from the Lyre upon the Dolphin, prolong it to the south, at the same distance from the Dolphin as the Dolphin is from the Eagle, and the constellation called *Aquarius*, or *Water-bearer*, will be situate a little to the east of that line. In descending from the Dolphin to Fomalhout, you pass through the whole length of Aquarius, beginning by the two bright stars that form his shoulders.

Protract a line from the Goat through the Pleiades, and it will lead you to a bright star that is the first in the head of the constellation called the *Whale*. A line drawn from Aldebaran through this last, carries you, through the body of the Whale, down to the bright star that forms its tail, after passing near those three remarkable ones that form its back.

Those two groups of stars that lie immediately south-east and north-east of the great star in the wing of Pegasus, are called the *Fishes*. The first bright star that lies south-east of these is termed the *Fishes Knot*, supposed to join them together. This last is the largest of this constellation, and is the first bright star situate to the south of the Ram's head.*

This general outline of the places of the constellations, may in time form a habitude of referring to any particular spot of the heavens. But the space is too wide for one grasp. I have only shown you the expansive field you have to work in.

When you behold the sky, the splendour of the stars, and that grand luminary which discovers all things that surround you, ask yourself from whence come all these things? Who has constructed this immense canopy of the heavens? Who has placed in this firmament those innumerable lights? Who has appointed those stars to move in such order, and that sun to enlighten and fertilize the earth? And your heart will answer, It was the power and wisdom of God that called all these things into being. What other than an intelligent and omnipotent power could have formed that grand and magnificent arch above us? Who could have given those globes that perpetual motion? Whence comes that connection, that harmony, that beauty displayed in all their parts? Who has appointed all these things with so much exactness in number, weight, and measure? Who has prescribed to these immense bodies those laws discoverable only to men of the greatest abilities and most profound wisdom?

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* Cometilla, or Views of Nature, p. 69 to 88.
De La Lande's Astronomie, vol. i.

All these questions refer to thee, most adorable Creator, self-existent, independent, infinite Being! to thee all the heavenly bodies owe their existence, their laws, their arrangements, their power, and all the advantages they are of to this world. Love prompted thy infinite wisdom to contrive, and employed thy omnipotence to execute the glorious universal plan; love still inclines the Almighty Maker to preserve his stupendous work, to uphold the perfect order running through the whole, to bless the creatures he has made, and under its unceasing influence will he proceed throughout all the boundless ages of eternity.*

END OF VOL. III.

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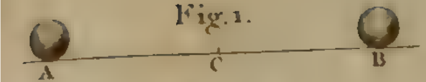
* *Tucker's* Light of Nature, vol. iv. part 2, p. 341.

Fig. 1.



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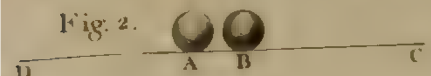

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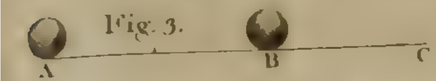

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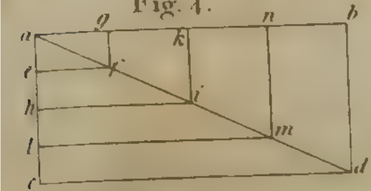

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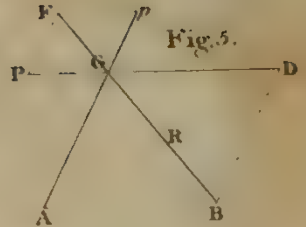

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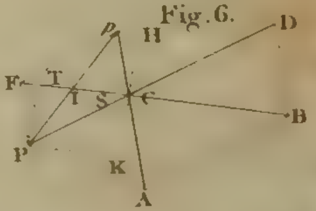

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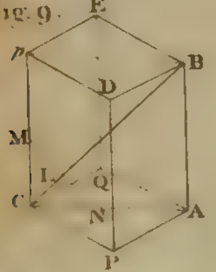

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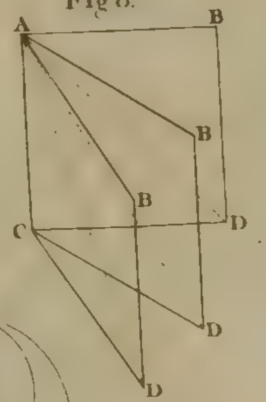

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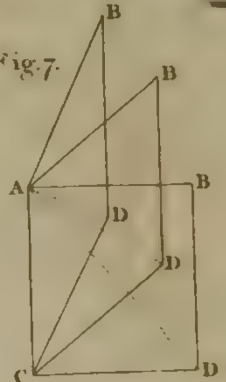

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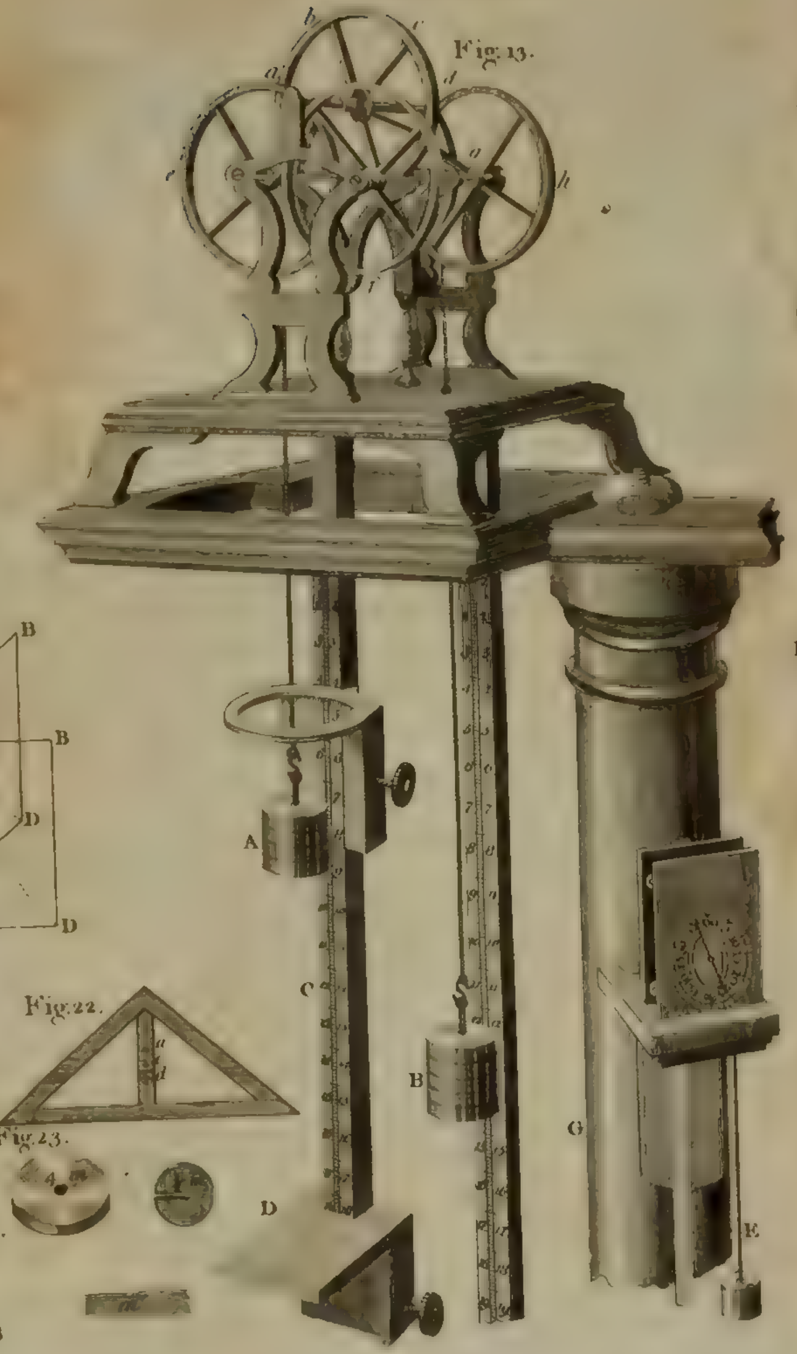

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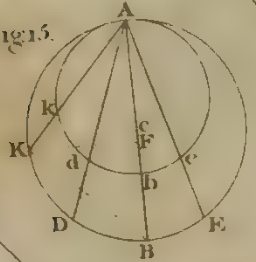
Fig. 15.



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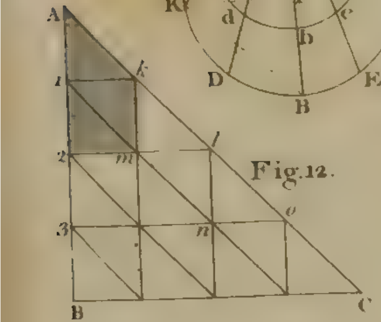

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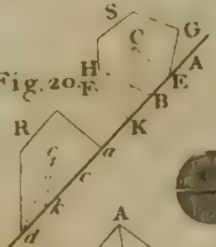

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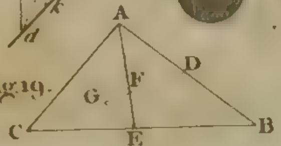

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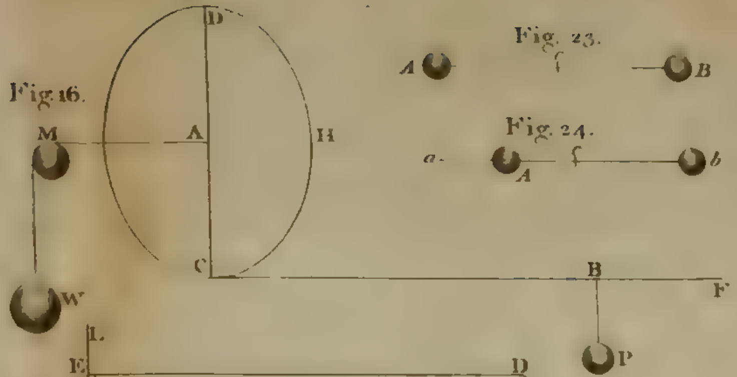

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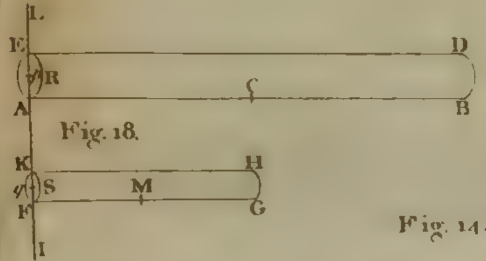
Fig. 24.

Fig. 18.



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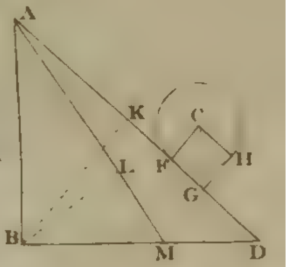

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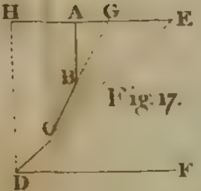

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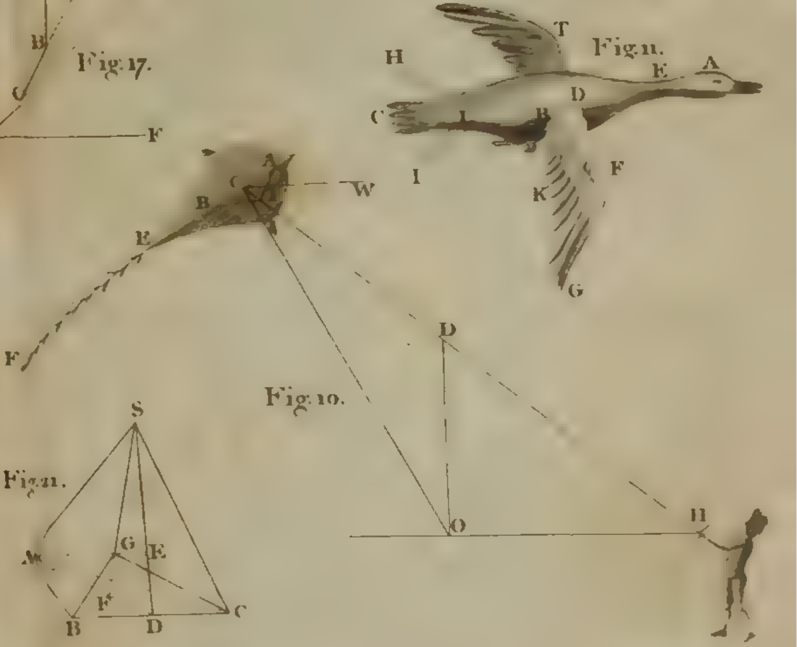

Fig. 10.

Fig. 21.

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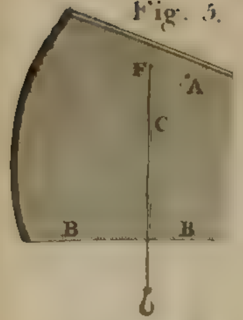
MECHANICS.

PLATE II. Vol. IV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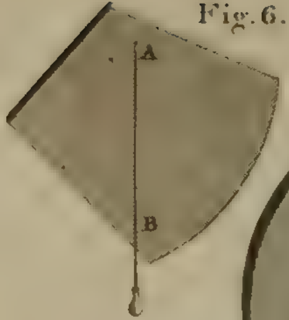
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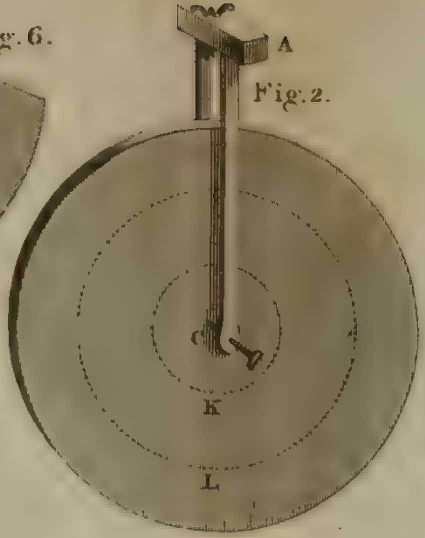
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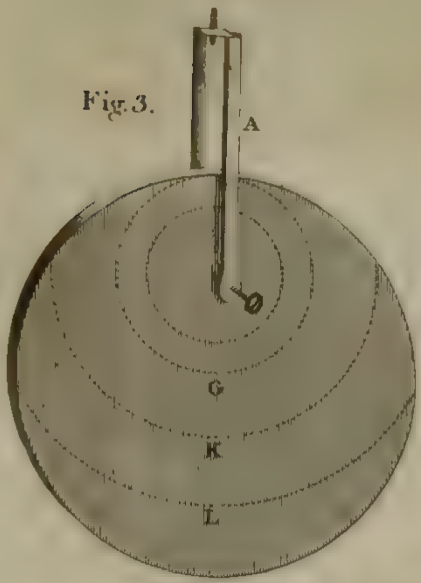
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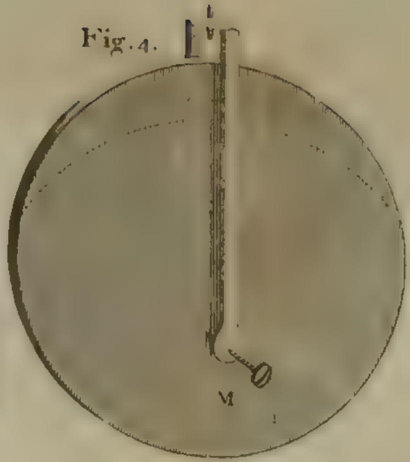
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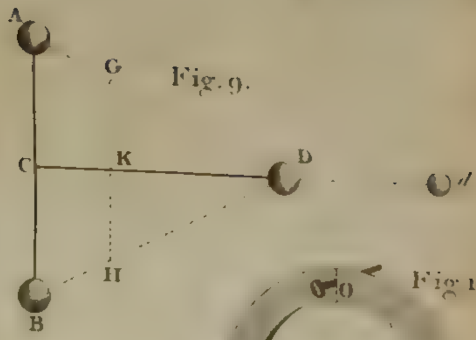
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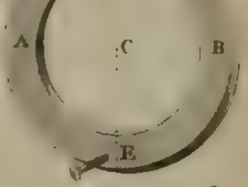
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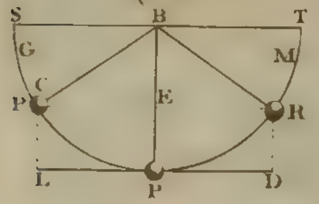
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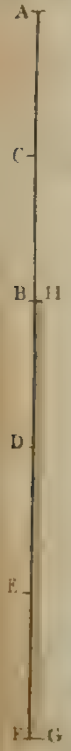
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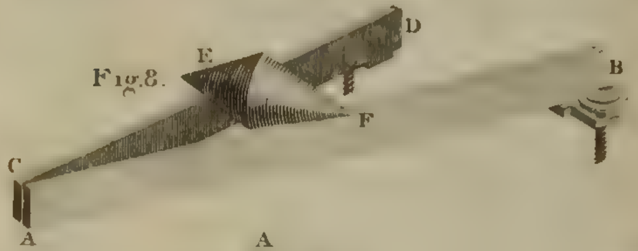
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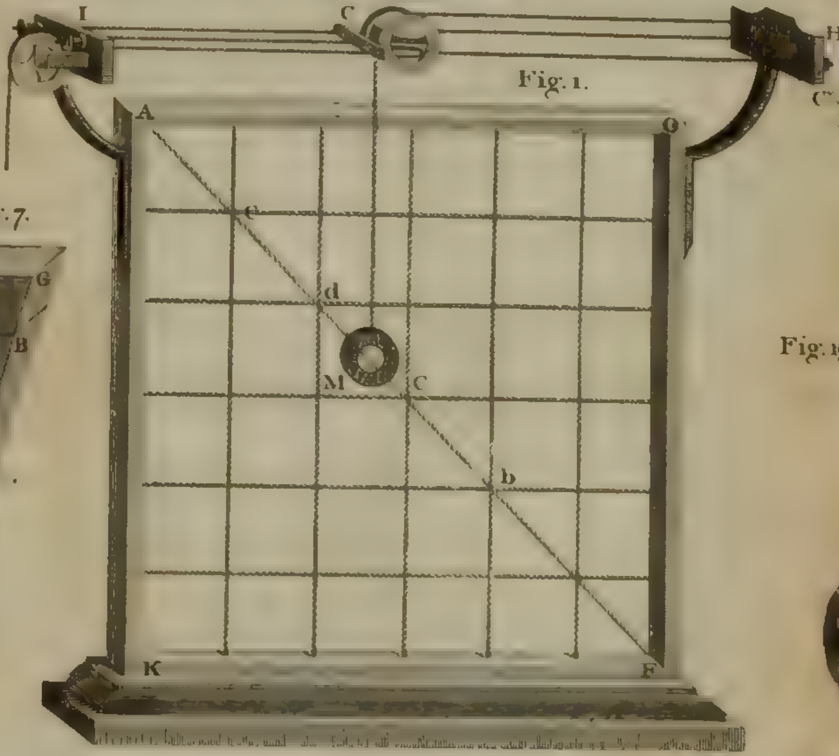
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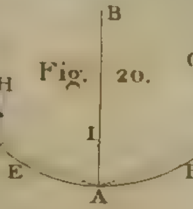
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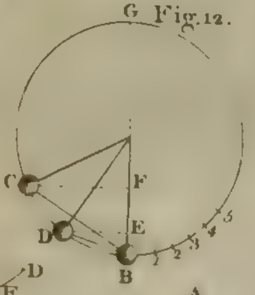
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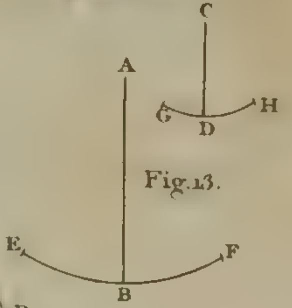
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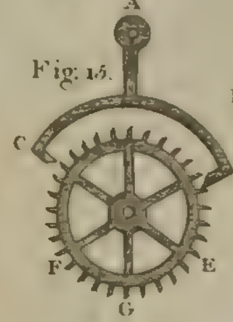
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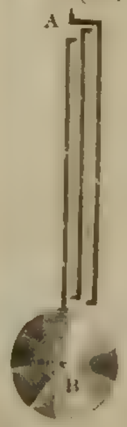
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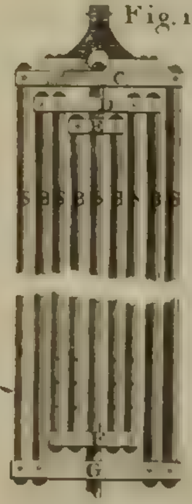
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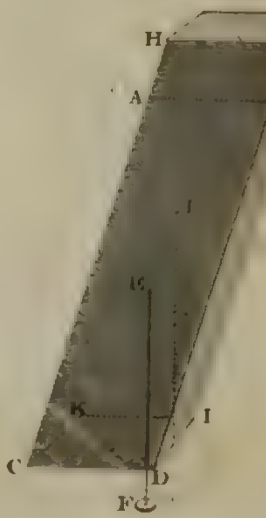
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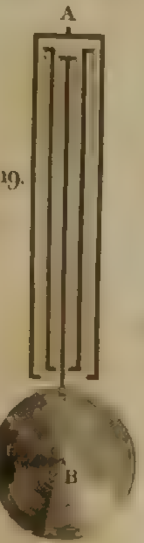
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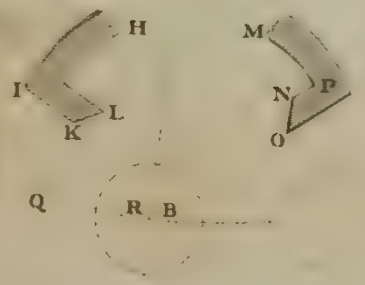

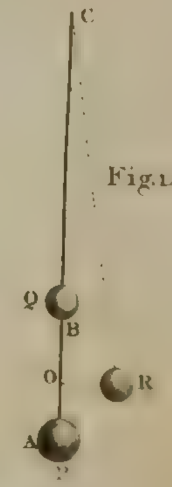
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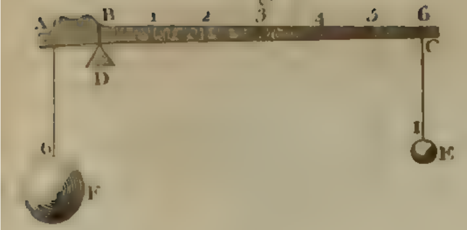

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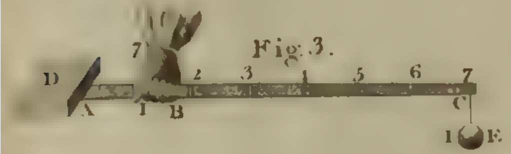

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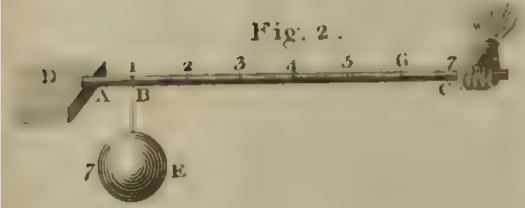

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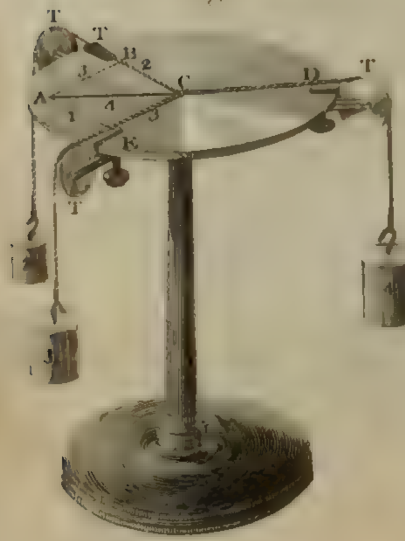

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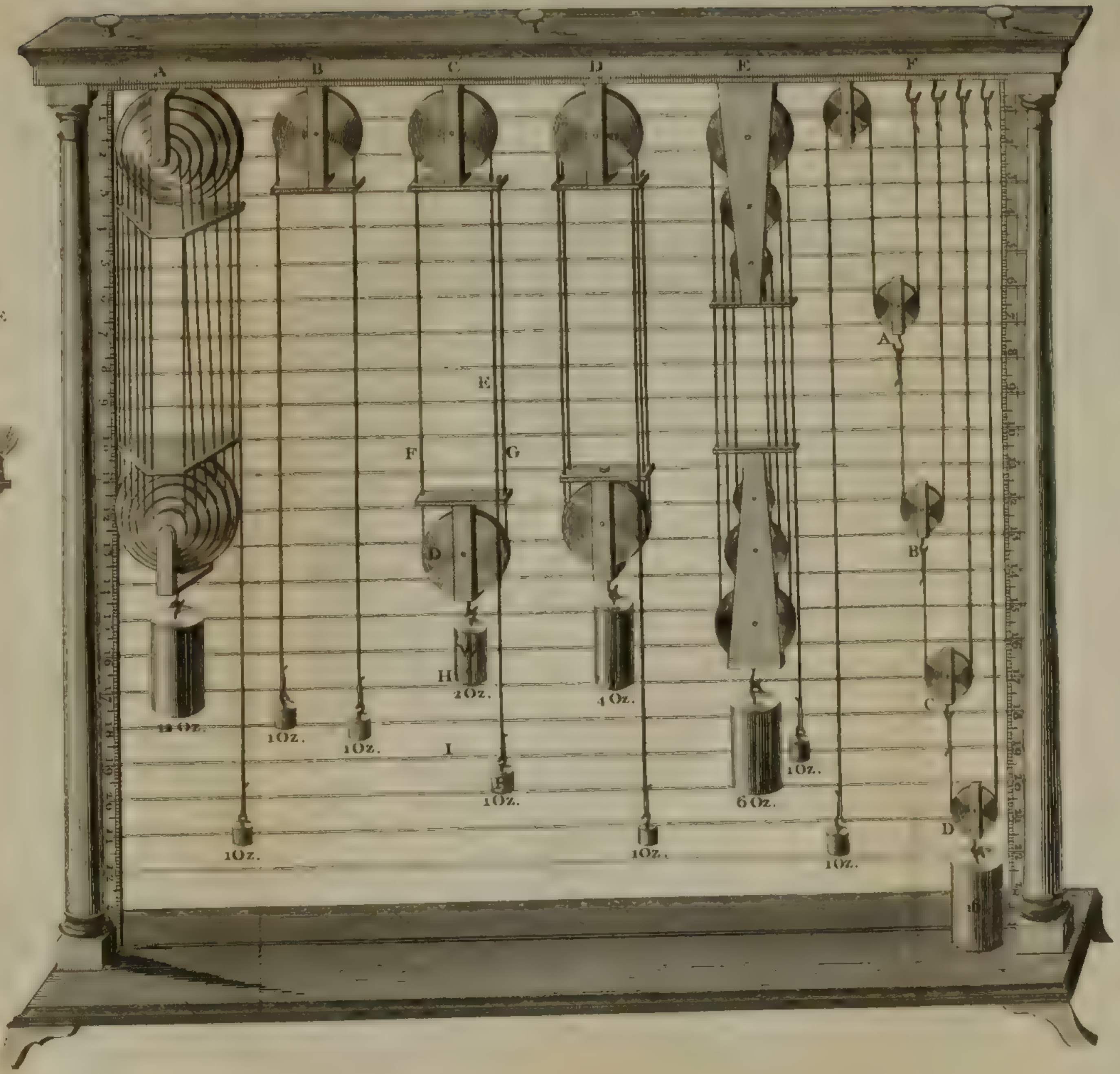

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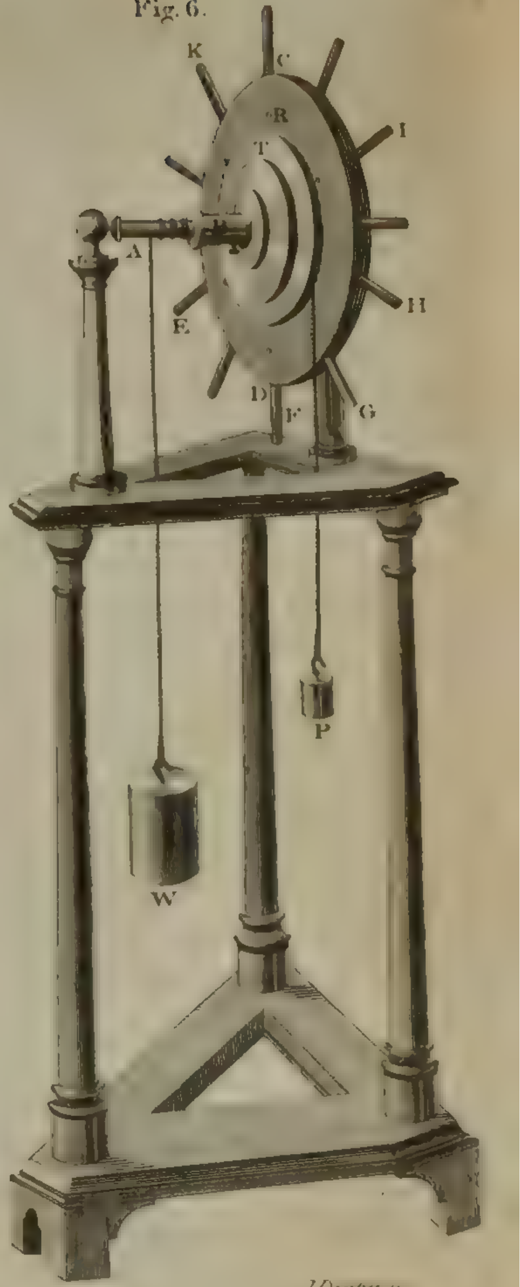

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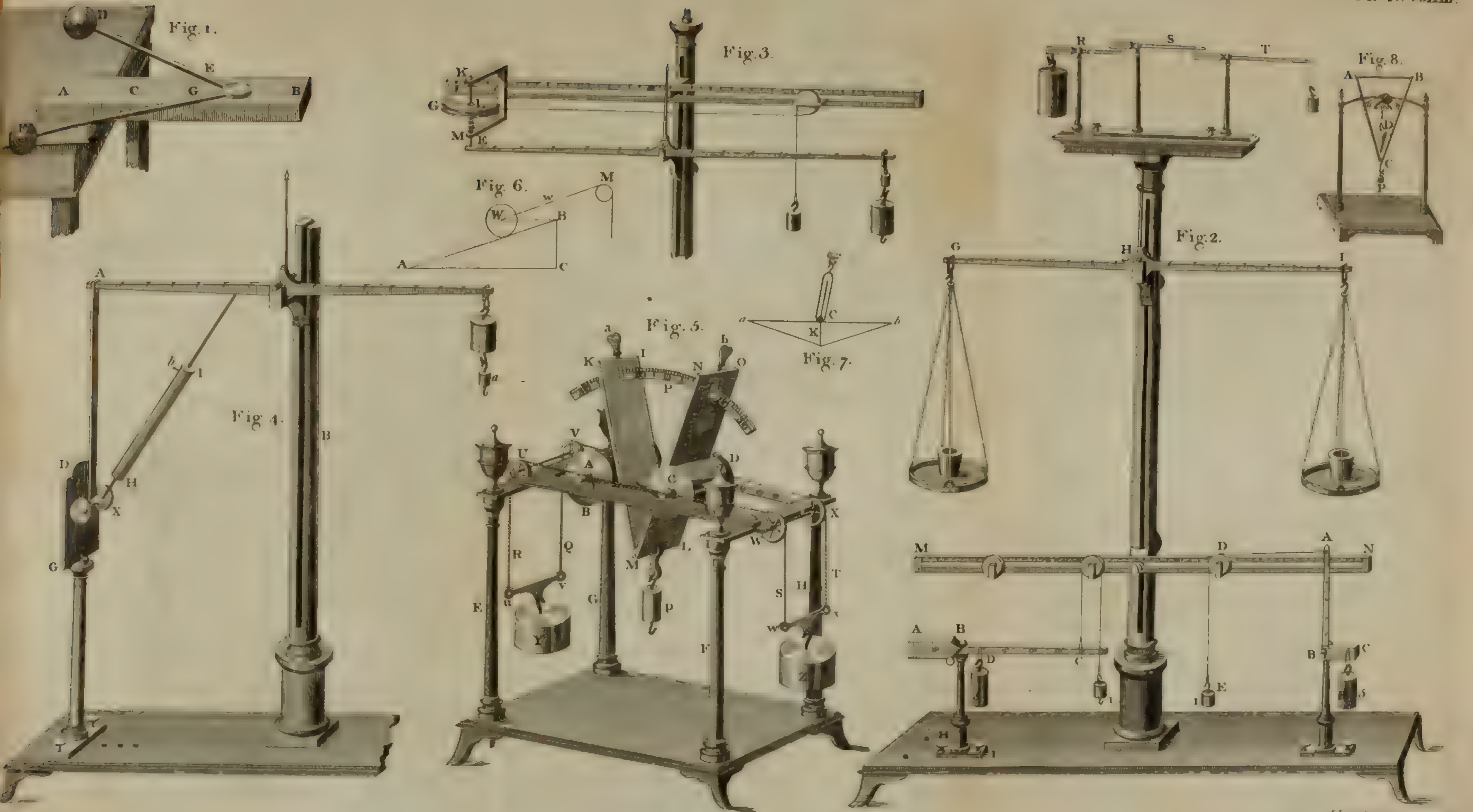
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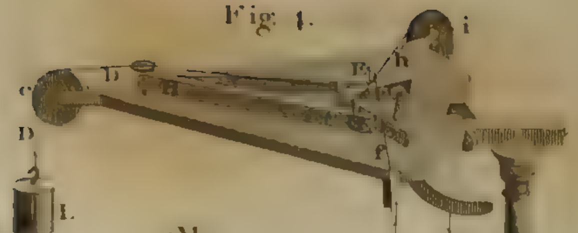
Fig. 4.



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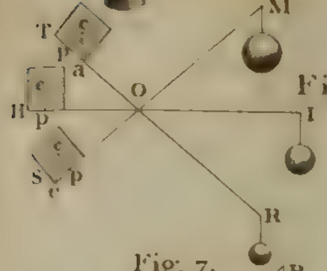

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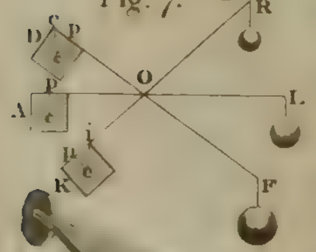

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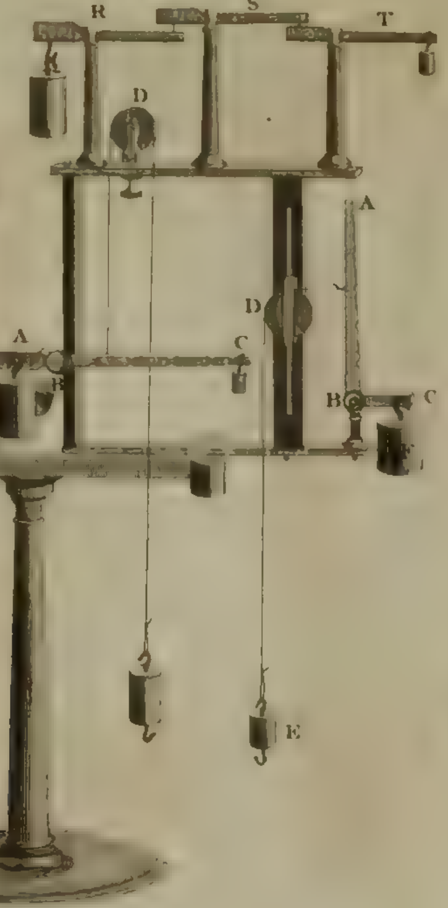

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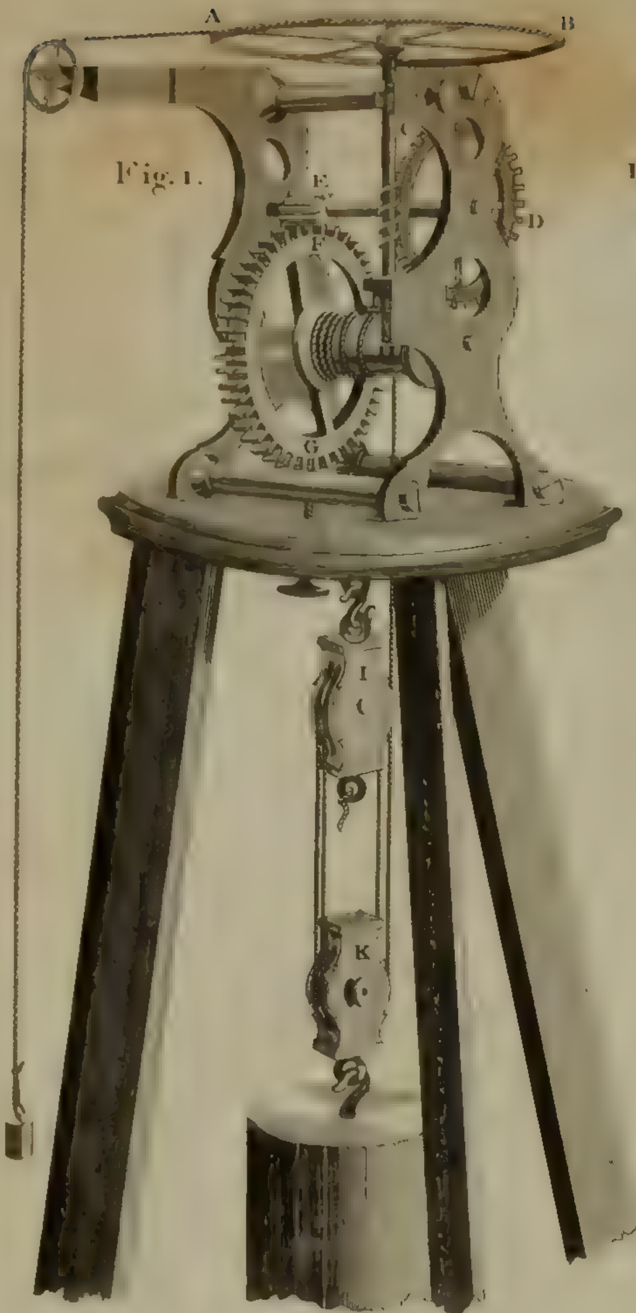

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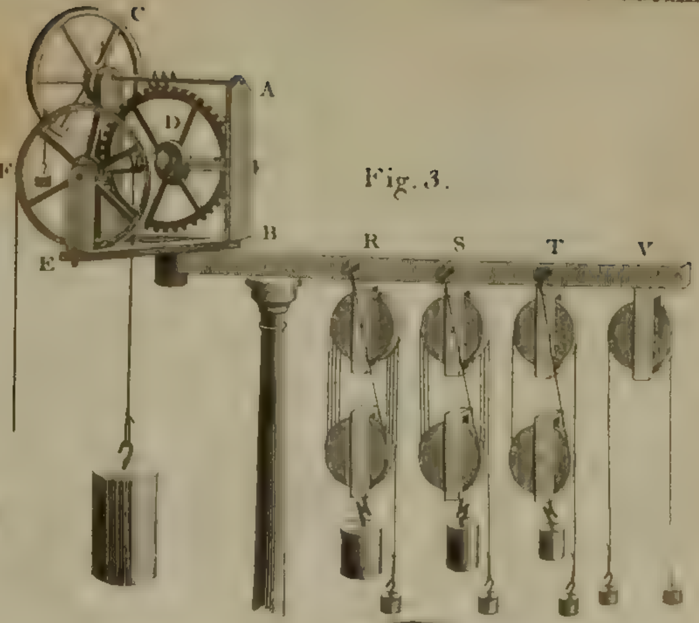

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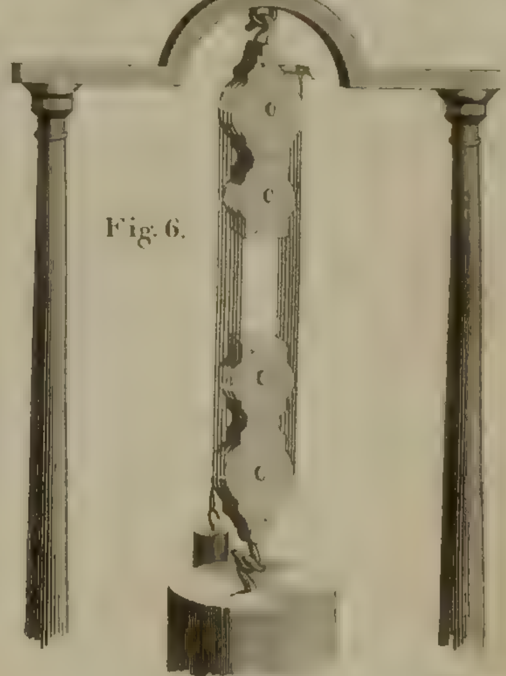

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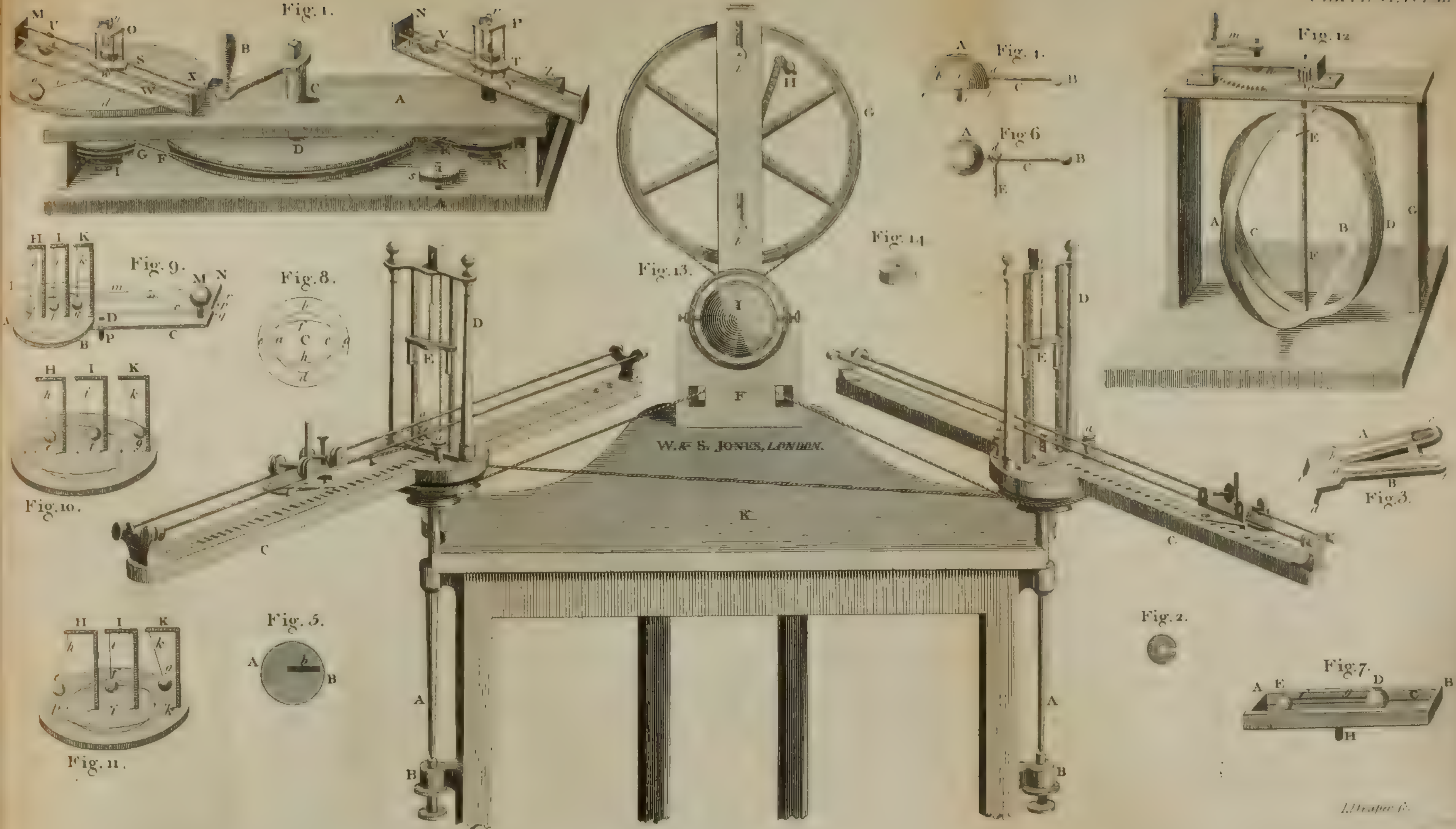

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Fig. 11.

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Fig. 7.

J. Draper sc.

HYDROSTATICS.

PLATE I. Vol. III

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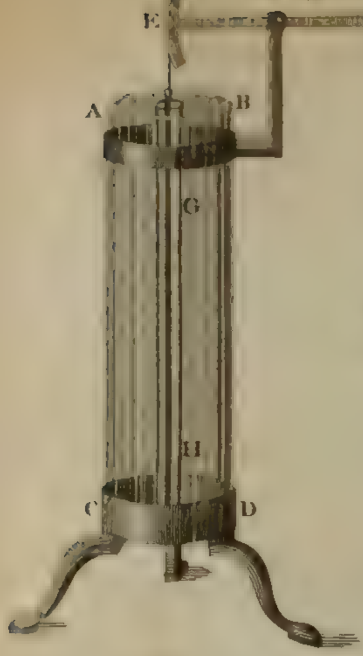

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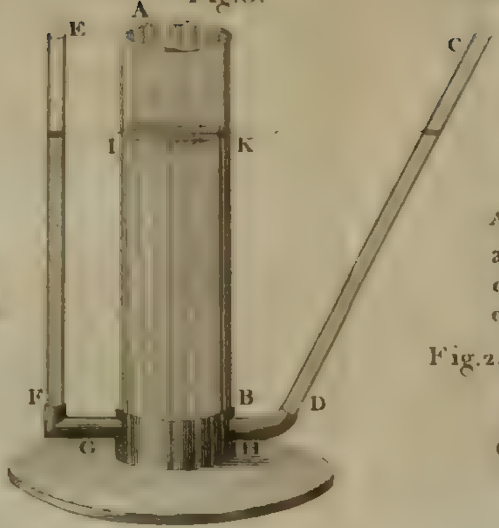

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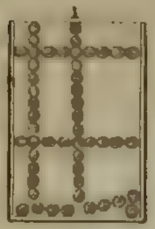

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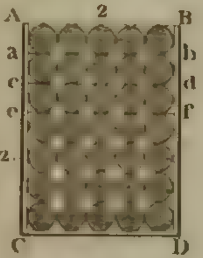

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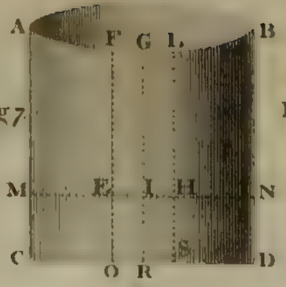

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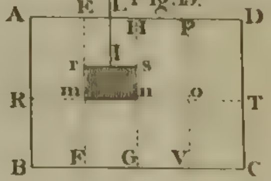

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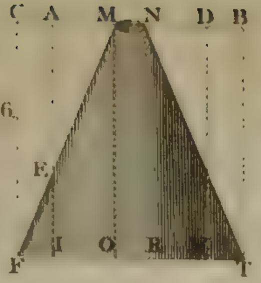

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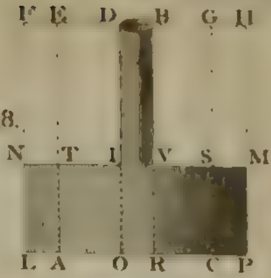

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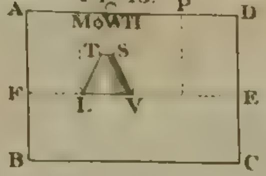

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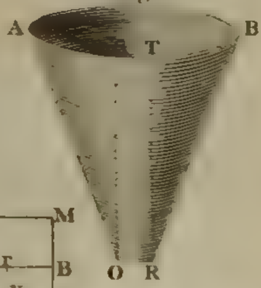

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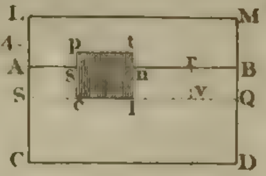

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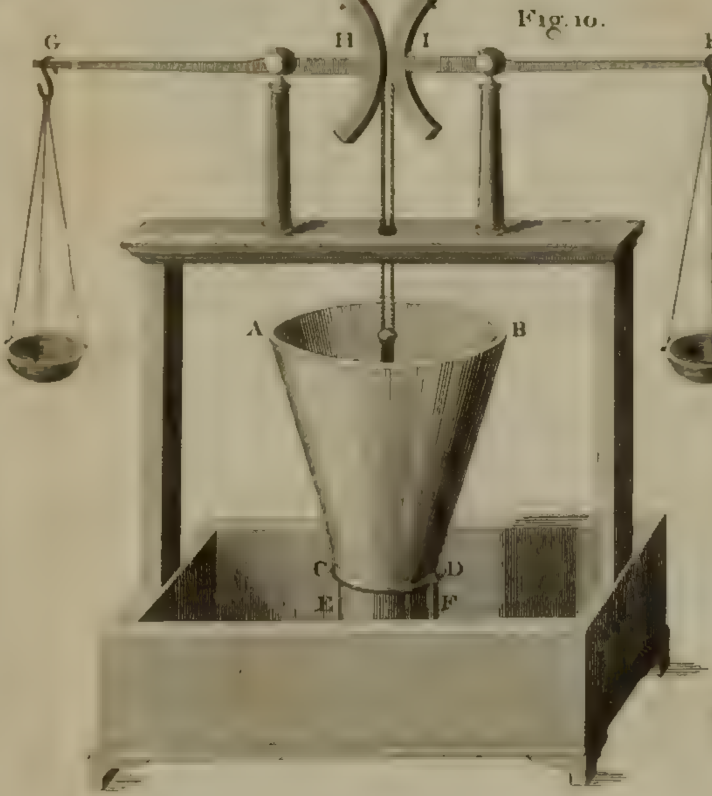

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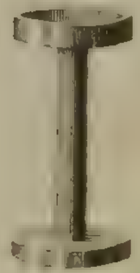

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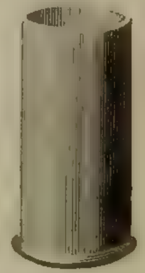
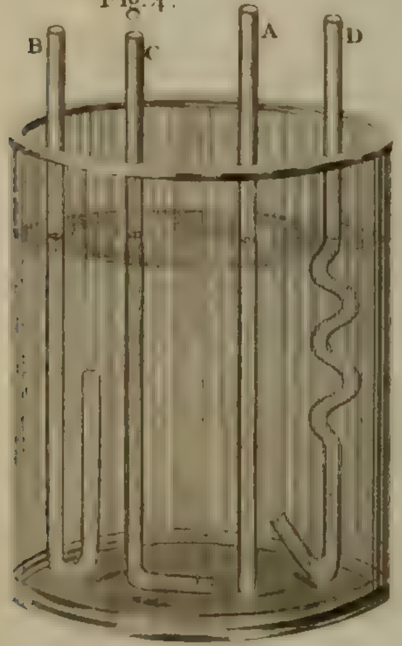


Fig. 4.



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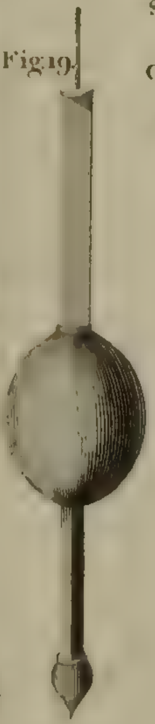

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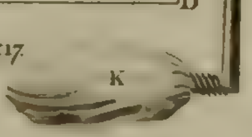

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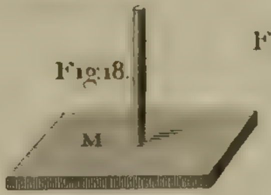

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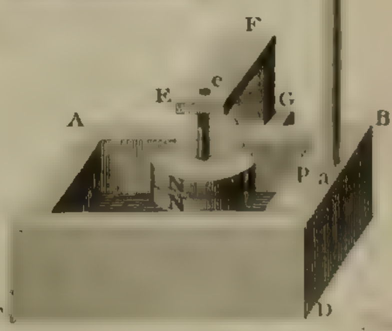
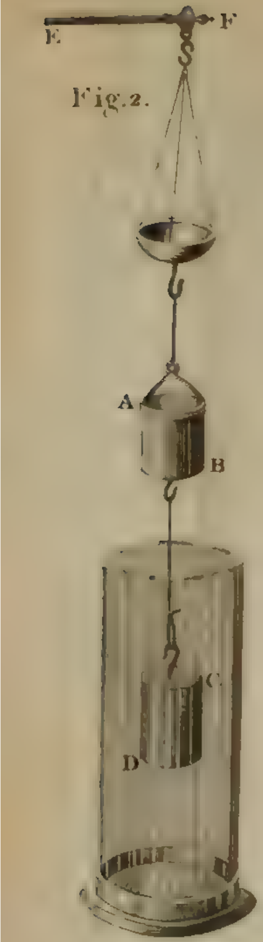


HYDROSTATICS.

Fig. 2.



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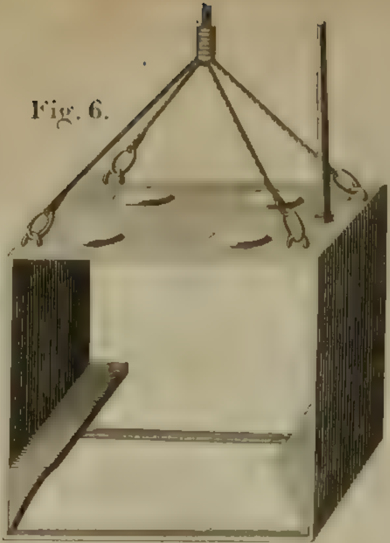

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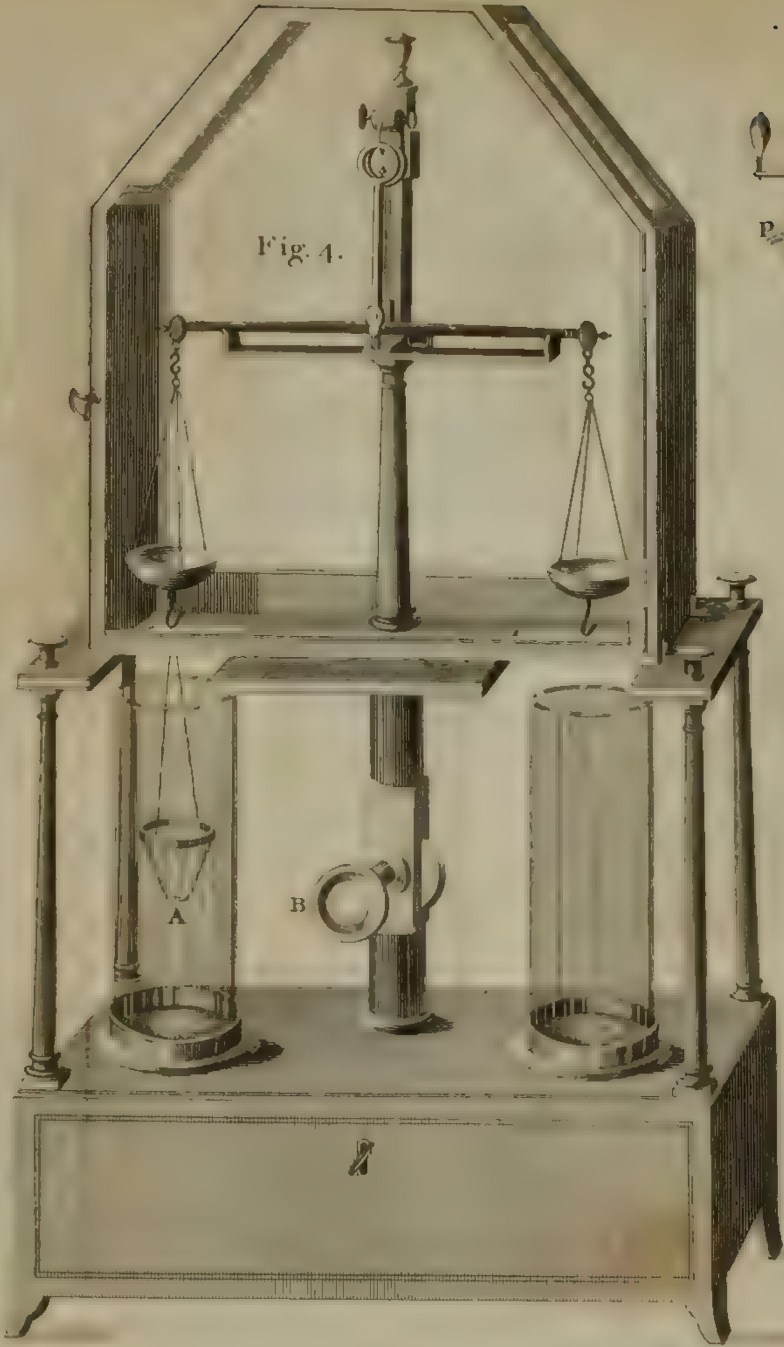

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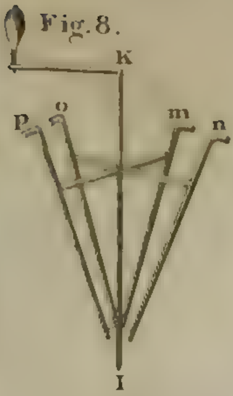

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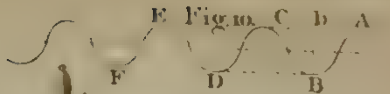

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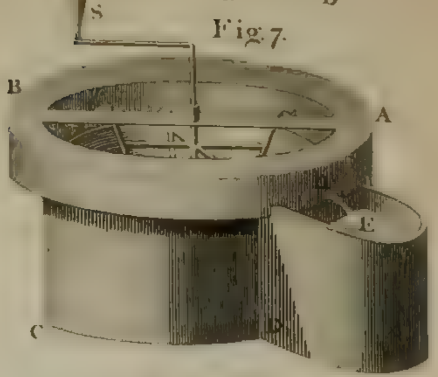

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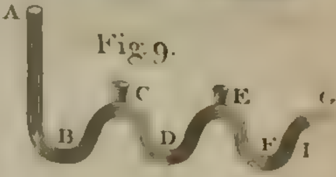

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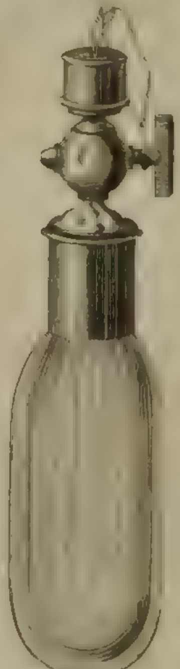
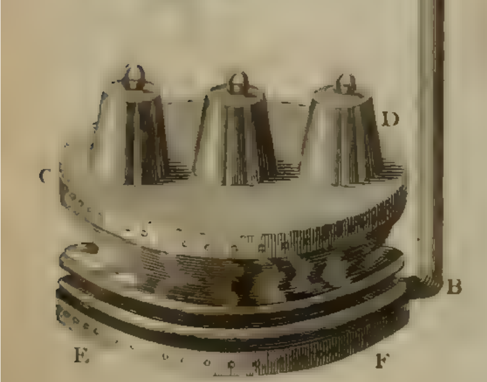


Fig. 1.



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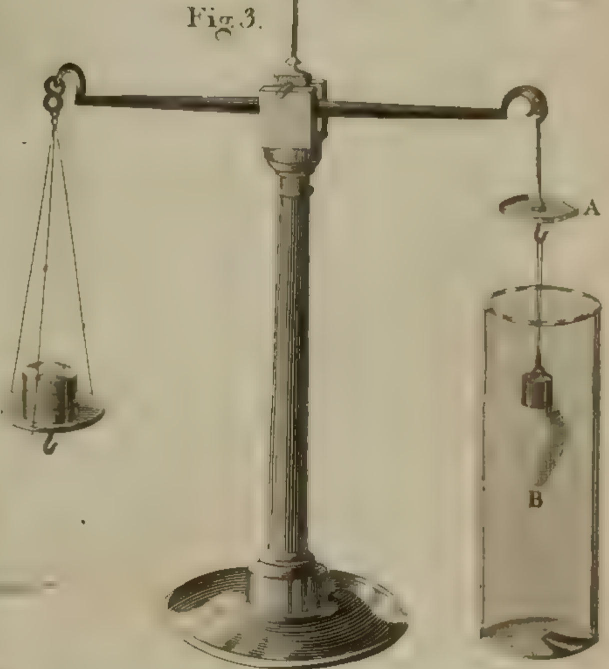

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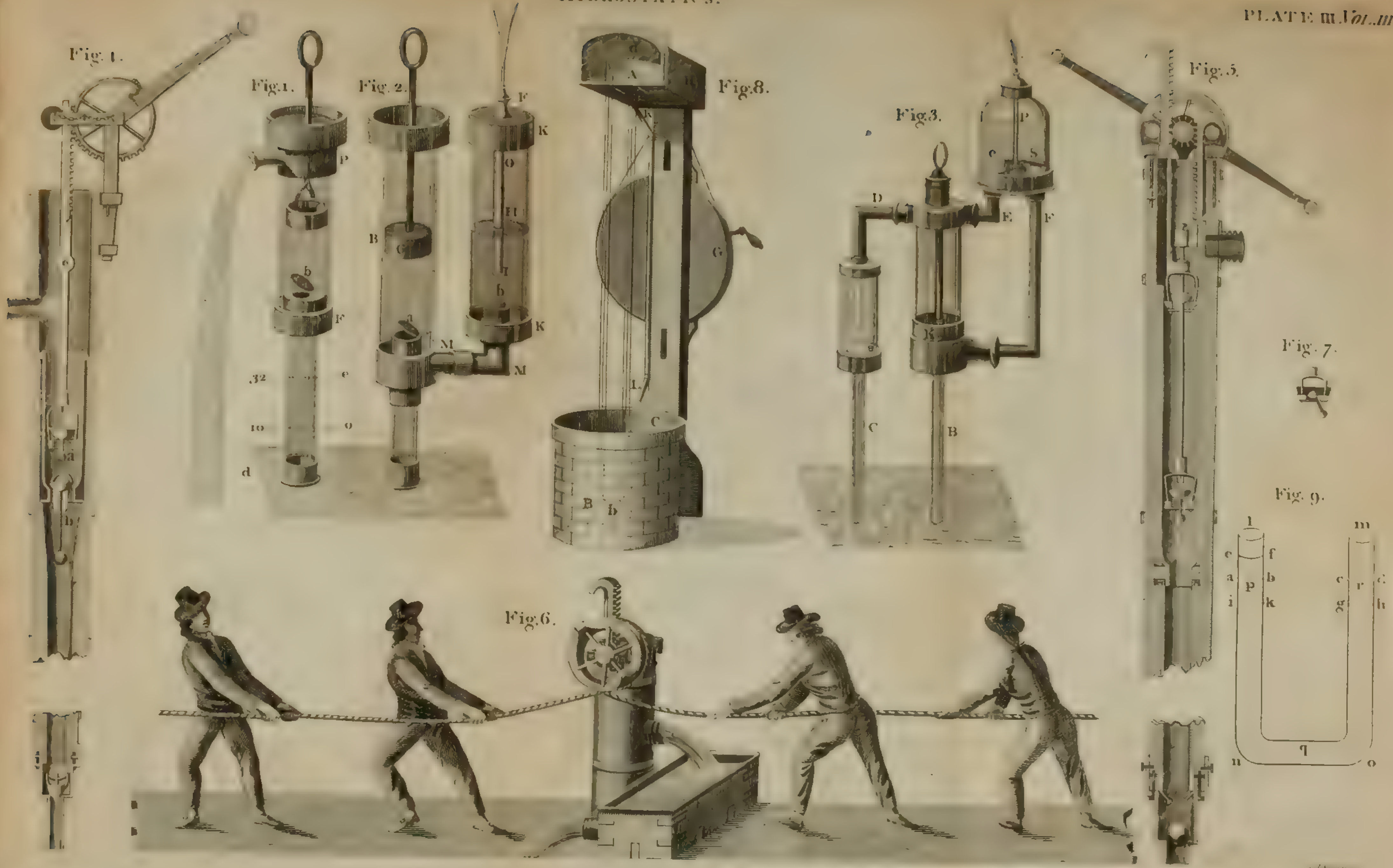
PLATE III. *Vol. III.*



J. Draper